Probability and Statistics
Hints/Solutions to Assignment No. 1

1. Required probability $= \dfrac{5 \times 4 \times \binom{5}{2} \times 3!}{5^5} = \dfrac{48}{125} = 0.384.$

2. Use definitions.

3. Let A → 5 appears, B → even number appears. Then
   A = {(1, 4), (2, 3), (3, 2), (4, 1)}. Also B will have 18 elements. So
   $P(A) = \frac{1}{9}$, $P(B) = \frac{1}{2}$. Now the P(5 appears first) $= \frac{1}{9} + \frac{14}{36}.\frac{1}{9} + \left(\frac{14}{36}\right)^2.\frac{1}{9} + ... = \frac{2}{11}.$

4. Given $P(C) = 0.6$, $P(H) = 0.3$, $P(C \cap H) = 0.2$.
   The required probability $= P(C \cap H^C) + P(C^C \cap H)$
   $= P(C) - P(C \cap H) + P(H) - P(C \cap H) = 0.5$

5. Let A → person is smoker, D → death due to lung cancer. Given P(A) = 0.2, P(D) = 0.006 and $P(D|A) = 10\, P(D|A^C) = 10$ x, say. Use theorem of total probability to get 10 x = 3/140 = 0.0214.

6. Use addition rule and the definition of the conditional probability.

7. 0.25.

8. (i) Use Bayes theorem, Reqd prob. $= \dfrac{2(1-\alpha)}{2(1-\alpha) + 2\beta + 3\gamma}.$
   (ii) Use theorem of total probability. Reqd prob. = $(\alpha + 2\beta + 3\gamma)/6$.
   (iii) Use theorem of total probability,
   P(digit 1 was received) = $(2 - 2\alpha + 2\beta + 3\gamma)/ 12$,
   P(digit 2 was received) = $(\alpha + 4 - 4\beta + 3\gamma)/ 12$,
   P(digit 3 was received) = $(\alpha + 2\beta + 6 - 6\gamma)/ 12$.

9. Let C → an automobile policyholder makes a claim,
   M → automobile policyholder is male
   F → automobile policyholder is female
   Given P(M) = α, P(F) = 1 – α., 0 < α < 1. $P(C|M) = p_m$, $P(C|F) = p_f$.
   Now $P(A_1) = P(C \mid M)\, P(M) + P(C \mid F)\, P(F) = \alpha\, p_m + (1 - \alpha)\, p_f$.
   $P(A_2 \mid A_1) = \dfrac{P(A_2 \cap A_1)}{P(A_1)} = \dfrac{\alpha p_m^2 + (1-\alpha)p_f^2}{\alpha p_m + (1-\alpha)p_f}.$

10. (i) False (ii) True (iii) False (iv) False

11. $\binom{13}{4} \Big/ \binom{52}{4}.$

12. Count the cases to find the required probability as 43/216.

13. P(getting all ‘Ex’ in one semester) = $\frac{1}{2^5} = \frac{1}{32}$. Reqd. prob. $= 1 - \left(\frac{31}{32}\right)^{10} = 0.272$.

14. Use definition.

15. $P(X = i) = \binom{n}{i} \Big/ 2^n$, $i = 0, 1, \ldots, n$. $P(A \subset B \mid X = i) = 2^{i-n}$. $P(A \subset B) = \left(\frac{3}{4}\right)^n$.

Also $P(A \cap B = \phi) = P(A \subset B^C)$.

16. P(student scores at least 8 marks) = P(scores 8 marks) + P(scores 9 marks) + P(scores 10 marks) = 5/512.

17. 7/17, 6/17, 4/17.

18. Let X → number of girls qualifying, Y→ number of boys qualifying.

$$p = P(X > Y) = P(X = n+1) + P(X = n+2) + \ldots + P(X = 2n)$$

$$= \left(\frac{1}{2}\right)^{2n}\left[\binom{2n}{n+1} + \ldots + \binom{2n}{2n}\right] = \left(\frac{1}{2}\right)^{2n}\left[\binom{2n}{0} + \ldots + \binom{2n}{n-1}\right]$$

$$= P(Y > X)$$

$$r = P(X = Y) = \binom{2n}{n}\left(\frac{1}{2}\right)^{2n}.$$

As $2p + r = 1$, we get $p = \frac{1}{2}\left\{1 - \left(\frac{1}{2}\right)^{2n}\binom{2n}{n}\right\}$.

19. Define, A_i = ith person gets back his own hat . So we need to find

P(No one gets back his own hat)

= P ($\bigcap_{i=1}^{n} A_i^c$)

=1 - P ($\bigcup_{i=1}^{n} A_i$)

=1- $\sum_{i=1}^{n} (-1)^{i-1} S_i$

where $S_i = \binom{n}{i}\frac{(n-i)!}{n!}$ = Prob. that i many people will get back their own hats.

20.

$$\binom{n}{k}\left(\frac{r}{R}\right)^k\left(1 - \frac{r}{R}\right)^{n-k}$$

21. $\dfrac{\binom{26}{3}\binom{26}{10}}{\binom{52}{13}}$

22. $\dfrac{\binom{13}{3}\binom{13}{4}\binom{13}{4}\binom{13}{2}}{\binom{52}{13}}$

23. Define Q_i = Queen of the ith suit drawn i= c, h, d, s.

Define K_i = King of the ith suit drawn i= c, h, d, s.

(a) $\dfrac{1}{52^4}(4!) = \dfrac{4}{52}\dfrac{3}{52}\dfrac{2}{52}\dfrac{1}{52}$

(b) $\left(\dfrac{4}{52}\right)^4$

24. 1st box will contain no ball. So we remove the first box.

From the resets one box must contain 2 balls in $\binom{n-1}{1}\binom{n}{2}$ ways. Left (n-2) ball can be distribute in (n-2) boxes in (n-2)! ways. So the probability of interest is

$$\dfrac{\binom{n-1}{1}\binom{n}{2}(n-2)!}{n^n}$$

25. (a) $\dfrac{n(n-1)^{r-1}}{n^r}$

(b) Choose r people from n people and spread the rumour in any order. So the probability of interest is $\dfrac{{}^nC_r}{n^r}$.