Journal of Structural Engineering
Vol. 34, No. 4, October-November 2007 pp. 284–290

No. 34-28

A system reliability based design equation for steel girder highway bridges

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Received 17 January 2007; Accepted 12 February 2007

Although structures are commonly designed and assessed on an element basis, the true measure of structural safety is its systems reliability. To be accurate, structural systems reliability must consider multiple failure paths, load sharing and load redistribution after member failures, and cannot be captured by element reliability analysis. The incremental loading method (ILM) in which the magnitude of the vector of external load variables is slowly increased from zero up to a pre-determined cut-off condition (while keeping the direction of the vector constant) and in which the structural state is updated within the confines of static equilibrium at each successive component failure is a versatile method for identifying failure sequences. This paper presents an improved procedure to assess the system factor to be used in a component-based design equation that will help achieve a target structural system reliability. System failure is defined as the union of strength and local instability failures. Adaptive importance sampling is used for system reliability analyses. A simple span five-girder steel highway bridge is selected for illustration.

KEYWORDS: System reliability; incremental loading method; statistical dependence; target relilability; bridge design; AASHTO LRFD.

Structural safety can most rationally be addressed using principles of structural reliability which consider uncertainties and incomplete knowledge about structural loads, strength, geometry and behavior in a probabilistic format. Structural reliability is defined as the probability that the structure will perform satisfactorily during a specified service life under given operation conditions. Accordingly, structural design codes have over the past three decades or so begun to move away from a deterministic safety factor based approach to a reliability-based one: at the current state of the art, most of these codes make use of design equations that involve partial safety factors calibrated to achieve an average target reliability and intended to reflect the relative uncertainties in the load and resistance parameters. Most of these design approaches also share a common feature that the design philosophy is element-based[1,2]. The safety check is performed on a component basis with the implicit expectation that the structural system will be safe as long as all its element reliabilities are satisfactory.

The term "element" in this paper is used in the logical rather than the physical sense: an "element" here means one critical cross-section or location of a structural member/component in exactly one failure mode (e.g, flexural failure, tensile failure or unstable crack growth etc.). A structural "component" or structural "member" (e.g., a beam or a column or a deck) typically has more than one critical cross section-failure mode combination, although only one may be dominant. A member thus can represent several elements of the system.

Until fairly recently, most bridges in the United States were designed using allowable stress methods (working stress design or WSD) in which uncertainties in loads and element resistance were taken into account using a single factor of safety. In 1994, as a result of NCHRP Project 12–33, the reliability-based AASHTO Load and Resistance Factor Design (LRFD) Highway Bridge Design Specifications[3] were published. The load and resistance factors were calibrated based upon a global population of bridges[4]. The benefit of LRFD over WSD is that safety checks can be performed beyond yield close to collapse conditions, and components designed using LRFD have a more uniform level of safety across a range of configurations.

Although these recent developments have served the profession well, element-based design makes it difficult to achieve uniform reliability in all structural systems designed to the given set of specifications. After all, it is the failure of the system that is of ultimate significance to the engineer and the public, hence, in addition to the elements having adequate reliability, it is essential that the system reliability meet its target value. The target must be commensurate with the consequences of system failure. A well-designed structural system should have adequate safety margin in intact condition; it should also be sufficiently indifferent to the failure of at least the first few elements. Such desirable behavior arises from structural indeterminacy coupled with excess capacity of the critical members, load redistribution, non-brittle member failure etc. A structural system may also possess system failure modes such as excessive global deformation that do

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(Discussion on this article must reach the editor before January 31, 2008)

not necessarily involve any component failure. Further, there is no assurance that the component-based safety criteria will be adequate if they are applied in relatively novel structures where there may be little experience and unexpected failure modes such as a progressive collapse sequence.

It is possible, in a practically appealing way, to modify the component design equation[1,5] that takes into account systems effects and ensures the target system reliability for the given structural configuration and loading. This technique requires a clear understanding of the system failure criteria, load sharing and load redistribution after initial component failures, post-failure material behavior, and possible dependence among the basic variables. This paper develops a methodology for analyzing the system reliability of a structure composed of ductile materials which is then used to derive a system factor to be used in a component design equation intended to achieve the desired system reliability. A steel highway girder bridge structure is selected for illustration.

