# Big Data Analysis
(MA60306)

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Lecture 19
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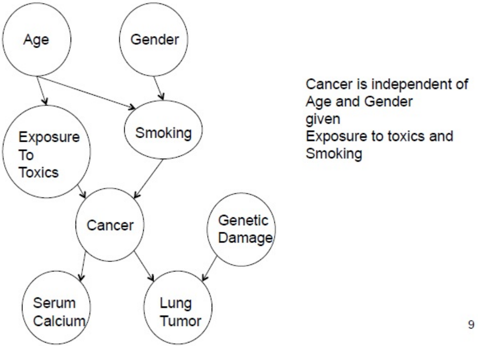
# Sampling methods



Cancer is independent of
Age and Gender
given
Exposure to toxics and
Smoking

# Sampling methods

Conditional independence Suppose $X, Y, Z$ are three rvs and the conditional distribution of $X$, given $Y$ and $Z$ does not depend on the value of $Y$ i.e.

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Any example?

In that case,

$$p(X, Y|Z) = p(X|Y, Z)\,p(Y|Z) = p(X|Z)\,p(Y|Z)$$

which means Conditioned on $Z$, the joint distribution of $X$ and $Y$ factorizes into product of the marginal distribution of $X$ and the marginal distribution of $Y$ (both conditioned on $Z$)

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which means Conditioned on $Z$, the joint distribution of $X$ and $Y$ factorizes into product of the marginal distribution of $X$ and the marginal distribution of $Y$ (both conditioned on $Z$) This further means $X$ and $Y$ are statistically independent given $Z$. We denote it as

$$X \perp\!\!\!\perp Y \,|\, Z$$

# Sampling methods

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$$p(X, Y) = \sum_{z} p(x, y, z) = \sum_{z} p(x|z)p(y|z)p(z)$$

so it does not factorize into the product $p(x)p(y)$, and so

$$X \not\perp\!\!\!\perp Y \,|\emptyset,$$

in general, where $\emptyset$ denotes the empty set

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Now if the value of $z$ is observed then

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- → Note that the graph is a path
- → The node corresponding to $Z$ is said to be tail-to-tail with respect to this path because the node is connected to the tails of the two directions, and the presence of such a path connecting $X$ and $Y$ causes these to be independent

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- → Thus the conditioned node 'blocks' the path from $X$ to $Y$ and causes $X$ and $Y$ to be conditionally independent

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Next suppose $Z$ is observed, and we condition on $Z$ then

$$p(x, y|z) = \frac{p(x, y, z)}{p(z)} = \frac{p(x)p(z|x)p(y|z)}{p(z)} = p(x|z)p(y|z)$$

i.e.

$$X \perp\!\!\!\perp Y \,|Z$$

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Another example Suppose the joint distribution of a graphical model is

$$p(x, y, z) = p(x)p(y)p(z|x, y)$$

Suppose none of the variables are observed. Then

$$p(x, y) = \sum_z p(x, y, z) = p(x)p(y) \sum_z p(z|x, y) = p(x)p(y),$$

so $X, Y$ are independent i.e. $X \perp\!\!\!\perp Y \,|\emptyset$ even if no variables are observed

# Sampling methods

Suppose we condition it on $Z$. Then

$$p(x, y|z) = \frac{p(x, y, z)}{p(z)} = \frac{p(x)p(y)p(z|x, y)}{p(z)}$$

which in general does not factorize into the product $p(x)p(y)$ and hence

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The node $Z$ is head-to-head wrt to the path from $X$ to $Y$

When the node $Z$ is unobserved, it ‘blocks’ the path and the variables $X, Y$ are independent

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Question What is the observation from the graph?

- The node $Z$ is head-to-head wrt to the path from $X$ to $Y$
- When the node $Z$ is unobserved, it 'blocks' the path and the variables $X, Y$ are independent
- When $Z$ is observes, it 'unblocks' the path and renders $X, Y$ dependent

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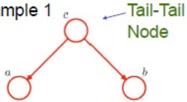
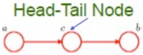
Example 1



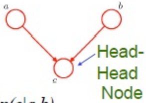
$p(a,b,c)=p(a|c)p(b|c)p(c)$

Example 2



$p(a,b,c)=p(a)p(c|a)p(b|c)$

Example 3



$p(a,b,c)=p(a)p(b)p(c|a,b)$