Indian Institute of Technology Kharagpur
Course: MA51014/MA41004 Topology
Spring Semester 2016
Class Test -01

Declaration: Answer without proper justification carries NO marks.

1. True or False. Justify your answer. (As always, to say 'true' you must prove it, while to say 'false' you must produce an appropriate counterexample.)
   (a) Let (X, τ) be any topological space. Then, there can be only a unique basis for the topology τ. [1]
   (b) The *infinite comb* is a closed subset of $\mathbb{R}^2$ with standard topology. [1]
   (c) The set $A = \{x \in \mathbb{R} \,|\, 0 < x < 1\}$ is an open subset of $(\mathbb{R}, \tau_l)$ where τ_l denotes the lower limit topology. [1]
   (d) The function $f : (\mathbb{R}, \tau_u) \to (\mathbb{R}, \tau_u)$, where τ_u denotes the standard topology, defined by $f(x) = |x - 1|$ is a continuous function. [1]
   (e) Quotient space of a Hausdorff topological space is Hausdorff. [2]

2. Let $X = \mathbb{R}$ with standard topology. Consider the partition
$$X^* = \{(n,\, n+1] \subset \mathbb{R} \ \ |\, n \in \mathbb{Z}\}.$$
   Describe the open sets in the resulting quotient topological space X^*. [2]

3. Prove that the subspaces 'unit open square' given by $S = \{(x, y) \in \mathbb{R}^2 : 0 < x < 1, 0 < y < 1\}$ and the 'open unit disc' of $\mathbb{R}^2$ with standard topology are homeomorphic. [2]

All The Best !!