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Ultimate Strength Analysis of Ship Hull Girder Under Random Material and Geometric Properties

The ultimate strength of a ship's hull depends on its material and geometric properties, some or all of which may be random in nature. In addition, initial imperfections in the form of initial deflection and residual welding stresses in plating between stiffeners can significantly affect the hull ultimate strength. In this paper, the effect of randomness in yield strength and in the initial imperfections on ultimate hull girder strength is determined. Different levels of statistical dependence between yield strength and initial imperfection of stiffeners and plating between stiffeners have been considered. The methodology is applied on a bulk carrier and a VLCC tanker. [DOI: 10.1115/1.4002738]

1 Introduction

A ship hull girder is a complex assembly of unstiffened/stiffened plates, frames, etc. and is subjected to longitudinal bending, transverse bending, and torsion. The ultimate hull girder strength is the maximum bending capacity that a ship hull girder can sustain under longitudinal bending. For structural reliability analysis (SRA), the uncertainties in ultimate hull girder strength must be estimated accurately. Due to the growing concern for safety of ships, lives at sea, and the marine environment, the International Association of Classification Societies (IACS) is considering a more refined set of rules.

The deterministic estimation of ultimate hull girder strength under longitudinal bending started with the so-called direct method, e.g., Refs. [1–3]. The progressive collapse analysis of hull girders can be found in Refs. [4–6]. The above approaches do not typically consider the effect of initial imperfections, which include initial deflections and the residual welding stresses for the plating between two longitudinals. These imperfections occur due to poor workmanship, improper handling of cargo during loading and unloading, improper use of grab, slamming, etc. Guedes Soares [7] reviewed various design equations for the design of ship plates under the compressive load considering the effect of plate slenderness, initial distortions, residual stresses, and boundary conditions. A new design equation was proposed, which included all the variables, and also the uncertainty associated with the use of this equation was quantified. Further, an approximate method was proposed by Gordo and Guedes Soares [8] for load shortening curves of stiffened plates, accounting the plate and stiffener distortions and residual stresses. A similar study [9] also estimates the ultimate strength and effective width of attached plating considering plate initial deflection and residual stresses. The assessment of the ultimate strength of the ship hull girder [10] uses the approximate methods for load shortening curves given in Ref. [8]. The present authors [11] analytically studied the effect of such initial imperfections on the ultimate hull girder strength analyzing a bulk carrier and a VLCC tanker. Bonello et al. [12] and Chryssanthopoulos [13] studied the effect of the random initial imperfections on the plates under axial compression. The references on the effects of initial imperfection on hull girder ultimate strength are very sparse. Kim [14] has estimated the ultimate strength of ten typical merchant ship hulls applying the idealized structural unit method (ISUM). Paik and Thayamballi [15] also applied the ISUM approach to obtain hull girder ultimate strength, considering the effect of initial deflection and welding residual stresses. Harada and Shigemi [16] have performed a series of nonlinear FEM analysis for a double hull VLCC and a cape size bulk carrier to obtain the ultimate longitudinal strength in hogging and sagging conditions; nevertheless, such analyses can be computationally demanding and may be prohibitive in many situations. The initial deflections of a stiffened panel due to welding have been considered in Ref. [16] while uncertainty in the welding induced residual stress has been ignored. The yield strength and member thickness were varied systematically from $\mu+3\sigma$ to $\mu-3\sigma$ in the FEA model.

Various available data on initial imperfections show that the initial deflections as well as residual welding stresses are random in nature. Moreover, they are likely to be correlated due to physical proximity, common material source, common welding practice, etc. The same holds for yield strength of different structural elements constituting the ship's hull. In this paper, we extend our previous work [11,17] to incorporate randomness (including correlation) in the initial imperfections in the ship hull plating between stiffeners and yield strength of stiffeners.

The results from this paper can lead to the realistic estimates of modeling uncertainty in the hull girder strength when randomness in the imperfection and the yield strength need to be taken into account. These additional uncertainties may be used to modify the design equation for new ships through appropriate partial factors. The proposed methodology is applied to a cape size bulk carrier and a double hull VLCC tanker that were used for the calibration of the hull girder longitudinal ultimate strength investigation in Ref. [18].

