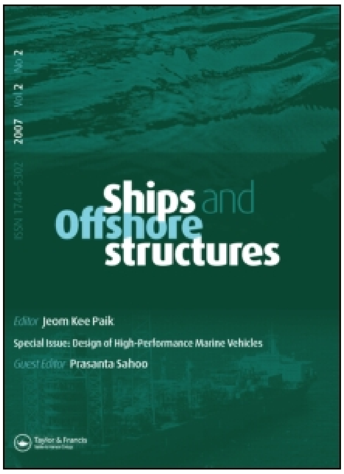

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Estimation of ultimate hull girder strength with initial imperfections

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Estimation of ultimate hull girder strength with initial imperfections

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The estimation of the ultimate strength of the ship hull is very important for its structural safety against applied loads. Various methodologies have been developed to evaluate the ultimate hull girder capacity, for example Caldwell (1965), Paik and Mansour (1995), International Association of Classification Societies Common structural rules (IACS CSR, 2006), etc. These methods do not usually include initial imperfections like initial deflection and initial residual welding stresses in the plating between stiffeners. The IACS CSR, introduced in April 2006, suggest the analytical incremental-iterative method for determining ultimate strength estimation of ship hull, which ignores welding residual stresses. In the present study, the stress–strain relationship for stiffened plate given in IACS CSR method is extended to account for the initial imperfections. Our previous work (Vhanmane and Bhattacharya 2007) on the stress–strain relationship of plate between stiffeners under axial loads, including imperfections like initial deflection and initial residual welding stresses, is used to determine the effective width of attached plating. The proposed methodology is applied to six benchmark cases: a double hull very large crude carrier and a capesize bulk carrier under three different levels of imperfection (slight, average, and severe) for both initial deflection and residual stresses. The ultimate strengths thus obtained are compared with published results that use two different methods (idealised structural unit method and finite element analysis) and it appears that the proposed methodology is simple yet robust in estimating hull girder ultimate strength under initial imperfections.

Keywords: ultimate strength; initial deflection; welding residual stress; ship hull girder; progressive collapse analysis

1. Introduction

A ship hull girder is a complex assembly of unstiffened/stiffened plates, longitudinals, frames, transverses, etc. Longitudinal bending, transverse bending, and torsion are the prime loads acting on the hull girder. Ultimate hull girder strength failure is the most critical failure mode in ship hull girder under these loading. The ultimate hull girder strength is the maximum bending capacity that a ship hull girder can sustain under longitudinal bending. Methods to evaluate the ultimate hull girder strength of ship's hull under longitudinal bending fall into two categories. The first is the direct method and other is the progressive collapse analysis of a hull girder. Though a more comprehensive review is available in Yao (1999), a brief historical introduction on analytical methods of ultimate strength evaluation is given in the following.

The category of direct methods started with Caldwell (1965) who theoretically evaluated the ultimate hull girder strength under longitudinal bending. His work yielded in the reduction factor at the compression side of hull girder. The bending moment produced by the reduced stress was considered as the ultimate hull girder strength. However, reduction in the capacity of structural members after attaining their ultimate strengths was not taken into consideration in Caldwell's work and the calculated ultimate strength was overestimated. Later on, improvements in the direct method had been brought about by Mansour et al. (1990). They proposed simple formulae to calculate the ultimate hull girder strength on the basis of experimental investigations. Paik and Mansour (1995) also contributed by proposing a simple prediction method for the ultimate hull girder strength under a vertical bending moment. For calculation of ultimate strength, a longitudinal stress distribution at the overall collapse state was assumed following the observation that all the side shells (under compression and tension) in the immediate vicinity of the neutral axis remain elastic and the stress distribution there is assumed to be as linear.

The direct methods described above do not take into account the strength reduction in the structural members beyond their ultimate strength. Also, the initial imperfections are ignored. These two aspects significantly affect the ultimate strength of the whole hull girder section and should be included in a more realistic description of collapse behaviour of the hull girder.

The second category, progressive collapse analysis, considers the strength reduction in structural members after attainment of their ultimate strength. Smith's (1977) method is very well-known in this category. The whole

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hull girder cross section was subdivided into combined stiffener-attached plating element and its average stress–strain relationships were derived performing finite element analysis (FEM) analysis before progressive collapse analysis. The main concern was initial imperfections (initial defection and welding residual stresses) in plating between stiffeners.

In the case of large number of structural members, FEM analysis can be expensive and time consuming to obtain stress–strain relationship of stiffened plate element. Keeping Smith's method as the outer shell of progressive collapse analysis, some advanced analytical methods also have been proposed to derive stress–strain relationship. Gordo and Guedes Soares (1993) applied analytical stress–strain relationship of a beam column subjected to axial compression to evaluate ultimate hull girder strength. An analytical method was proposed by Yao and Nikolov (1991, 1992) to derive average stress–strain relationship for the element composed of a stiffener and attached plating. Paik et al. (1999) developed a simple analytical model to calculate the ultimate strength of a stiffened panel under uniaxial compression. The methods by Yao and Paik have considered the effect of initial imperfection for plating and for stiffener.

