IIT Kharagpur
TS70006: Quantum Mechanics and Quantum Computing
Quantum Computing Assignment - 5

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1. Consider the following generalized CNOT gates

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

   Show that all four generalized CNOT gates can be constructed using the standard CNOT gate and single qubit gates. Implement the corresponding quantum circuits using Qiskit.

2. Shaw that the SWAP gate can be constructed using generalized CNOT gates. Implement the corresponding quantum circuits using Qiskit.

3. Construct a CNOT gate using one CZ gate and two Haramard gates.

4. The CMINUS gate is defined by CMINUS = CPHASE(π). Prove that CMINUS gate can be constructed by CNOT and H gates. Implement the corresponding quantum circuits using Qiskit. How can we construct a CNOT gate in terms of a CMINUS and H gates?

5. Construct a C-U gate for $U = R_x(\theta)$ and $U = R_y(\theta)$ using only CNOT and single qubit gates.

6. Using CNOT and quantum Toffoli gates construct a quantum circuit to perform this transformation

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

   Also implement the corresponding quantum circuit in Qiskit.

7. Consider the transformation $U_f : \mathbb{C}^{2^{n+1}} \to \mathbb{C}^{2^{n+1}}$ given by

$$U_f \left| x_{n-1}, x_{n-2}, \ldots, x_0 \right\rangle \left| y \right\rangle = \left| x_{n-1}, x_{n-2}, \ldots, x_0 \right\rangle \left| y \oplus f(x_{n-1}, x_{n-2}, \ldots, x_0) \right\rangle$$

   where $f : \{0,1\}^n \to \{0,1\}$ is any map. Then show that U_f is a unitrary map. Any idea about the matrix representation of this map?

8. Design a quantum circuit which implements the function $f(x) = x^3$ when $x \in \{0,1\}^2$. Can you give a "tight" lower bound on the minimum number of universal gates which are required to design the circuit? Any idea about generalizing it for the function $f(x) = x^n$, $x \in \{0,1\}^2$, where n is any positive integer.