2 The Effect of Initial Imperfections

The effective width of attached plating, which along with the stiffener profile forms the longitudinal section, is the major element contributing to the ultimate hull girder strength. We first present the methodology for its determination; the effect of correlated random initial imperfections will be reflected in the random effective width of the attached plating.

Our previous work [17] developed the average stress-strain relationship for plating between stiffeners under the effects of applied axial load and initial imperfections, where two different methods: (i) the membrane stress method by Ref. [19] involving large elastic deformation theory and (ii) the rigid plastic collapse

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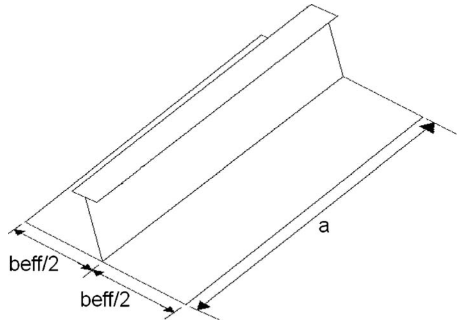

Fig. 1 Stiffened plate element, combination of stiffener and attached plating

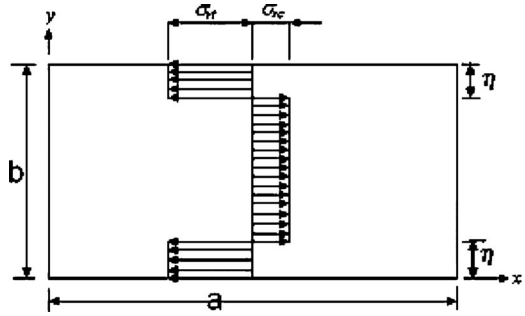

Fig. 3 Idealized welding residual stress distribution of plating between stiffeners

mechanism theory by Ref. [5] are combined. The former governs the stress-strain relationship up to ultimate strength while the latter is used beyond ultimate strength. The plating between stiffeners is analyzed under axial load and by varying the plate aspect ratios. This formulation is used to obtain the effective width b_{eff} of the attached plating of the stiffener in the present study by the following formula:

$$\frac{b_{\text{eff}}}{b} = \frac{\sigma_{\text{xav}} + \sigma_r}{\sigma_{\text{max}}} \tag{1}$$

where b is the breadth of plating between longitudinal stiffeners, σ_r is the welding induced residual stress in the plating, σ_{xav} is the average value of the applied axial compressive stress, and σ_{max} is the maximum value of axial compressive stresses (equal to yield strength of the plate material). The maximum compressive membrane stresses for plating with welding residual stresses are determined at $y=\eta$ or $y=b-\eta$ (shown in Fig. 3) as

$$\sigma_{\text{max}} = \sigma_{\text{xav}} + \sigma_r - \frac{m^2\pi^2 E}{8a_2}(A^2 - A_0^2)\cos\frac{2\pi\eta}{b} \tag{2}$$

where A is the unknown amplitude of the added deflection and A_0 is the initial deflection amplitude. The method to calculate the effective width given by Ref. [17] has been used in the present study to form a combined stiffener-attached plating structural element, as shown in Fig. 1. Accordingly, the stress-strain relationships given by IACS CSR are modified for various modes of stiffener failure: beam-column buckling, torsional buckling of stiffeners, web local buckling of flat bars, and web local buckling of flanged profiles of the stiffeners. The two aspects of initial imperfections—initial deflection and welding residual stresses—are described next.

2.1 Initial Deflection Amplitude. Figure 2 shows the initial deflected shape of plating possibly due to an imperfect manufacturing process. The plating between stiffeners is considered as simply supported at all edges. The initial deflection w_0 of the plating is expressed as

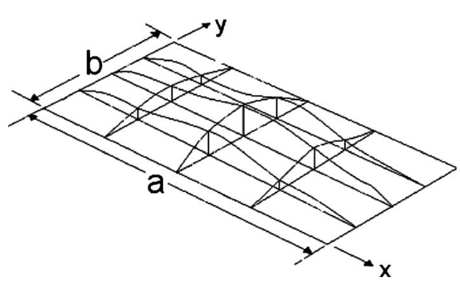

Fig. 2 Initial deflected shape of plating between the stiffeners

$$w_0 = A_0 \sin\frac{m\pi x}{a}\sin\frac{\pi y}{b} \tag{3}$$

where A_0 is defined in Eq. (2) and m is the buckling mode half-wave number in the x direction, $a/b \le \sqrt{m(m+1)}$. For relatively thin plates (slenderness ratio, $\beta > 1.9$), the lowest bifurcation mode plays a dominant role. In case of thick plates (slenderness ration, $\beta < 1.9$), the buckling mode deformation may not appear before the plate collapses due to elastoplastic response before the inception of an "original" buckling wave mode that normally takes place in the elastic regime. In this case, it is approximated as one half-wave mode, $m=1$.

