Indian Institute of Technology Kharagpur Course: MA41024/ MA60020/ MA60262 Information and Coding Theory Spring Semester 2022-23 Assignment - I Last date of submission : February 15, 2023

- 1. True or False. Justify your answer. (As always, to say 'true' you must prove it, while to say 'false' you must produce an appropriate counterexample.)
 - (a) Let an unbiased coin is tossed until the first head occurs. If X denotes the number of tosses then the entropy of X is 2 bits.
 - (b) Let X denote a ramdom variable on a finite sample space. Then $H(X) < H(2^X)$.
 - (c) If H(Y | X) = 0 then Y is a function of X.
 - (d) Let $X_1 \to X_2 \to X_3 \ldots \to X_n$ be a Markov chain. Then $I(X_1; X_2, \ldots, X_n) = I(X_1; X_2)$.
 - (e) Let X, Y be independent rvs, and Z = X + Y. Then H(Z) = H(X) + H(Y).
 - (f) For any two pmfs p and q on a finite sample space \mathcal{X} , $D(p \parallel q) \neq D(q \parallel p)$
 - (g) Let X, Y be two random variables on finite sample spaces. Then I(X;Y) = I(X) I(X|Y), where I(X) = I(X;X)
- 2. Let a_i be a symbol with probability p_i , i = 1, ..., m. Suppose a_i s are encoded into strings from a set of d code letters $\{0, 1, ..., d-1\}$ with a prefix free coding scheme. If m = 6 and the codeword lengths are $(l_1, l_2, ..., l_6) = (1, 1, 2, 3, 2, 3)$, then find a good lower bound on d.
- 3. Consider a sample space of 8 symbols whose probabilities are as follows:

А	В	С	D	Е	F	G	Н
1/2	1/4	1/8	1/16	1/32	1/64	1/128	1/128

What is the entropy of the each of the above symbol set? Construct a uniquely decodable prefix free code for these symbols and explain why those are uniquely decodable and have the prefix free property.

4. Let P(x, y) denote the pmf of the joint random variables (X, Y) with finite sample spaces. Then show that

 $\mathbb{E}_X[\|P(Y|X=x) - P(Y)\|_1] \le \sqrt{2\ln 2 \cdot I(X;Y)}.$