RELIABILITY ANALYSIS OF REDUNDANT SYSTEMS

A component reliability problem arises in the case of a single structural element (or, more generally, a critical cross-section of an element) in a single failure mode (such as tensile or flexural). For practical purposes, it is the availability of a single, differentiable and closed-form performance function that separates a component reliability problem from a systems one. Although computationally very attractive, it is often not possible to cast the performance of a structural system also in terms on a single limit state (say, using approximate numerical techniques such as a response function fit) and thereby take advantage of the speed, elegance and accuracy of component reliability solution techniques. One should also add that structural system failure events are thankfully so rare and-since structural systems can hardly be deemed to constitute a nominally identical sample-that a frequentist interpretation of structural system reliability is simply not feasible.

The need to estimate system reliability of bridge structures has long been recognized. Liu and Moses[6] considered reliability of bridges after they sustain damage to one or more girders. Estes and Frangopol[7] developed a lifetime repair strategy for bridges that minimized total lifetime repair costs while maintaining a minimum system reliability. Moses[2] proposed a general method of optimization based on AASHTO LRFD code considering system reliability effects. A comprehensive analysis of redundancy of bridge superstructures was performed in NCHRP Project 12–36[8]. Bridge system reliability was considered in intact as well as damaged conditions.

System reliability computation for structures is not straightforward since the component failures are not mutually independent events on account of (i) active redundancy in the structure leading to load sharing, (ii) load path dependence in case of successively applied multiple yet sustained loads, (iii) load redistribution after initial member failures for redundant structures, (iv) non-linear behavior and non-brittle failure of the components, (v) failure sequences of different probabilities for the same cut set in a progressive collapse or incremental loading situation, and (vi) possible statistical dependence among the basic variables. On top of these, the fact that system failures are extremely rare events often requires use of numerical simulation with special variance reduction properties, as used later in this paper.

Several techniques, each tailored to address some of the above issues, have been developed over the years for tackling structural systems reliability problems. The first order reliability method (FORM), which maps the limit state equation from the basic variable space to the rotationally symmetric uncorrelated standard normal space (using methods of various sophistication that preserve the dependence structure), can be easily extended to simple system reliability problems of the pure parallel or series kind. For more general systems, the greatest challenge is to identify the minimal cut sets (at least the dominant ones), particularly in light of the circumstances peculiar to structural systems mentioned above. If the cut sets $C_i, i = 1, \cdots, n_c$ can be identified for the system, the failure probability can be expressed as:

$$P_{f,sys} = P\left[\bigcup_{i=1}^{n_c} C_i\right] = P\left[\bigcup_{i=1}^{n_c}\left\{\bigcap_{j=1}^{n_i} g_{ij} \leqslant 0\right\}\right] \tag{1}$$

where g_{ij} is the j^{th} limit state in cut set i. Exact solution of Eq.(1) may be impossible. Bounds on system reliability, based on marginal events[9], pairs of joint events[10] or triplets of joint events[11] are available. Cut sets, without regard to an ordering of failure events, are possible to be determined for elastic-perfectly plastic structures.

For a structure with n binary components, 2^n mutually exclusive system states are possible: only some of these produce system failure. If the order in which these components fail are important and need to be enumerated (either due to modeling convenience or if the physical process of failure is indeed such), the total number of distinct sequences increases to $\sum_{r=1}^{n} n!/(n-r)!$ which can be a very large number even for moderately sized structures. Only some of these sequences are failure sequences, and, only a subset of these failure sequences usually are dominant. It is thus clear that efficient methods of identifying the dominant failure sequences are necessary for all but the simplest structures.

Depending on the structural complexity and desired accuracy of the solution, the dominant failure sequences can be found in a variety of ways. The assumption of rigid perfectly plastic material behavior is fairly popular in structural system reliability analysis as it eliminates load history dependence. It is well-known that deterministic plastic mechanism analysis can lead to collapse mode identification in case of rigid-plastic framed structures, although the number of modes generated quickly becomes huge[12,13]. Such deterministic rules have been variously adapted to search for the probabilistically dominant collapse modes by (i) creating linear combinations of those basic mechanisms that have the lowest reliability indices (the beta-unzipping method[14]), (ii) using linear programming[15], (iii) stochastic programming[16], (iv) genetic algorithms[17] etc. The probabilistically dominant failure sequences can be searched using truncated enumeration schemes that include the incremental loading method, which is the method adopted in this paper, and is described in the following.