The literature on the effects of initial imperfection on hull girder ultimate strength is very sparse. Kim (2003) has estimated ultimate strength of 10 typical merchant ship hulls applying the idealised structural unit method (ISUM). Paik and Thayamballi (2003) also applied ISUM approach to obtain hull girder ultimate strength considering the effect of initial deflection and welding residual stresses. The initial imperfection was applied in three levels discussed in later part of this article.

Though use of FEM analysis is expensive and time consuming in evaluating hull girder ultimate strength, Harada and Shigemi (2007) have done a series of nonlinear FEM analysis for double hull very large crude carrier (VLCC) and capesize bulk carrier to obtain the ultimate longitudinal strength in hogging and sagging conditions. The initial

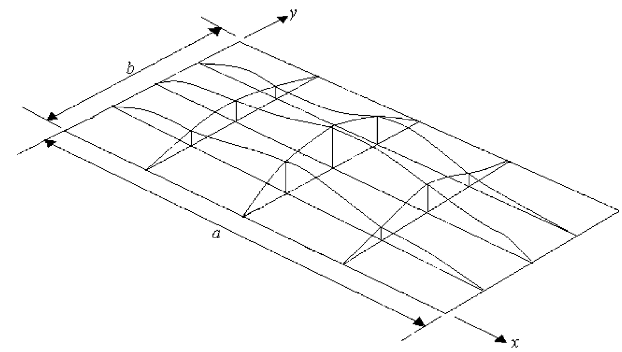

Figure 2. Initial deflected shape of plating between the stiffeners.

deflections of stiffened panel because of welding have been considered while uncertainty in the welding induced residual stress has been ignored.

The IACS CSR method (IACS CSR, 2006) is a simplified incremental-iterative method to evaluate hull girder ultimate strength using analytically derived stress–strain curves for stiffened plate element, hard corners, and plate elements. Classification societies will be using this method to check hull girder ultimate strength. This method also ignores the effect of welding residual stresses.

In a large majority of the progressive collapse analysis methods listed above and in IACS CSR, the whole ship hull girder is assumed to be an assembly of combined stiffener-attached plating structural member. The calculated effective width of attached plating differs among the individual author(s). The effective width of plating between stiffeners is very important as the sectional properties of stiffener-attached plating depend mostly on this parameter. Along with the initial imperfections, the effective width formulation should consider the effect of applied compressive load on the plating. We have proposed an analytical formulation (Vhanmane and Bhattacharya 2007) for average stress-average strain relationship of plating between stiffeners combining the theory of elastic large

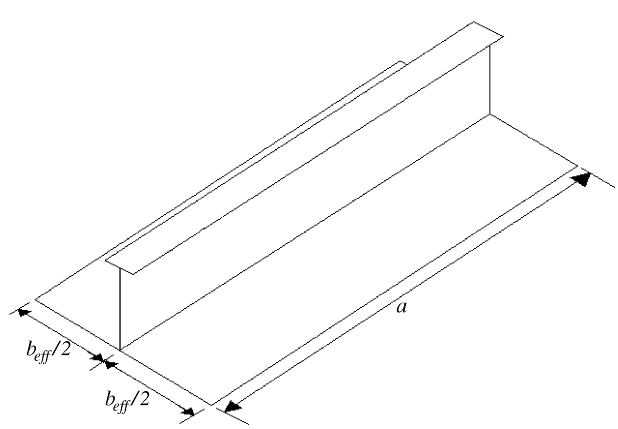

Figure 1. Stiffened plate element, combination of stiffener and attached plating.

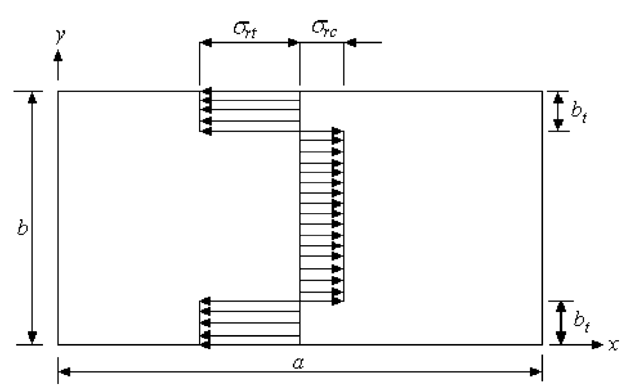

Figure 3. Idealized welding residual stress distribution of plating between stiffeners.

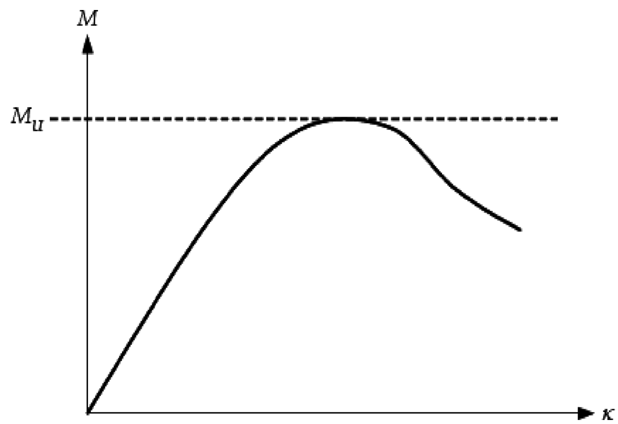

Figure 4. Typical Moment-Curvature Relationship.

deformation and the theory of rigid plastic mechanism. The stress–strain curve considers the effect of applied axial load, initial deflection, and welding residual stresses in the plating.

