Indian Institute of Technology Kharagpur
Course: MA41024/ MA60020/ MA60262 Information and Coding Theory
Spring Semester 2022-23
Assignment - II
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1. Consider the following map from $\{0,1\}^3$ to $\{0,1\}^5$ defined as follows.

$$(000) \mapsto (00000),\ (001) \mapsto (11011),\ (010) \mapsto (10110),\ (011) \mapsto (01101)$$
$$(100) \mapsto (01001),\ (101) \mapsto (10010),\ (110) \mapsto (11111),\ (111) \mapsto (00100).$$

   Does this map correspond to a linear block code? If yes, determine a generator matrix. Is the generator matrix unique? Find a standard array of the code.

2. Encode an information sequence (10101010101) for $(15, 11)$ Hamming code. How many errors can the code detect and correct? Suppose (111111000000) is a received coreword for this codeing scheme. Then determine the correct codeword.

3. Conside a linear code with the generator matrix

$$G = \begin{bmatrix} 111000 \\ 100110 \\ 010101 \end{bmatrix}.$$

   Then determine generator matrix and parity-check matrix in the systematic form. What is the minimum distance of the code?

4. Find the dual code of a code whose generator matrix is given by

$$G = \begin{bmatrix} 100100000000 \\ 010000000010 \\ 001000000011 \\ 000100000001 \\ 000010000011 \\ 000001000000 \\ 000000100011 \\ 000000010001 \\ 000000001010 \\ 000000110100 \end{bmatrix}$$

5. Consider the cyclic code with block code length 7 and the generator polynomial $g(x) = 1 + x + x^3$. Then determine the generator matrix in systematic form, parity-check polynomial, and the parity-check matrix. Is the code equivalent to a Hamming code? Justify your answer.

6. Let a $g(x)$ be a (binary) cyclic code C with $g(1) \neq 0$. Then show that the code with generator polynomial $(x-1)g(x)$ is a subcode of C having sum of the entries of a codeword as 0.

7. How many cyclic codes of block code length 8 can exist over F_3? Give a generator polynomial of each such code.