INCREMENTAL LOAD METHOD APPLIED TO STEEL GIRDER BRIDGES

Incremental loading of a structure in which the magnitude of the vector of external load variables is slowly increased from zero up to a pre-determined cut-off condition (while keeping the direction of the vector constant) and in which the structural state is updated within the confines of static equilibrium at each successive component failure is a versatile method for identifying failure sequences[2,18]. Some or all the basic variables in such incremental analysis may be set at their mean values; alternately, random samples of the basic variables may be generated repeatedly until the dominant sequences have been determined. The incremental loading method is particularly useful (and often the only way out) when component failure is multistate instead of the usual binary[19], material behavior is brittle, semi-brittle or non-linear instead of ideal plastic[20], and system failure occurs not due to formation of a mechanism, but due to excessive deformation or a specified drop in structural stiffness with regard to specified degrees of freedom. One potential drawback of the incremental analysis method is its quasi-static assumption of structural behavior: the load duration needs to be sufficient to allow potential redistribution of load effects throughout the system.

We assume that the structure behaves linearly and elastically between two consecutive component failures. Geometrical second-order effects to structural failure are neglected. The system fails if the structure reaches its ultimate capacity under the external loads. The external load $L = L(X_1, \cdots, X_m)$ is increased along a proportional static load path, i.e., $L = \gamma \cdot L(X_1, \cdots, X_m)$, $\gamma: \rightarrow 1.0$. The ultimate capacities for all components are random variable $R_1, \cdots, R_N$. The system fails as soon as $n(n \leq N)$ components fail forming a mechanism.

The performance function for the j^{th} collapse mode can be expressed as the linear combination:

$$g_j = \sum_k C_{jk} R_k^e - \sum_i b_{ji} S_i = R_s - \sum_i b_{ji} S_i \tag{2}$$

where R_k^e is the k^{th} component strength, S_i is the i^{th} load term, and R_s is the system resistance at collapse. C_{jk}, b_{ji} are coefficients from structural analysis. Thus, the system behavior may be expressed as

$$\begin{bmatrix} R_1^e \\ R_1^e \\ \vdots \\ R_m^e \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m1} & \cdots & a_{mm} \end{bmatrix} \cdot \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_m \end{bmatrix} \tag{3}$$

where a_{ij} is the normalized force (with respect to the component strength R_i^e) in component i due to unit increase in load j[18]. r_i is load increment in the i^{th} load incrementing step, in which component i fails. System fails occurs when component m fails. The sum of all load increments gives the system resistance R_s at collapse as

$$R_s = \sum_i r_i \tag{4}$$

Correctly identifying the failure sequence is crucial in ILM for system reliability. Moses[5] suggested that the failure candidate in each stage is the one with the largest change in a_{ij} in Eq. (3) neglecting the force accumulated in the component. The failure sequence generated may not always be a realistic one. Karamchandani[21] and Karamchandani and Cornell[20] suggested to select failure candidate with the minimum load factor. This reflects the maximum ratio of the external loads sustained by structure in each linear elastic state. Caution needs to be exercised, however, when some of the load factors are negative.

In the following, $\alpha_{ij}^{(k)}$ is defined as influence coefficient of load effect in component i under the effect of unit external load j in the k^{th} loading stage (k failed components) of structure. $\gamma_i^{(k)}$ is the load factor for component i in the k^{th} loading stage.

Staring from the 0^{th} stage, the influence coefficient for component i in the intact structure under unit load j is denoted as $\alpha_{ij}^{(0)}, i = 1, 2, \cdots, N\ j = 1, 2, \cdots, m$. Calculate every load factor $\gamma_i^{(0)}$:

$$\gamma_i^{(0)} = R_i \Bigg/ \sum_{j=1}^{m} \left(\alpha_{ij}^{(0)} X_j \right) \tag{5}$$

and identify component i_0 with the minimum load factor $\gamma_{i_0}^{(0)}$. If $\gamma_{i_0}^{(0)} < 1$, component i_0 has failed. The load effects D_i^0 in all remaining components at the instant when i_0 fails are

$$D_i^0 = \gamma_{i_0}^{(0)} \cdot \sum_{j=1}^{m} \left(\alpha_{ij}^{(0)} X_j \right) \qquad i \neq i_0 \tag{6}$$