Chryssanthopoulos [13] proposed a normal probability model for A_0 normalized by b

$$A_0/b \sim \text{normal}(\mu, \sigma) \tag{4}$$

with mean μ given by

$$E[A_0/b] = 0.12b\sigma_0/(tE) \tag{5}$$

and the coefficient of variation (V, equal to the standard deviation divided by the mean) given by

$$V[A_0/b] = 0.675 - 0.004b/t \tag{6}$$

The initial deflection amplitude, A_0, of the hull platings is assumed to be jointly normal.

2.2 Welding Residual Stresses. In the idealized welding induced residual stress distribution used in the present method, the tensile residual stresses σ_{rt} are developed at the edges of the plating, i.e., along the welding line; the residual compressive stresses σ_{rc} are developed in the middle of part of the plating, as shown in Fig. 3. The breadth of the tensile residual stress may reach the yield stress but a somewhat reduced 80% of the yield stress tensile residual stress may be used. Hence, the residual stress distribution may be expressed as [15]

$$\sigma_r = \sigma_{rt} \quad \text{for} \quad 0 \le y < \eta$$

$$\sigma_r = \sigma_{rc} \quad \text{for} \quad \eta \le y < b - \eta$$

$$\sigma_r = \sigma_{rt} \quad \text{for} \quad b - \eta \le y \le b \tag{7}$$

Chryssanthopoulos [13] proposed the following model of uncertainty in compressive residual stress:

$$\frac{\sigma_r}{\sigma_y} = \frac{2\eta}{\left[\frac{b}{t} - 2\eta\right]} \tag{8}$$

where σ_r is the residual stress, σ_y is the material yield strength, and η is the nondimensional width of tensile stress block equal to (as shown in Fig. 3) having a normal distribution with mean

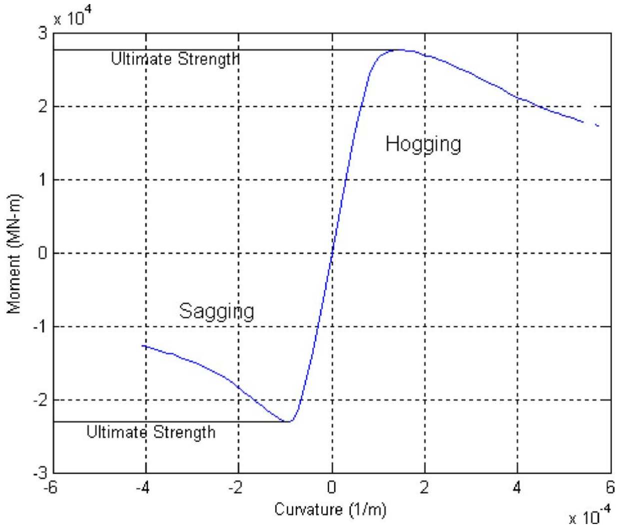

Fig. 4 Typical moment-curvature relationship of ship hull girder

$$E[\eta] = 1.20 + 0.06\frac{b}{t} \tag{9}$$

and a standard deviation given by

$$\sigma_{\Delta\eta} = 0.04\frac{b}{t} \tag{10}$$

3 Simplified Incremental-Iterative Method

The ultimate hull girder bending moment capacity (in hogging or sagging conditions) M_u, is defined as the peak value of the vertical bending moment M, versus the curvature κ, as shown in Fig. 4. The simplified incremental-iterative method [20] simulates the progressive strength reduction of each structural element (stiffeners, unstiffened/transversely stiffened plate panels, and hard corners) in the hull girder section and thus represents the real collapse behavior of the ship hull girder. The possible modes of failure of stiffened plate in interframe collapse mode are beam-column buckling, torsional buckling of stiffeners, web local buckling of flat bars, and web local buckling of flanged profiles of the stiffeners. The minimum stress among these failure modes is assumed as the governing stress and the same is considered to calculate element force. The theoretical formulations of the stress-strain relationship of stiffened plate under compression are described in Table 1.

