IIT Kharagpur
TS70006: Quantum Mechanics and Quantum Computing
Quantum Computing Assignment - 2

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1. Is the Fredkin gate reversible? Justify your answer.
2. Is it possible to dintinguish the following functions just by drwaing them in your favorite programming language? $f_1(n) = n$, $f_2(n) = n^2$, $f_3(n) = n^3$, $f_4(n) = \log(n)$, $f_5(n) = n\log(n)$, $f_6(n) = n!$, $f_7(n) = 2^n$, $f_8(n) = 2^{\frac{n}{10}}$, $f_9(n) = e^n$.
3. Suppose $g(n)$ is a polynomial of degree k. Show that $g(n)$ is $\mathcal{O}(n^l)$ for any $l \geq k$.
4. Justify the following.
   (a) $log(n)$ is $\mathcal{O}(n^k)$ for any $k > 0$.
   (b) n^k is $\mathcal{O}(n^{\log(n)})$ for any k, but that $n^{\log(n)})$ is never $\mathcal{O}(n^k)$.
   (c) c^n is $\Omega(n^{\log(n)})$ for any $c > 1$, but that $n^{\log(n)}$ is never $\Omega(c^n)$.
5. Suppose $X_1, X_2, \ldots, X_n$ are independent and identically distributed random variables, each taking the value 1 with probability $\frac{1}{2} + \epsilon$, and the value 0 with probability $\frac{1}{2} - \epsilon$. Then
$$P\left(\sum_{i=1}^{n} X_i \leq \frac{n}{2}\right) \leq e^{-2\epsilon^2 n}.$$
6. (Award question): Write a python function which accepts an user-given list $[x_1, x_2, \ldots x_n]$ where $x_i \in \{0, 1\}$ of arbitrary length n and return the state vector $|x_1 x_2 \ldots x_n\rangle = |x_1\rangle \otimes |x_2\rangle \otimes \cdots \otimes |x_n\rangle$. The use of any python library specially designed for quantum computing, for instance Qutip, and Qiskit is restricted. Give your function 9 random inputs and produce 9 outputs.
7. Justify: A unitary operator can always be written as $\exp(iX)$ for some Hermitian operator X. If yes, cany you sketch a way to determine X?