The structure now enters the 1^{st} loading stage and the remaining external load $(1 - \gamma_{i_0}^{(0)})$. $L(X_1, \cdots, X_m)$ needs to be redistributed among the remaining components with influence coefficients $\alpha_{ij}^{(1)}$ after failure of component i_0. The load $(1 - \gamma_{i_0}^{(0)})$. $L(X_1, \cdots, X_m)$ is increased proportionally from 0 until all load is exhausted or another component fails. Denote this component as i_1, i.e.,$\gamma_{i_1}^{(1)}$ is the minimum among all the surviving components:

$$\gamma_{i_0}^{(0)} \cdot \sum_{j=1}^{m} \left(\alpha_{i_1 j}^{(0)} X_j \right) + \gamma_{i_1}^{(1)} \left(1 - \gamma_{i_0}^{(0)} \right) \cdot \sum_{j=1}^{m} \left(\alpha_{i_1 j}^{(1)} X_j \right) = R_{i_1} \tag{7}$$

The first part in the left side of the inequality is the accumulated load effect in component i_1 at the time when component i_0 fails in the 0^{th} stage; the second part is the load effect imposed on component i_1 during the redistribution.

Following similar analysis for subsequent events as above, as the structure evolves to the k^{th} stage, load factor for component i_k will be

$$\gamma_{i_k}^{(k)} = \frac{R_{i_k} - \sum_{q=0}^{k-1} \left[\gamma_{i_q}^{(q)} \left(\prod_{p=1}^{q} \left(1 - \gamma_{i_{p-1}}^{(p-1)} \right) \cdot \sum_{j=1}^{m} \left(\alpha_{i_k j}^{(q)} X_j \right) \right) \right]}{\prod_{p=0}^{k-1} \left(1 - \gamma_{i_p}^{(p)} \right) \cdot \sum_{j=1}^{m} \left(\alpha_{i_k j}^{(k)} X_j \right)} \tag{8}$$

As before, if $\gamma_{i_k}^{(k)}$ is the minimum load factor among all for the remaining components, and $\gamma_{i_k}^{(k)} < 1$, component i_k will be selected as the failure candidate. Otherwise, if the minimum load factor $\gamma_i^{(k)} > 1.0$ in the K^{th} stage and structure is still maintaining its stable configuration, the incremental analysis stops. The structural performance function for ultimate limit state is

$$g(R_i, X_j) = \sum_{p=i_k}^{i_N} \left(R_p - D_p^k - \prod_{q=0}^{k} \left(1 - \gamma_{i_q}^{(q)} \right) \cdot \sum_{j=1}^{m} \left(\alpha_{pj}^{(k)} X_j \right) \right), \quad i = 1, 2, \cdots N,\ j = 1, 2, \cdots m \tag{9}$$

where R_p is the resistance of component $p\ (p = i_k, \cdots, i_N)$, and D_p^k is the accumulated load effect in component $p\ (p = i_k, \cdots, i_N)$ in the k^{th} stage.

The deflection during the incremental analysis can be computed using similar analysis procedures. Since linear elastic behavior of structure in each loading stage is assumed, the deflection at a given location will be accumulated at each loading stage. For instance, at the end of the k^{th} stage, the accumulated deflection at φ is,

$$U_\varphi^k = \sum_{q=0}^{k} \left[\gamma_{i_q}^{(q)} \left(\prod_{p=1}^{q} \left(1 - \gamma_{i_{p-1}}^{(p-1)}\right) \cdot \sum_{j=1}^{m} \left(\beta_{\varphi j}^{(q)} X_j \right) \right) \right] + \prod_{p=0}^{k} \left(1 - \gamma_{i_p}^{(p)}\right) \cdot \sum_{j=1}^{m} \left(\beta_{\varphi j}^{(k)} X_j \right) \quad (10)$$

where the first part for U_φ^k is the total deflection of φ accumulated at the time when component i_k fails, the second part is the additional deflection due to the redistribution of the remaining part of external loads at the end of the k^{th} stage. $\beta_{\varphi j}^{(q)}$ is the deflection influence coefficient of deflection φ due to unit external load j on the structure in the $q^{\text{th}}(q = 0, 1, 2, \cdots, k)$ stage. $\gamma_{i_q}^{q}$ is the load factor in the $q^{\text{th}}(q = 0, 1, 2, \cdots, k)$ stage.