The curve $M-\kappa$ is obtained by means of an incremental-iterative approach where curvature is applied incrementally and the adjustment for the instantaneous neutral axis is achieved iteratively. The bending moment M, which acts on the hull girder section due to the imposed curvature κ, is calculated for each step of the incremental procedure. This incrementally imposed curvature corresponds to an incremental angle of rotation of the hull girder transverse section about its effective horizontal neutral axis. Due to this, an increment in the axial strain ε is induced in each structural element in the section. In the sagging condition, the structural elements below the neutral axis are elongated, while elements above the neutral axis are compressed and vice versa in the hogging bending condition. The stress σ induced in each structural element by the strain ε is obtained from the average stress-strain relationship $\sigma-\varepsilon$ of the element in the nonlinear elastoplastic domain. The force in each structural element is obtained from its area times the stress at its centroid and these forces are summed to derive the total axial force on the transverse section.

The element area is taken as the gross cross-sectional area of the structural element. At the first iteration, this total force will not be zero as the effective neutral axis moves due to the nonlinear response. Hence, an adjustment in the neutral axis position is made by recalculating the element strains, forces, and total sectional force. The adjustment in the neutral axis position is continued until the total force on the section is equal or less than a prescribed tolerance.

Once the position of the new neutral axis is known, then the stress distribution in the structural elements can be obtained. The bending moment M about the new neutral axis due to the imposed curvature κ is then obtained by summing the moment contribution given by the force in each structural element.

Table 1 Stress-strain relationship of stiffened plate under compression

Failure mode	Description	Stress-strain relationship formula (N/mm^2)
01	Beam-column buckling	$\sigma = \Phi\sigma_{C1}\left(\frac{A_s + b_{\text{eff}} \times t_p}{A_s + b \times t_p}\right)$
02	Torsional buckling	$\sigma = \Phi\left(\frac{A_s \times \sigma_{C2} + b \times t_p \times \sigma_{CP}}{A_s + b \times t_p}\right)$
03	Web local buckling of flat bars	$\sigma = \Phi\sigma_y\left(\frac{b_{\text{eff}} \times t_p + d_w \times t_w + b_f \times t_f}{b \times t_p + d_w \times t_w + b_f \times t_f}\right)$
04	Web local buckling of flanged profiles	$\sigma = \Phi\left(\frac{A_s \times \sigma_{C4} + b \times t_p \times \sigma_{CP}}{A_s + b \times t_p}\right)$

Parameter description

Φ=edge function $= \frac{\varepsilon}{\varepsilon_y}$
ε=element strain
ε_y=strain at yield stress
σ_{Ci}=critical stress in failure mode i ($i=1,2,3,4$)
A_s=stiffener area without attached plating
b_{eff}=effective width of attached plating defined in Eq. (1)

b=breadth of plating between longitudinal stiffeners (equal to stiffener spacing)
t_p=attached plating thickness in mm
σ_{CP}=ultimate strength of the attached plating
d_w=depth of web
t_w=web thickness
b_f=flange width
t_f=flange thickness
σ_y=element material yield stress