In the present study, an attempt to improve the IACS CSR method is made by incorporating initial imperfections: both initial deflection and welding induced residual stresses in the plating between stiffeners. The formulation developed in Vhanmane and Bhattacharya (2007) is applied to obtain the effective width of the attached plating. The attached plating and stiffener form the stiffened plate structural member (Figure 1) in the evaluation of the ultimate strength of the hull girder. The stress–strain curves are derived for stiffener-attached plating combined structural element under axial load. The longitudinal bending of the hull girder is applied incrementally and the induced axial force in each structural member is determined at its centroid using calculated stress–strain curve. The new position of the neutral axis is found iteratively. The bending moment is calculated by adding force contribution from all structural elements. The peak value of moment on the moment-curvature curve is the ultimate strength of the ship hull girder. The proposed method is demonstrated on a double hull VLCC and a capesize bulk carrier.

2. Effective width of attached plating

The average stress–strain curve for plating between stiffeners under the effects of applied axial load and initial imperfections has been developed in our previous work (Vhanmane and Bhattacharya 2007). There we combined two different methods that are as follows: (i) the membrane stress method by Paik et al. (2000) involving large elastic deformation theory and (ii) the rigid plastic collapse mechanism theory by Yao and Nikolov (1991). The former governs the stress–strain relationship up to ultimate strength while the latter is used beyond ultimate strength. The plating between stiffeners is analysed under axial load for different aspect ratios. This formulation is used to obtain effective width, b_{eff}, of the attached plating in the present study by the following formula:

$$\frac{b_{\text{eff}}}{b} = \frac{\sigma_{\text{xav}} + \sigma_r}{\sigma_{\max}} \tag{1}$$

where b = breadth of plating between longitudinal stiffeners, σ_{r} = welding induced residual stress in the plating, σ_{xav} = average value of the applied axial compressive stress, $\sigma_{\max}$ = maximum value of axial compressive stresses (equal to yield strength of the plate material).

The maximum compressive membrane stresses for plating with welding residual stresses are determined at $y = b_t$ or $y = b - b_t$ (see Eq. 5 for b_{t}) as

$$\sigma_{\max} = \sigma_{\text{xav}} + \sigma_{\text{r}} - \frac{m^2\pi^2 E}{8a_2}\left(A^2 - A_0^2\right)\cos\frac{2\pi b_{\text{t}}}{b} \tag{2}$$

Table 1. Stress strain relationship of stiffened plate under compression.

Failure mode	Description	Stress–strain relationship formula, N/mm^2
01	Beam-column buckling	$\sigma = \Phi\sigma_{C1}(\frac{A_s+b_{\text{eff}}\times t_p}{A_s+b\times t_p})$
02	Torsional buckling	$\sigma = \Phi(\frac{A_s\times\sigma_{C2}+b\times t_p\times\sigma_{CP}}{A_s+b\times t_p})$
03	Web local buckling of flat bars	$\sigma = \Phi\sigma_y(\frac{b_{\text{eff}}\times t_p+d_w\times t_w+b_f\times t_f}{b\times t_p+d_w\times t_w+b_f\times t_f})$
04	Web local buckling of flanged profiles	$\sigma = \Phi(\frac{A_s\times\sigma_{C4}+b\times t_p\times\sigma_{CP}}{A_s+b\times t_p})$

Parameter description

Φ = edge function = $\frac{\varepsilon}{\varepsilon_y}$	b =breadth of plating between longitudinal stiffeners (equal to stiffener spacing)
ε = element strain	t_p = attached plating thickness in mm
ε_y = strain at yield stress	σ_{CP} = ultimate strength of the attached plating
σ_{Ci} = critical stress in failure mode i (i = 1,2,3,4)	d_w = depth of Web
A_s = stiffener area without attached plating	t_w = Web thickness
b_{eff} = effective width of attached plating defined in equation 1	b_f = flange width
	t_f = flange thickness
	σ_y = element material yield stress