The above procedure implicitly assumes that the minimum load factor in each loading stage is positive: $0 < \gamma_i^{(k)} < 1$. Care should be taken in situations the minimum load factor turns negative caused by the redistribution of the remaining external loads. Assume that some component i has experienced always $positive$ load increments up to stage k, i.e., $0 < \gamma_{i_q}^{q} < 1, q = 0, 1, 2, \cdots, k$, and its resistance is not exhausted in the k^{th} stage, i.e., $R_i - D_i^k > 0$. At the end of the k^{th} stage, $negative$ increment may arise in component i when redistributing the remaining external load portion of $\prod_{p=0}^{k} \left(1 - \gamma_{i_p}^{(p)}\right) . L(X_1, \cdots, X_m)$; obviously, this is equivalent to unloading of the load effect in component i causing $\gamma_i^{(k+1)} < 0$. Component i in this case is a $spurious$ candidate for the failure sequence in the $(k+1)^{\text{th}}$ stage and should be excluded.

NUMERICAL EXAMPLE

We now apply the methodology in Section 3 to an example involving a simply-supported 5-girder steel bridge. Only the superstructure is considered in this analysis. The bridge is illustrated in Fig. 1, girders are numbered from 1 through 5. Only flexural failure is considered, defined as complete plastification of its critical cross-section. The girders are nominally identical and no aging effect is considered. Composite action is neglected, no source secondary stiffness is considered either. Component failure is irreversible. The material behavior is elastic perfectly plastic, hence once a plastic hinge is formed, a girder does not carry any additional load. Live load is applied incrementally and proportionately until the system fails or the load reaches its final value without causing global instability. The spread of plasticity through an element after failure of a section is neglected. System reliability of the bridge is evaluated by adaptive important sampling. Effects of dependency among resistance of girders on system reliability is also analyzed.

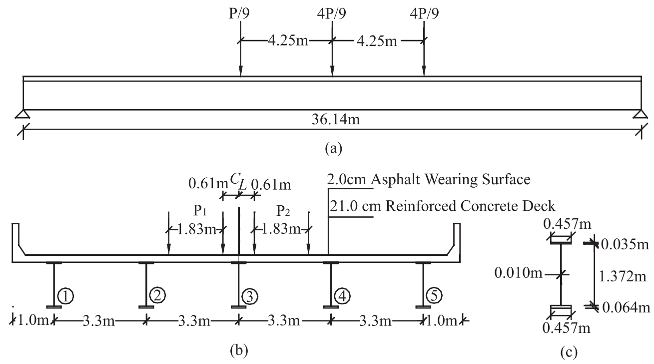

Fig. 1 Bridge elevation, typical cross section, and truck position

There is a certain amount of subjectivity in the definition of system failure of any structure, and bridges are no exception. Cho and Ang[22] defined the dominant stable configuration as either no overstressed girder or one overstressed beam. Nowak and Zhou[23] suggest that system failure occurs when a group of adjacent girders fails. Tabsh and Nowak[24] considered several girders must reach their ultimate loads before the structure collapses in a series-parallel system. Moses[2] generalized a system formulation in which the system fails either by reaching a maximum load level or by attaining unserviceable large displacements. Ghosn and Moses[8] defined the ultimate capacity limit as the maximum possible truck load that can be applied on the structure before the formation of a collapse mechanism. Moses[5] proposed a general solution to find out multi-collapse mode. Estes and Frangopol[7] considered any three adjacent girders out of the simplified five-girder bridge model tantamount to system failure. Enright and Frangopol[25] analyzed several system failure models for a five girder bridge.

A recent NCHRP Project[26] concluded that deformations that cause bridge damage are relative deflections between adjacent girder members, local rotations and deformations. An early deformation based failure definition for bridges by ASCE was based on avoiding the undesirable structural effects and undesirable psychological reaction[27] and a limit of span/800 for steel bridges having simple and continuous spans under live load plus impact was suggested. Based on "the limit of visual observation", Galambos et al[28] proposed a maximum permanent or residual deflection equal to span/300 as serviceability limit state in bridge inelastic rating. Ghosn and Moses[8] considered span/100 as "dangerously high levels" of deformation.