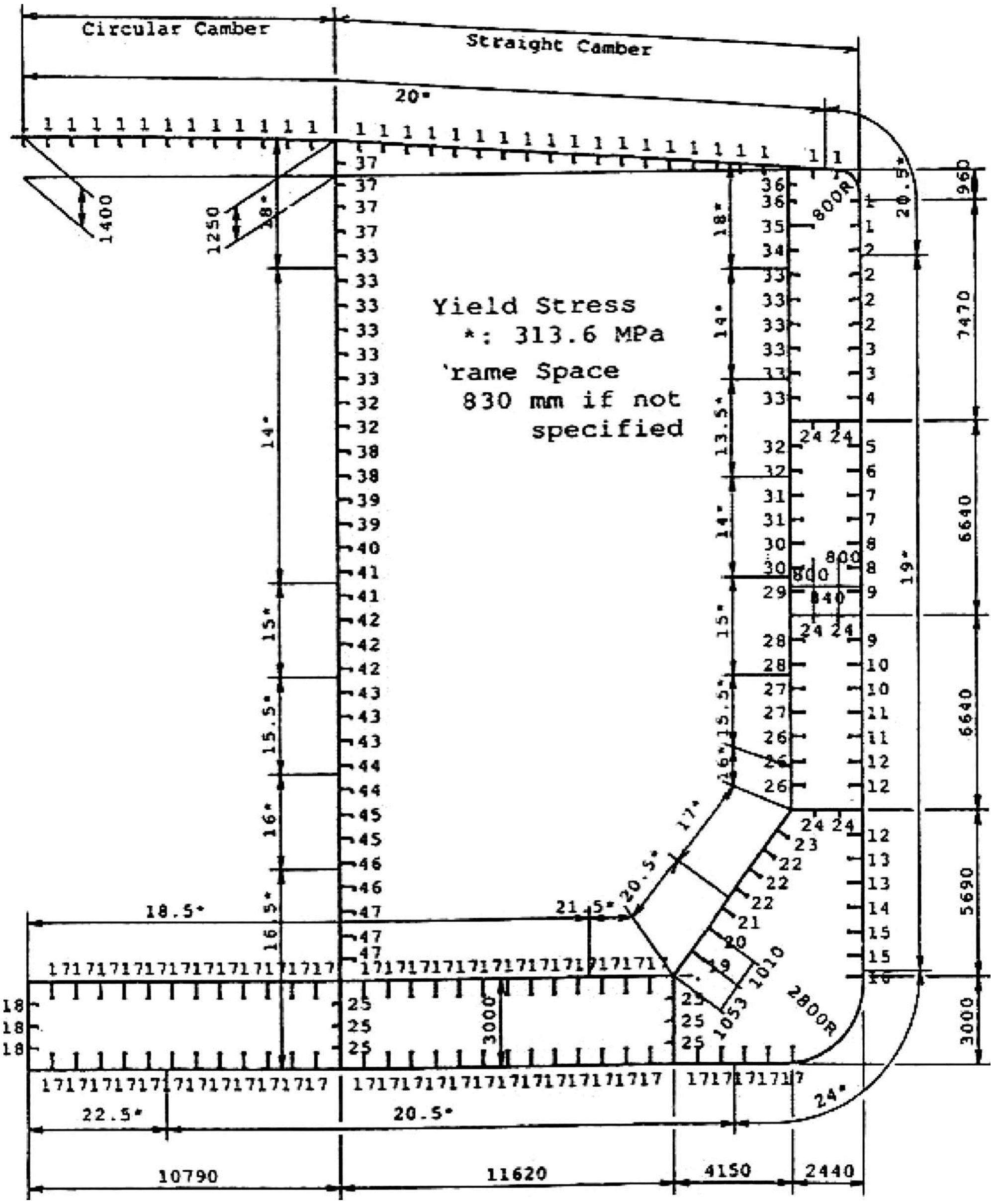

Fig. 5 Midship section of tanker

4 Numerical Examples

The above methodology is applied to two ship structures—one double hull tanker (Fig. 5) and one bulk carrier (Fig. 6)—taken from the benchmark study by the special task committee of Ref. [18]. The simplified incremental-iterative method has been used to evaluate the ultimate strength of the ship hull. For each ship, 1000 samples of ultimate strength are generated through Monte Carlo simulations. The dimensions of the two ships considered in the present work are given in Table 2. Figure 5 shows the midship section of the tanker and Fig. 6 shows the bulk carrier midship section. The nominal ultimate strength values for the above mentioned ship structures are given in Table 3. The nominal ultimate strength is determined, ignoring the effect of initial imperfections (both initial deflection and welding residual stress).

4.1 Correlated Random Material Yield Strength (Alone). The hull girder ultimate strength is rather sensitive to the variation in the material yield stress. The material yield strength of each structural member (stiffener, plate panels, or hard corner) in the midship section of the hull girder is considered to be random.