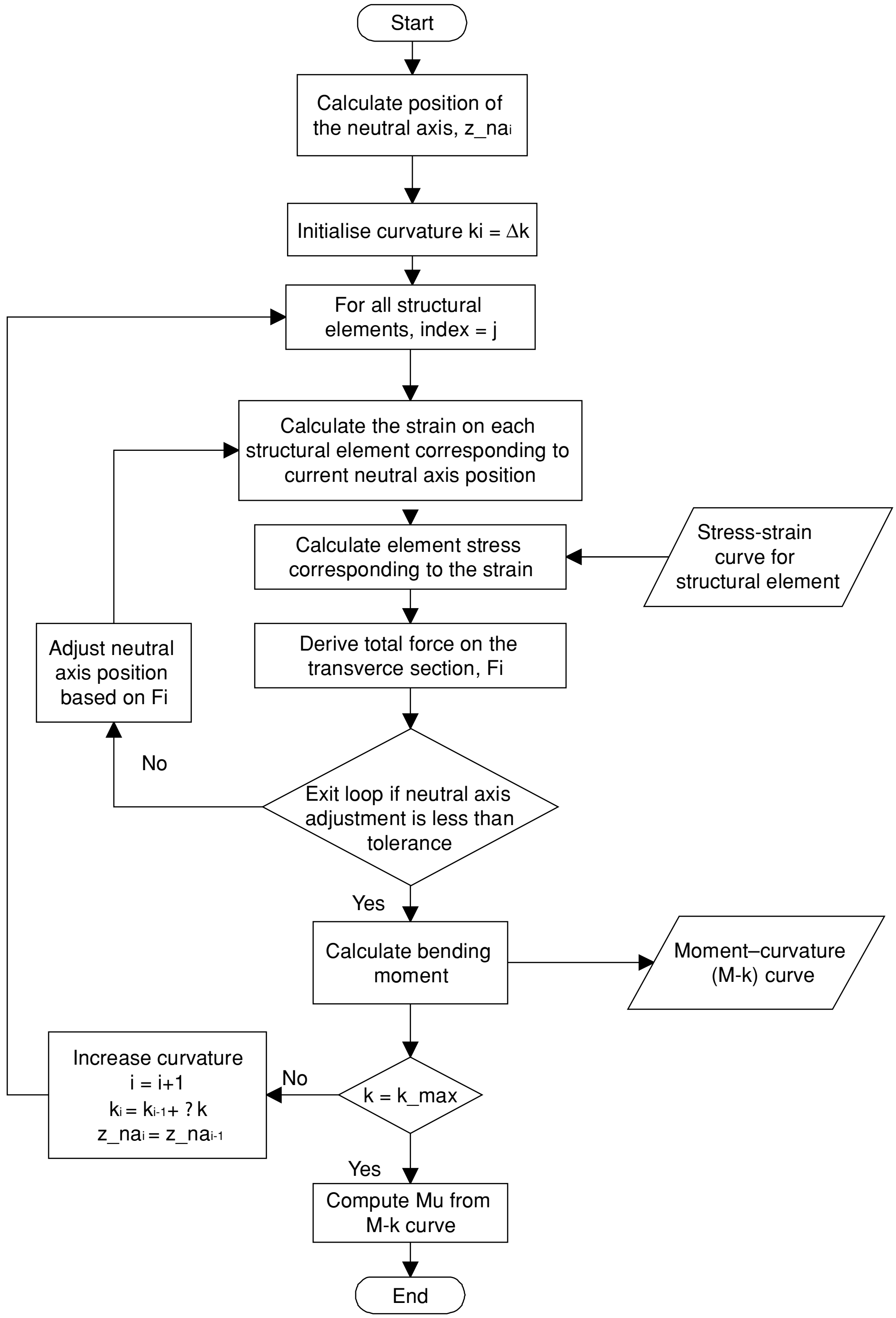

Figure 5. Flow chart of the simplified incremental-iterative method.

where A = unknown amplitude of the added deflection and A_0 = initial deflection amplitude. The effective width calculated by Vhanmane and Bhattacharya (2007) has been used in the present study to form combined stiffener-attached plating structural element, shown in Figure 1. Accordingly, the stress–strain relationships given by IACS CSR are modified for various modes of stiffener failure: beam-column buckling, torsional buckling of stiffeners, Web local buckling of flat bars, and Web local buckling of flanged profiles of the stiffeners.