Based on the above discussion, we define failure of the bridge system to be the union of (i) the yielding of all 5 girders and (ii) maximum deflection of the deck exceeding span/300. We do not consider any uncertainty in the failure criteria in this paper; the effect of uncertainty has been investigated by Bhattacharya et al[29].

Using ANSYS 7.1[30], the five girders are modeled as Timoshenko beams (beam188). Shell63 elements are used to model deck integrated with girders. Coupling degree of freedoms (DOFs) in common nodes among girders and shells are used to simulate the behavior of girder with elastic perfectly plastic material. Before girder reaches its resistance in the critical section, all DOFs (3 translational DOFs and 3 rotational DOFs) in the common nodes between girder and shell element will be coupled completely, as shown in Fig. 2(a). However, proper rotational DOFs will be released after girder reaches its resistance to simulate the plastic hinge formed in girder, as shown in Fig. 2(b), where rotational DOF around x axis is released. Deck is assumed to resume its force-transferring function along the analysis. The secondary components, such as asphalt and carrier, are only considered as the dead load imposed on the structure. The bridge structure is thus approximated as longitudinal girders with deck overlaid. The influence coefficients for moment in midspan of each girder and for deflections along midspan of bridge deck, at each linear elastic state due to unit external load was determined using APDL provided by ANSYS 7.1[30]. By coupling or releasing proper DOFs in the common nodes among beam and shell elements, structural stiffness could be changed along with every stable configuration.

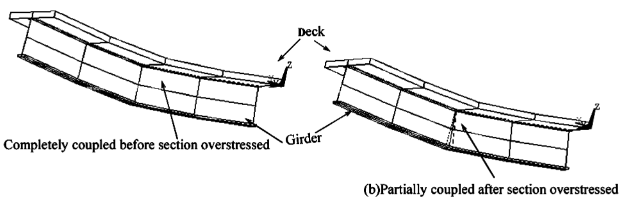

Fig. 2 Completely and partially coupled DOFs in a simple span girder under self-weight
(a) Completely coupled before section overstressed
(b) partially coupled after section overstressed

We consider only one loading case involving two trucks side by side in the bridge. No lane load is considered. The transverse and the longitudinal layout of trucks on the bridge are shown in Fig. 1. Trucks are placed transversely symmetric on bridge in this example.

In using ILM, all dead loads and live loads are considered as external loads. The statistical properties of the load random variables are as follows: the uniformly distributed load from the deck has a mean 5.82 kN/m^2 and a coefficient of variation (c.o.v.) of 5% and that from the steel girder has a mean of 4.54 kN/m with a c.o.v. of 5%. Dead loads are assumed to be Normally distributed. The weight of AASHTO HS-20 trucks is used as nominal live load. The weight of each truck is assumed to be Gumbel distributed with mean 320 kN and c.o.v. 12%. The bias factor for live load is assumed as 1.0. The resistance of each girder is taken to be Lognormally distributed. According to the AASHTO LRFD Bridge Design Specifications[3], the required nominal resistance for each girder in bridge structure is about 10,000 kNm. The resistance bias is assumed to be 1.08 and c.o.v. is assumed to be 8%. Statistical dependence among the girder resistances is also considered in this example: the resistance vector, X, have been assumed to be jointly lognormal with the distribution function:

$$F_X(x) = \Phi\left(z;\hat{\rho}\right), \text{ where } z_i = (\ln x_i - \mu_i)/\sigma_i \tag{11}$$

where ρ is the correlation matrix of X, Φ is the multidimensional standard Gaussian distribution function, $\hat{\rho}$ is the matrix of correlation coefficients for z; μ_i and σ_i are respectively the mean and standard deviation of ln X_i, and $\hat{\rho}_{ij}$ needs to be derived from ρ_{ij}, as suggested by der Kiureghian and Liu[31].

