Big Data Analysis
(MA60306)

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Lecture 24
March 30, 2023

Random walks

Transition matrix A random process $\{X_0, X_1, \ldots\}$ with finite state space $S = \{s_1, s_2, \ldots, s_k\}$ is a (homogeneous) Markov chain with transition matrix $P = [p_{ij}] \in \mathbb{R}^{k\times k}$ if

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for all t, $i, j \in \{1, 2, \ldots, k\}$

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Example:

$$P = \begin{bmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{bmatrix}$$

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Question Does the transition matrix have any special property?

Sampling methods

m-step transition matrix $[P^m]_{ij} = p(X_{t+m} = j | X_t = i)$.

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Stationary distribution A row vector $\pi = (\pi_1, \ldots, \pi_k)$ is said to be a stationary distribution fro the Markov chain if

(i) $\pi_i \geq 0$ and $\sum_{i=1}^{k} \pi_i = 1$, and

(ii) $\pi P = \pi$, i.e. $\sum_{i=1}^{k} p_{ij} = \pi_j$, for $j = 1, \ldots, k$

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Question Is every Markov chain stationary? Can there exist more than one stationary distributions of a Markov chain?

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Observation If $\pi_i p_{ij}$ is interpreted as the amount of probability mass flowing at any time instant from the state s_i to s_j, then for reversible Markov chain, it is equal to the amount of probability mass flowing from s_j to s_i. (a sense of equilibrium/balance!)

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Theorem If π is a reversible distribution then it is also a stationary distribution

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Observation A finite Markov chain is irreducible if and only if its graph representation is a strongly connected graph.

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Observation For a reversible MC,

$$\sum_{j\in S} p_{ji}\pi(j) = \sum_{j\in S} p_{ij}\pi(i) = \pi(i)\sum_{j\in S} p_{ij} = \pi(i)$$

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Aperiodicity The period of a state $s_i \in S$ is defined as

$$gcd\{t \geq 1 : [P^t]_{ii} > 0\}.$$

Thus the period of s_i is the greatest common divisor of the set of times that the chain can return (i.e., has positive probability of returning) to s_i , given that we start with $X_0 = s_i$. If the period $s_i = 1$, then we say that the state s_i is aperiodic.

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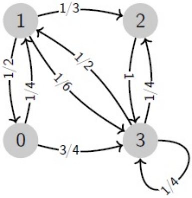
A Markov chain is said to be aperiodic if all its states are aperiodic. Otherwise the chain is said to be periodic

Sampling methods

Homework Consider the following Markov chain:

$$P = \begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{3}{4} \\ \frac{1}{2} & 0 & \frac{1}{3} & \frac{1}{6} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$



Determine the periodicity, regularity and irreduciblity property of the Markov Chain.

Sampling methods

Existence and uniqueness Any irreducible and aperiodic Markov chain has exactly one stationary distribution.

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Example of a MC model Assume that a mosquito hops between the forehead, the left check and the right check of a person

- There are three states
- Suppose the rule is: At some time t, if the mosquito is sitting on the foreheaad then it will hop to left check at time $t+1$
- if it sits on the left check, it will stay there or move to the right check with probability 0.5 for each, and if it is on the right check then it will stay there or move to the forehead with probability 0.5 each.

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Then the transition matrix is

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \end{bmatrix}$$

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Homework Determine the MC properties of the walk of the mosquito in the last example.

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GOAL How should we use the MC concept for simulating a pdf?

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GOAL How should we use the MC concept for simulating a pdf? MCMC was placed in the top 10 most important algorithms of the 20th century (SIAM news).

- $\rightarrow$ Suppose $p(\mathbf{x})$ is the given pdf
- $\rightarrow$ Define a MC on a state space whose stationary distribution is the target density

Sampling methods

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Sampling methods

Math framework of MCMC algo To draw simulations from a target distribution π on a countable set S, define a stationary Markov chain or a transition matrix P which is irreducible and aperiodic such that the reversibility condition $p_{ij}\pi_i = p_{ji}\pi_j$ holds.

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Theorem (Justification of approximating the true average by sample average) Let X_n, $n \geq 0$ be an irreducible stationary Markov chain on a discrete state space S. Suppose X_n possess a stationary distribution π. If $\phi : S \to \mathbb{R}$ is such that $\mathbb{E}_\pi[\phi(X)]$ exists then for any initial distribution μ,

$$\frac{1}{n}\sum_{k=1}^{n} \phi(X_k) \to^{\text{a.s.}} \mathbb{E}_\pi[\phi(X)]$$

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Then the entries of the transition matrix are given by

$$
\begin{aligned}
p_{ij} &= \theta_{ij}\gamma_{ij},\ i, j \in S,\ j \neq i \\
p_{ii} &= 1 - \sum_{j \neq i} p_{ij}
\end{aligned}
$$

where θ_{ij} is the probability that j is picked as a candidate state, and γ_{ij} is the probability of moving to the state j

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Homework Verifying the assumptions!