Indian Institute of Technology Kharagpur
Course: MA20107 Matrix Algebra (Section-1)*
Autumn Semester 2017
Assignment -01

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1. True or False. Justify your answer.
   (a) If $\mathcal{R}(A) \subseteq \mathcal{R}(B)$ then $\mathcal{R}(AC) \subseteq \mathcal{R}(BC)$ for any matrix $C$, where $\mathcal{R}(X)$ denotes the row space of $X$.
   (b) If $\mathcal{R}(A) = \mathcal{R}(B)$ then $\text{rank}(A) = \text{rank}(B)$.
   (c) If $AB$ and $BA$ are defined then $\text{rank}(AB) = \text{rank}(BA)$.
   (d) The eigenvalues of a permutation matrix are real numbers.
   (e) If a matrix $A$ is diagonalizable then $A^k$ for any positive integer $k$ is also diagonalizable.
   (f) A matrix is nilpotent if and only if all its eigenvalues are 0.
   (g) $A\exp(A) = \exp(A)A$ for any square matrix $A$.
   (h) Eigenvalues of $AB$ are same as the eigenvalues of $BA$ where $A, B$ are square matrices of same order.
   (i) Any real symmetric matrix of rank $r$ is a sum of $r$ rank-one symmetric matrices.
   (j) Let $m_A(x)$ be the minimal polynomial of a matrix $A$. Then $m_A(x)$ divides any annihilating polynomial of $A$.

2. Let $A = \begin{bmatrix} 4 & 1 & 6 & 0 \\ 0 & 1 & 2 & -4 \\ 1 & 0 & 1 & 1 \end{bmatrix}$. Determine matrices $X$ and $Y$ such that $A = XY$ for which $\mathcal{C}(A) = \mathcal{C}(X)$ and $\mathcal{R}(A) = \mathcal{R}(Y)$.

3. If $\mathcal{C}(A) \subseteq \mathcal{C}(B)$ and $\mathcal{R}(A) \subseteq \mathcal{R}(D)$ then prove that there exists a matrix $C$ such that $A = BCD$.

4. Determine the orthogonal projection of the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ onto the plane spanned by $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix} \right\}$.

5. Find the distance from the point $(3, 2, 3)$ to the plane $x - 2y + 3z = 0$.

6. Find an orthonormal basis for the column space of the following matrix.

$$\begin{bmatrix} 2 & 6 & 4 \\ -1 & 1 & 1 \\ 0 & 4 & 3 \\ 1 & -5 & -4 \end{bmatrix}.$$

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7. Let $A$ be a normal matrix. Then show that every vector in the column space of $A$ is orthogonal to every vector in the null space of $A$.

8. Derive a spectral decomposition of $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$.

9. Prove that the minimal polynomial of a diagonal block matrix $\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ where $A$ and $B$ are square matrices, is the lowest common multiple of the minimal polynomials of $A$ and $B$.

10. Give an example of a real non-symmetric/skew-symmetric matrix of order $4 \times 4$ that is unitarily diagonalizable.

11. Find the minimal polynomial of the all-one matrix of order $n$.

12. Let $p(x) = 1 + 2x - 3x^2 + x^3$. Define a matrix $A$ for which $p(x)$ is the characteristic polynomial of $A$.

All The Best!!