Adaptive importance sampling[21] is used to evaluate system reliability in this example. The estimate of the failure probability is:

$$\hat{P}_f = \frac{1}{N}\sum_{i=1}^{N}\left[I_D(x^{(i)})\frac{f_x(x^{(i)})}{h(x^{(i)})}\right] \tag{12}$$

where D is the failure domain, X is the vector of basic variables and f_x is the original density function of the basic variables. $I_D(x)$ is the indicator function,

$$I_D(x) = \begin{cases} 1 & x \in D \\ 0 & x \notin D \end{cases} \tag{13}$$

which is evaluated for each realization of the basic variable vector,$x^{(i)}$, using the finite element based ILM procedure described above. $h(x)$ is the multimode density function which will be updated after a point in failure domain is generated, and is developed as:

$$h(x) = \sum_{j=1}^{k}\left(\omega_j f_X^{(j)}(x)\right) \tag{14}$$

where $f_X^{(j)}(x)$ is the original density function but with mean shifted to $\hat{x}^{(j)}$, and $\omega_j = f_X(\hat{x}^{(j)})\Big/\left(\sum_{r=1}^{k} f_X(\hat{x}^{(r)})\right)$ are the normalizing weights. The set of k representative points $\{\hat{x}^{(1)}, \hat{x}^{(2)}, \cdots, \hat{x}^{(k)}$ in the failure domain are selected to develop the multimodal density. These points in the set are all believed to have relatively large probability density in f_x, and each of them satisfies some prescribed minimum distance from each other.

Figure. 3 shows the bridge system reliability as a function of statistical dependence among girder resistances. The reliability index is defined as $\beta = \Phi^{-1}(1 - P_f)$. As is well known, the system reliability decreases with increasing dependence among the girder strengths: $\beta = 5.95$ when girder strengths are independent and reduces to $\beta = 4.79$ when the girders are fully dependent.

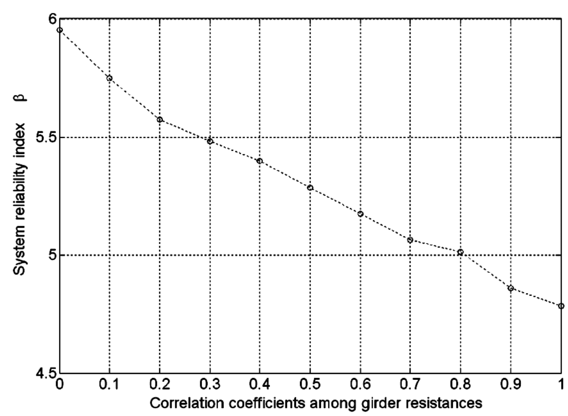

Fig. 3 System reliability index as a function of correlation among girder resistances

We now look at the effect of the system factor, φ_s, commonly used to modify the component design equation to achieve a target reliability:

$$\varphi_s \varphi R_n = \gamma_D D_n + \gamma_L L_n \tag{15}$$

In this example, a target system reliability index of 5.5 is selected based on Nowak et al[32]. Figure 4 shows the system factor, φ_s, required to achieve this target for different degrees of correlation among the girder resistances. It is clear that the bridge has more than adequate component capacity to achieve the target system reliability.

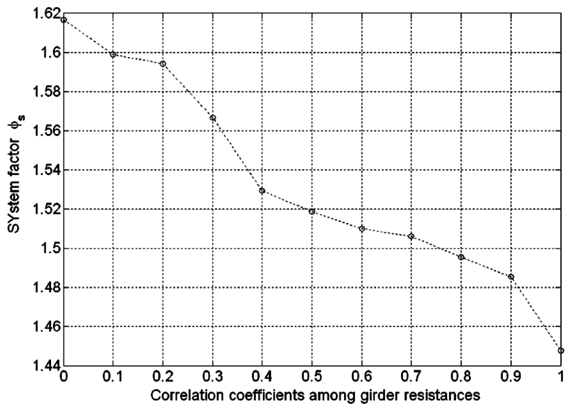

Fig. 4 System factor as a function of correlation among girder resistances to achieve target system reliability index of $\beta = 5.5$

CONCLUSIONS

This paper presented an improved procedure for evaluating the system reliability of ductile structures using the incremental loading method that eliminates spurious failure sequences on account of unloading during load redistribution. The incremental analysis is embedded in an adaptive Monte Carlo importance sampling simulation scheme for estimating system failure probability. The methodology was illustrated on a 5 girder steel highway bridge: system failure was defined as either the formation of a mechanism or the accumulation of excessive inelastic deformation or both. Statistical dependence among girder strengths was considered. The system factor for modifying the component design equation for achieving a desired system reliability was also derived.

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