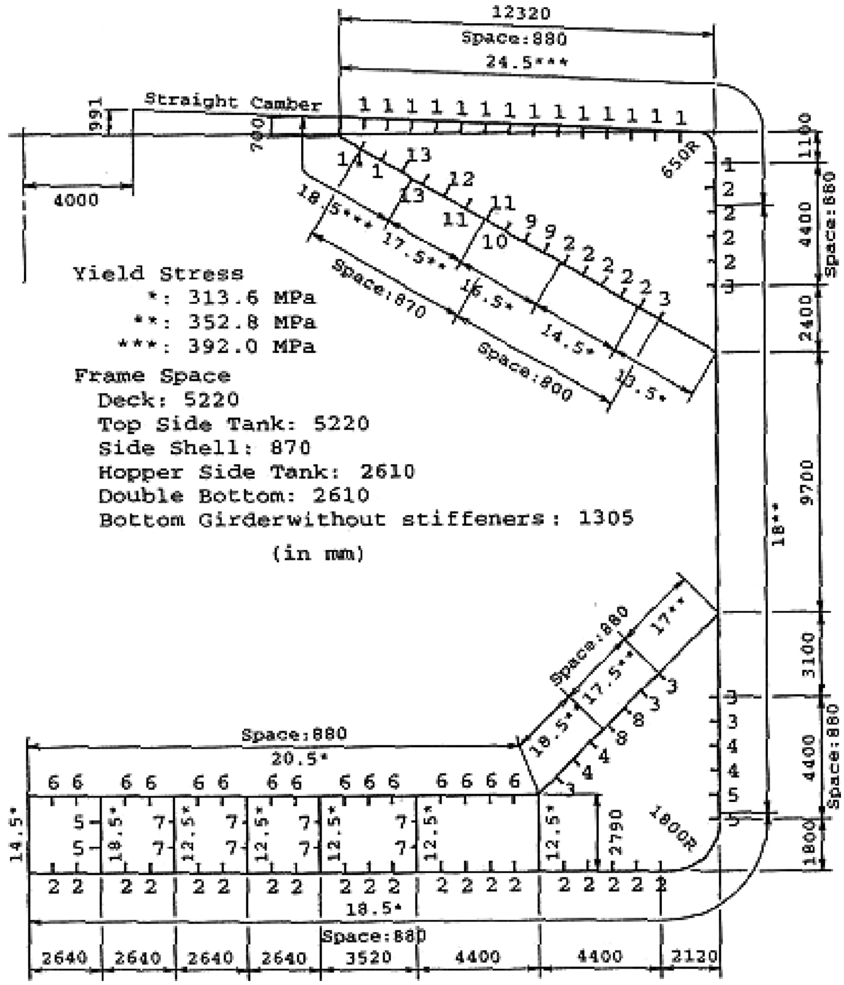

Figure 6. Midship section of bulk carrier.

Effect of initial deflection

Figure 2 shows the initial deflected shape of plating possibly because of imperfect manufacturing process. The plating between stiffeners is considered as simply supported at all edges. The initial deflection, w_0, of the plating is expressed as:

$$w_0 = A_0 \sin \frac{m\pi x}{a} \sin \frac{\pi y}{b} \tag{3}$$

where $A_0 =$ initial deflection amplitude, $m =$ buckling mode half wave number in the x direction.

Table 2. Ship particulars (ISSC, 2000).

Type of ship	LBP (m)	Beam Width (m)	Depth (m)	Block coeff.	DWT
Bulk carrier	285.00	50.00	26.70	0.826	170,000
Double hull VLCC	315.00	58.00	30.30	0.823	313,000

Similarly, the deflection due to axial compressive load is given by

$$w = A \sin \frac{m\pi x}{a} \sin \frac{\pi y}{b} \tag{4}$$

where $A =$ initial deflection amplitude.

Effect of residual stresses

In the idealized welding-induced residual stress distribution used in the present method, the tensile residual stresses, σ_{rt}, are developed at the edges of the plating, i.e. along the welding line; the residual compressive stresses, σ_{rc}, are developed in the middle of part of the plating. The breadth of the tensile residual stress zone is obtained by equilibrium condition, Figure 3, as follows:

$$2b_{\mathrm{t}} = \frac{\sigma_{\mathrm{rc}}}{\sigma_{\mathrm{rc}} - \sigma_{\mathrm{rt}}} b \tag{5}$$

The tensile residual stress may reach the yield stress but a somewhat reduced (80% of the yield stress) tensile residual

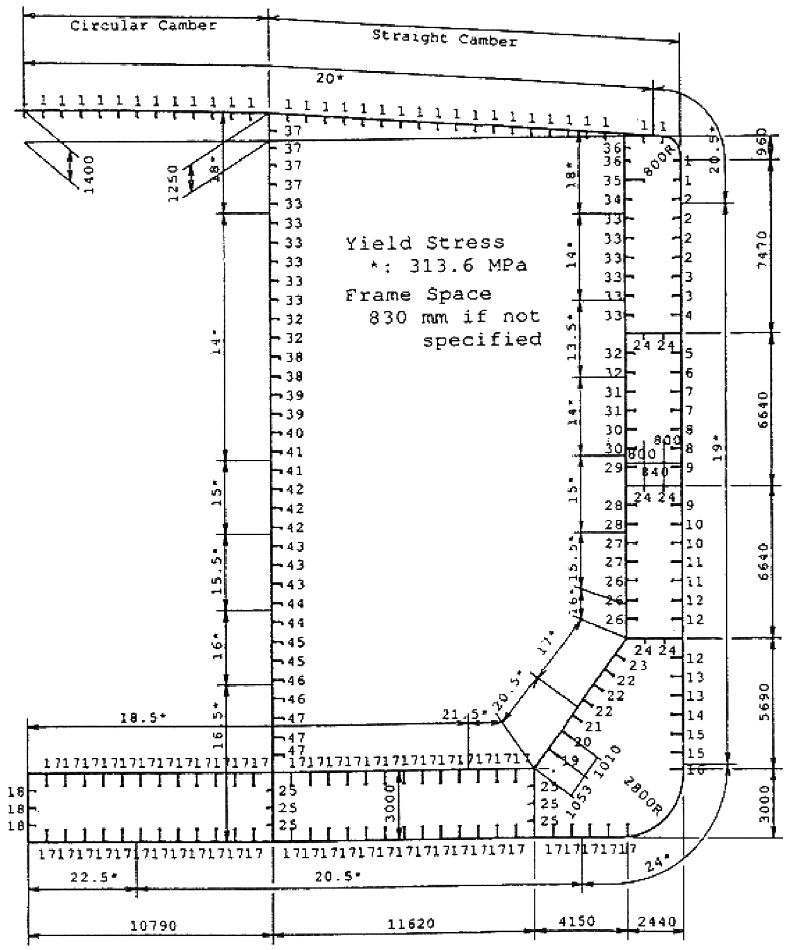

Figure 7. Midship section of double hull VLCC.

