Indian Institute of Technology Kharagpur
Course: TS70006 Quantum Mechanics and Quantum Computing
Spring Semester 2019-20
Mid Semester Examination

Declaration:

- **Answer without proper justification carries NO marks.**
- **NO query will be entertained during the examination.**

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1. True/False? Justify your answer.
   (a) There exist $O(2^{n2^n})$ distinct functions $f : \{0,1\}^n \to \{0,1\}^n$ of n bits. [1]
   (b) A (classical) circuit composed of n NAND gates can implement at most $O(n^{2n})$ distinct functions $f : \{0,1\}^n \to \{0,1\}^n$. [2]
   (c) AND, OR gates can be created using one or more Fredkin gates. [2+2]

2. Let
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle)$$
be the joint state of a pair of spin particles, first one of which is with Alice and the second one is occupied by Bob. Let $p(a+, b+)$ denote the probability that Alice obtains $\sigma_a^{(A)} = +1$ and consequently Bob obtains $\sigma_b^{(B)} = +1$, $a, b \in \{x, y, z\}$ after measuring the observables $\sigma_a^{(A)}, \sigma_b^{(B)}$ of their particles. Here $\sigma_a^{(A)}$ and $\sigma_b^{(B)}$ denote the polarization of the spin particles occupied by Alice and Bob along the axes a, b respectively. Then,
   (a) using the postulates of quantum mechanics, determine [4]
$$p(x+, y+),\ p(x+, z+),\ p(z+, y+).$$
   (b) determine an inequality concerning the probabilities $p(x+, y+), p(x+, z+), p(z+, y+)$, assuming the locality principle of EPR paradox. Besides, assume that there are N number of such pair of particles shared with Alice and Bob. [2]
   (c) verify if the probabilities obtained using the quantum postulates match with the inequality obtained assuming the EPR locality principle. Justify your answer. [1]

All The Best !!