Indian Institute of Technology Kharagpur
Course: MA51014/MA41004 Topology
Spring Semester 2016
Class Test -02

**Declaration: Answer without proper justification carries NO marks.**

1. True or False. Justify your answer. (As always, to say 'true' you must prove it, while to say 'false' you must produce an appropriate counterexample.)

   (a) Let $X$ be any topological space. Let $A_n, n \in \mathbb{N}$ be a connected subspace of the topological space $X$ such that $A_n \cap A_{n+1} \neq \emptyset$ for all $n$. Then

   $$A = \bigcup_n A_n$$

   is connected. [1]

   (b) A Hausdorff topological space is regular. [1]

   (c) Every connected metric space having more than one point is uncountable. [1]

   (d) Consider the topology $\tau$ on $\mathbb{R}$ consisting of all sets $A$ such that $\mathbb{R} \setminus A$ is either countable or all of $\mathbb{R}$. Then $[0,\, 1]$ is a compact subspace of $\mathbb{R}$. [2]

   (e) Any metrizable space has a countable basis. [1]

   (f) A unit circle with center as origin and a line segment passing through origin in $\mathbb{R}^2$ are homeomorphic. [1]

2. Let $X$ be a nonempty set. Let $p \in X$. Define a topology $\tau$ on $X$ as follows

   $$\tau = \{S \subseteq X : p \in S \,\text{or}\, S = \emptyset\}.$$

   Then show that $(X, \tau)$ is locally compact. [1]

3. Let $X$ be a regular topological space. Then prove that every pair of points of $X$ have neighborhoods whose closures are disjoint. [2]

**All The Best !!**