stress may be used. The magnitude of compressive residual stress is given in Table 3 for three different classes of imperfections. Hence, the residual stress distribution may be expressed as in Paik and Thayamballi (2003):

$$\begin{aligned} \sigma_r &= \sigma_{\mathrm{rt}} \quad \text{for } 0 \leqslant y < b_{\mathrm{t}} \\ \sigma_r &= \sigma_{\mathrm{rc}} \quad \text{for } b_{\mathrm{t}} \leqslant y < b - b_{\mathrm{t}} \\ \sigma_r &= \sigma_{\mathrm{rt}} \quad \text{for } b - b_{\mathrm{t}} \leqslant y \leqslant b \end{aligned} \tag{6}$$

3. Simplified incremental-iterative method

In this simplified method, the ultimate hull girder bending moment capacity (in hogging or sagging conditions), M_{u}, is defined as the peak value of the vertical bending moment, M, versus the curvature, κ, plot of the ship cross section as shown in Figure 4. This method simulates the progressive strength reduction of each structural element in the hull girder section and thus represents real collapse behaviour

Table 3. Comparison of M_{u} with ISSC (2000) benchmark calculations.

Midship section	Ultimate moment in 10^4 (MN-m)	Present*	Chen (ISUM)	Cho	Masaoka (ISUM)	Rigo	Soares	Yao (HULLST)
Bulk carrier	Sagging	1.54	1.53	1.44	1.68	1.50	1.37	1.58
	Hogging	1.90	1.87	1.85	1.89	1.91	1.74	1.77
Double hull VLCC	Sagging	2.55	2.38	2.21	2.68	2.06	1.98	2.12
	Hogging	2.96	2.82	2.92	3.08	2.89	2.76	2.92

*Without any initial imperfection.

Note: The initial imperfection considered by the authors of ISSC (2000) did not involve welding residual stress. However, they did consider a small initial deflection of $A_{\mathrm{i}} = 0.01 \times t$. Nevertheless, this initial deflection is negligible compared to even the "slight" imperfection level given in Table 4: since plate slenderness ration, β, for ship plating ranges from 2 to 4, this initial deflection is at least an order of magnitude smaller than the "slight" case. Hence, it is appropriate to assume that initial imperfection was not considered in the ISSC (2000) study.

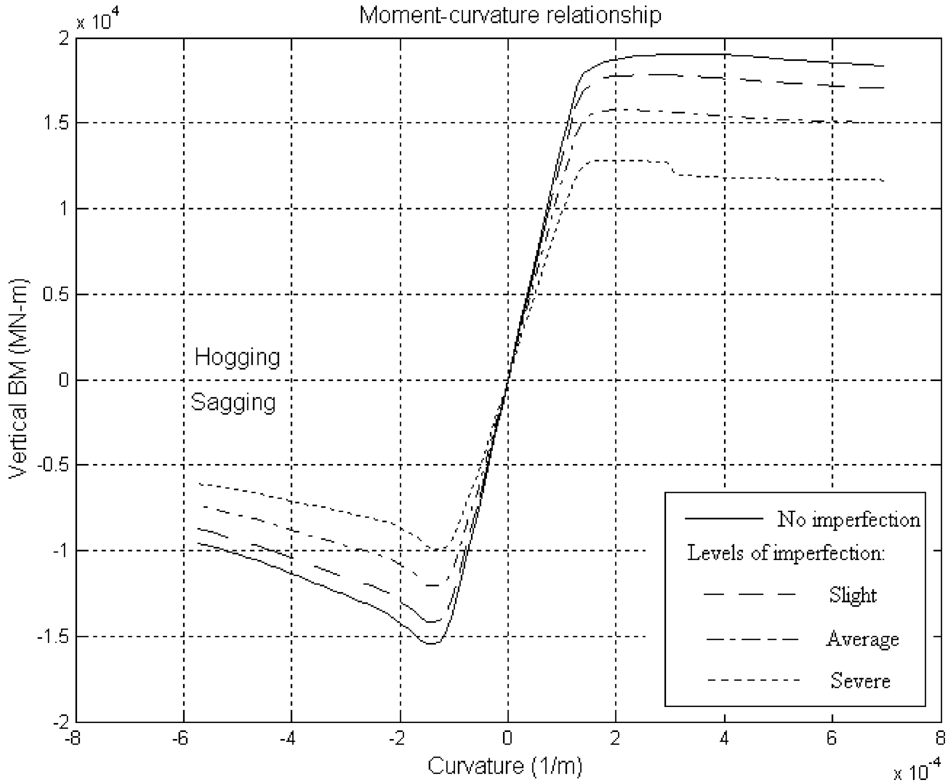

Figure 8. Effect of imperfection on moment-curvature relationship for bulk carrier.

