Indian Institute of Technology Kharagpur
Course: MA41024/MA60020 Information and Coding Theory
Spring Semester 2020-21
Time : 25 minutes
Class Test - III

Declaration:

- **Each question carries 4 marks.**
- **NO query will be entertained during the examination.**
- **There may be multiple options correct for a problem. Full marks is given only when all the correct options are identified.**
- **Once a problem is passed, it will not appear in your screen again and hence if a problem appears in your screen then identify the correct option and then go for the next problem.**

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1. Consider the linear code C defined by the generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

Then the number of elements of the dual code of C is ——-

Ans. 4

Tol: 0.01

The parity check matrix corresponding to C is given by

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

The elements of $C^{\perp}$, the dual code of C are linear combinations of rows of H. Thus

$$C^{\perp} = \{00000, 10010, 11101, 01111\}.$$

2. Suppose the parity-check matrix H for a code C is given by

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Then the minimum weight of C is —-

Ans. 4

Tol: 0.01

A codeword c in the code satisfies $Hc^T = 0$. Observe that the rightmost 6 columns of H are the complements of the leftmost six columns. Thus the sum of the first and last columns is the all-ones vector, as is the sum of the second and next to last column. The sum of the first two and last two columns is zero, which corresponds to the following weight 4 codeword:

$$(1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1).$$

Another weight 4 codeword, which corresponds to a linear dependence of the first six columns of H, is

$$(0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0).$$

Easy to see there can not be the codewords of less weight (apart from $c = 0$).

3. Consider the linear code C defined by the generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

Then which one of the following can be a coset leader for the standard array of C?

(a) (110000)
(b) (100010)
(c) (100001)
(d) All the above

Ans. b)

Note that there are $2^{6-3} = 8$ cosets and $d_{\text{min}} = 3$. Since every weight 1 vector is a coset leader, there are 6 cosets with code leaders having weight 1. Now there are $\binom{6}{2}$ weight 2 vectors in $\{0, 1\}^6$. In order to be in a coset $e_l + C$ with $w(e_l) = 1$, a vector must be at a distance 1 from one of the codewords of C. Since

$$C = \{000000, 100101, 010110, 110011, 001011, 101110, 011101, 111000\},$$

the only weight 2 vectors with this property are:

$$000101, 100001, 100100, 000110, 010010, 010100, 000011, 001001,$$
$$001010, 011000, 101000, 110000.$$

The remaining weight 2 vectors are $100010, 010001$ and 001100 must lie in the 8th coset, and any of these 3 may be chosen as the coset leader.

4. Let the parity-check equations of a $(8, 4)$ linear code C be given by

$$\begin{aligned} y_0 &= x_1 + x_2 + x_3 \\ y_1 &= x_0 + x_1 + x_2 \\ y_2 &= x_0 + x_1 + x_3 \\ y_3 &= x_0 + x_2 + x_3, \end{aligned}$$

where x_j are message digits and y_j are parity-check digits, $0 \leq j \leq 3$. Then the number of columns with Hamming weight 2 of the parity-check matrix corresponding to C is ___

Ans. 0

Tol: 0.001

The generator and parity check matrices are given by:

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

5. Let C be a $(21, 11)$ cyclic code generated by the polynomial $g(x) = 1 + x^2 + x^4 + x^6 + x^7 + x^{10}$. Then the Hamming weight of the codeword for the input $1 + x + x^3 + x^9$ is ___

   Ans. 10

   Tol. 0.001

   The codeword corresponding to the input $u(x)$ is $v(x) = x^{n-k}u(x) + r(x)$ where $r(x)$ is the remainder obtained by dividing $x^{n-k}u(x)$ by the generator polynomial $g(x)$. Then note that

   $$v(x) = 1 = x + x^2 + x^4 + x^5 + x^9 + x^{10} + x^{11} + x^{13} + x^{19}$$

   and hence $v(x) = (11101100011101000001)$.