

IIT Kharagpur
TS70006: Quantum Mechanics and Quantum Computing
Quantum Computing Assignment - 1

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1. Given unary representation of two positive integers as an input, write an algorithm to add them in a Turing machine. Can you implement this algo in your preferable programming language?
2. Write a program in your favorite programming language in your own computer such that the program does not halt by itself, that is, it can run for infinite time (assuming infinite resource is available).
3. Design circuits to implement the following functions f_1, f_2 , and f_3 using the universal 2-bit logic gates.

| x | y | z | f_1 | f_2 | f_3 |
|-----|-----|-----|-------|-------|-------|
| 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 |

4. Construct a reversible circuit which, when two bits x and y are input, outputs $(x, y, c, x \oplus y)$, where c is the carry bit when x and y are added.
5. What is the smallest number of Fredkin gates needed to simulate a Toffoli gate? What is the smallest number of Toffoli gates needed to simulate a Fredkin gate?
6. (Award question) Show that there exist Boolean functions on n inputs which require at least $2^n/\log n$ gates to compute.

Suggested readings

1. Digital Design by M. Morris Mano, Michael D. Ciletti.
2. Introduction to the theory of computation by Michael Sipser.