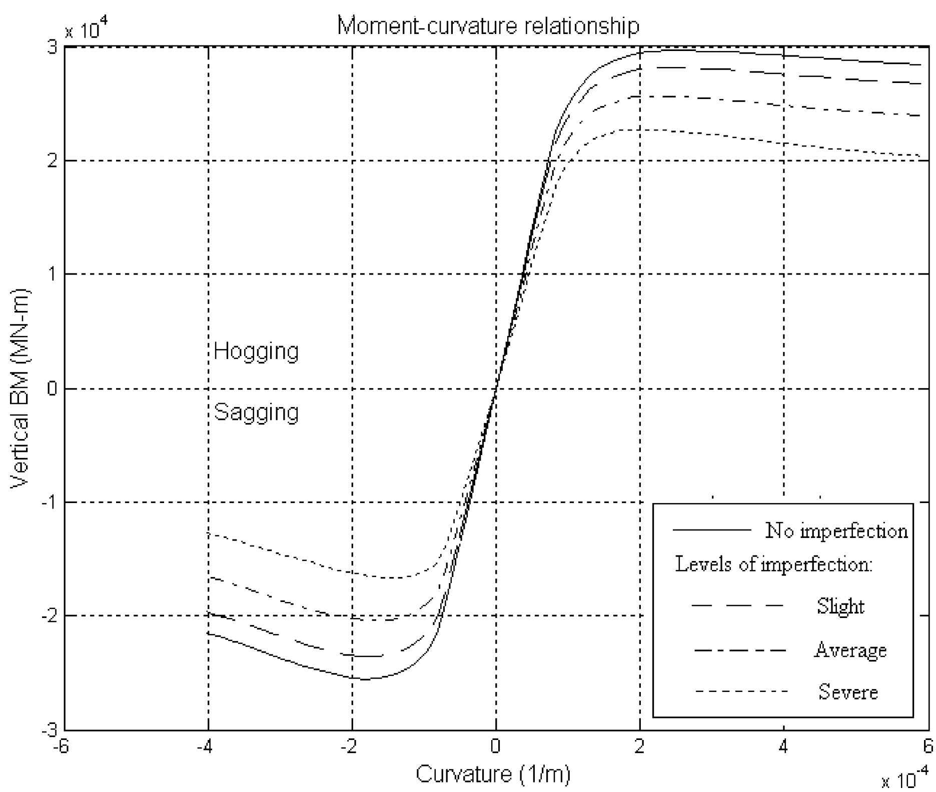

Figure 9. Effect of imperfection on moment-curvature relationship for VLCC.

Table 4. Initial imperfection levels (Paik and Thayamballi, 2003).

Level	Initial deflection (A_i)	Welding residual stress (σ_{rc})
Slight	$0.025\beta^2$t	$0.05\sigma_y$
Average	$0.1\beta^2$t	$0.15\sigma_y$
Severe	$0.3\beta^2$t	$0.3\sigma_y$

β = late slenderness parameter; $\beta = b/t(\sigma_y/E)^{1/2}$, σ_y = material yield stress, t = thickness of attached plating.

of the ship hull girder. The possible modes of failure of stiffened plate in interframe collapse mode are beam-column buckling, torsional buckling of stiffeners, Web local buckling of flat bars, and Web local buckling of flanged profiles of the stiffeners. The minimum stress among these failure modes is assumed as the governing stress and the same is considered to calculate element force. The theoretical formulations of stress strain relationship of stiffened plate under compression are described in Table 1.

The curve *M*-κ is obtained by means of an incremental-iterative approach where curvature is applied incrementally and the adjustment for the instantaneous neutral axis is achieved iteratively. This method is illustrated in the flow chart in Figure 5. The bending moment, M, that acts on the hull girder section due to the imposed curvature κ is calculated for each step of the incremental procedure. This incrementally imposed curvature corresponds to an incremental angle of rotation of the hull girder transverse section about its effective horizontal neutral axis. Because of this, an increment in the axial strain, ε, is induced in each structural element in the section. In the sagging condition, the structural elements below the neutral axis are elongated, whereas elements above the neutral axis are shortened; and vice versa in hogging bending condition.

The stress, σ induced in each structural element by the strain, ε, is obtained from the average stress–strain relationship, σ-ε of the element in the nonlinear elastoplastic domain. The force in each structural element is obtained from its area times the stress at its centroid and these forces are summed to derive the total axial force on the transverse section.

The element area is taken as the gross cross-sectional area of the structural element. At the first iteration, this total force will not be zero because the effective neutral axis moves due to the nonlinear response. Hence, an adjustment in the neutral axis position is made by recalculating the element strains, forces, and total sectional force. The adjustment in the neutral axis position is continued until the total force on the section is equal or less than a prescribed tolerance.

Once the position of the new neutral axis is known, then the correct stress distribution in the structural elements can be obtained. The bending moment, M, about the new neutral axis because of the imposed curvature, κ, is then obtained by summing the moment contribution given by the force in each structural element.

4. Examples

Two ship structures —one bulk carrier (Figure 6) and one double hull tanker (Figure 7) —taken from the benchmark study by the special task committee of International Ship and Offshore Structures Congress (ISSC, 2000) are analysed in this section. The object ship structures considered in the present study are identical to those considered in Paik and Thayamballi (2003) and Harada and Shigemi (2007) in terms of structural scantlings, material, and structural arrangements. The main particulars of the ship are given in Table 2.

The bulk carrier transverse frame spacing for deck and topside tank is 5.22 m. Its side shell extends 0.87 m in the longitudinal direction. The structural elements in the double bottom and hopper side tank extend 2.61 m longitudinally. The analytical model of bulk carrier is discretised into 188 longitudinal and 30 hard corner elements. It is assumed that hard corner elements will follow elastic-perfectly plastic paths in both tension and compression with the same absolute value of the yield strength.

Table 5. Ultimate hull girder strength.

		Ultimate strength (M_u)$\times 10^4$ (MN-m)						
		Harada and Shigemi (2007)	Paik and Thayamballi (2003)			Present		
Midship section	Condition	(1)*	(1)	(2)	(3)	(1)	(2)	(3)
Bulk carrier	Sagging	1.55	1.65	1.55	1.25	1.41	1.20	1.0
	Hogging	2.04	1.46	1.24	0.98	1.78	1.57	1.28
Double hull VLCC	Sagging	2.24	2.05	1.91	1.45	2.35	2.03	1.66
	Hogging	3.38	2.47	2.35	1.90	2.81	2.56	2.27

Imperfection level: (1) slight, (2) average, (3) severe.
*Without welding residual stresses.

The transverse frame spacing of double hull VLCC is 4.95 m uniformly throughout the length. The analytical model of the section is discretised into 396 longitudinal and 30 hard corner elements. The hard corners are treated as mentioned above. The simplified incremental-iterative analysis described above is performed to determine the hull girder ultimate moment of the bulk carrier and double hull VLCC.

Table 3 shows the comparison between the present method and the six different methods listed in ISSC (2000) for evaluating the hull girder ultimate strength of these two ships without any initial imperfection (please see note below Table 3). The results are scattered due to differences among the approaches to estimating the ultimate strength, the assumed stiffened plate collapse modes, the effective width of attached plating, treatment given to the initial imperfections and hard corners in the section, rotational restraints applied to the plating, etc. The different methods give somewhat different buckling strengths of the deck and bottom in sagging and hogging. The ultimate strength estimated by Soares seems to be undervalued because the effect of hard corners in the structure is not considered. According to ISSC (2000) benchmark calculations for stiffened plate element characteristics, it is found that Soares's method does not consider tripping of stiffeners and reduction in capacity after ultimate strength. Otherwise there is not so large a difference in the ultimate hull girder strength and the present results are within the scatter range.

We now include initial imperfection in the analysis. Three different levels of initial deflection and stress pair are chosen on the basis of Paik and Thayamballi (2003) and are mentioned in Table 4.

Figure 8 shows the effect of initial imperfection on bulk carrier moment-curvature relationship. In case of double hull VLCC, the effect of initial imperfections on moment-curvature curve with different levels of imperfection are shown in Figure 9. The estimated hull girder ultimate strength of the two ships using the proposed method are shown in Table 5 and are compared with those estimated by Harada and Shigemi (2007) and Paik and Thayamballi (2003). In the former, the authors applied FEA, whereas Paik and Thayamballi (2003) used ISUM elements like plate units, beam-column units, and stiffened panel units to obtain the ultimate strength. As expected, the ultimate strength (both in hogging and sagging condition) decreases with increase in the level of imperfection. As the basic method used by each author to estimate ultimate hull girder strength is different, the results do not match exactly. Paik and Thayamballi (2003) did not consider any hard corners in the structure, whereas Harada and Shigemi (2007) used the four-noded shell elements in their FEA model where, apparently, the failure mode of stiffened plate and its stress–strain relationship cannot be identified. Harada and Shigemi (2007) did not consider any welding residual stresses either.

5. Conclusions

We have proposed an improvement in the IACS CSR (2006) in predicting the ultimate ship hull girder strength by incorporating two important factors that are as follows: (i) initial deflection and (ii) welding induced residual stresses in plating between stiffeners. The proposed method is analytical-based as it uses the progressive collapse method in conjunction with the calculated average stress–strain relationship for stiffened plate behaviour. The influence of the initial imperfections is introduced in the model through the effective width of the attached plating in stiffener-plating combined element. Furthermore, the stress–strain relationship for the above element is derived for each relevant failure mode (beam-column buckling, torsional buckling of stiffeners, Web local buckling of flat bars, and Web local buckling of flanged profiles of the stiffeners).

A double hull VLCC and a capesize bulk carrier — taken from existing benchmark studies — are analysed to obtain hull girder ultimate strength under three levels of imperfection. As expected, the ultimate hull girder strength reduces with higher level of imperfection; for severe imperfections the reduction can be as much as 35%. The present results are compared with available published results that use two different methods (ISUM and FEA), and it is clear that the proposed method is able to estimate hull girder ultimate strength accounting effects of imperfections.

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