According to the existing literature on the statistics of yield strength of structural steel used in the marine industry, the mean strength bias (or B=mean/nominal) is between 1.00 and 1.15 (Table 4). The coefficient of variation (cov) (or V=standard deviation/mean) varies between 6% and 10%.

In this paper, we assume that the material yield strength is lognormal with cov 6% and mean value of 1.1 times the nominal value. The specified nominal values of the material properties used are described in Ref. [18]. In the bulk carrier section, the deck plating, deck longitudinals, and a sheer strake have nominal yield strength of 392 $\mathrm{N/mm^2}$ and members between sheer strake and inner bottom are built with 352.8 $\mathrm{N/mm^2}$ yield strength. The bottom structure yield strength is 313.6 $\mathrm{N/mm^2}$. All structural members in the tanker section are 313.6 $\mathrm{N/mm^2}$.

The yield strengths of all stiffeners in the section have been assumed to have some degree of statistical dependence due to physical proximity, common material source, and common welding practice: A set of correlation coefficients (0.0, 0.2, 0.5, 0.8, and 1.0), for the purpose of parametric study, has been assumed

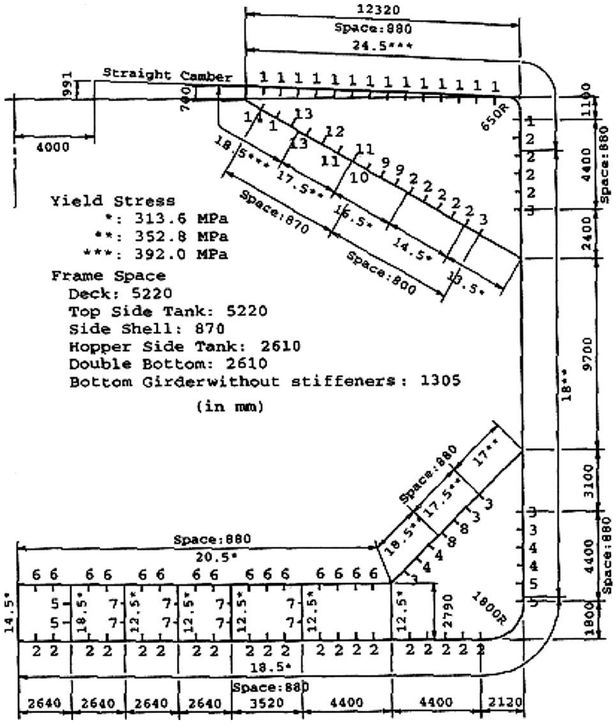

Fig. 6 Midship section of bulk carrier

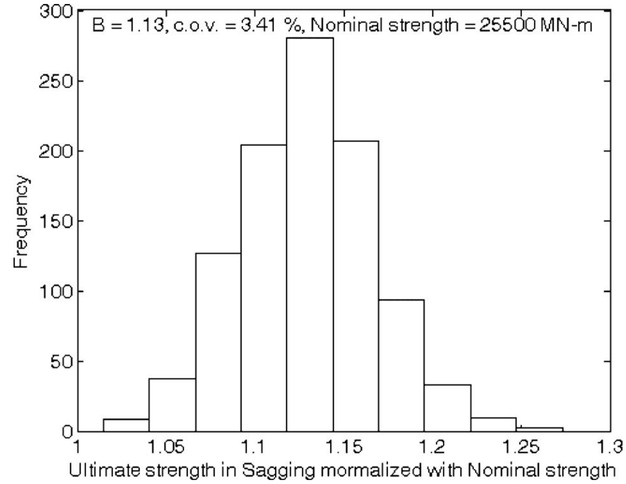

Fig. 7 Random yield strength (alone) for tanker-sagging (1000 cases, ρ=0.5)

between the yield strengths of any pair of stiffeners. Statistical dependence in yield strength is assumed to be absent between two different element groups—stiffeners, unstiffened plates, and hard corners. Further, yield strengths of the unstiffened plates are independent of each other as those of the hard corners.

The resultant histograms of ultimate strength in sagging and hogging conditions with only correlated (ρ=0.5) random yield strength are shown in Figs. 7–10. The bias and cov of each case are mentioned in the respective figure. The sample size in each case is 1000.

Table 5 lists the bias and cov of ultimate strength due to randomness in yield strength alone with different degrees of correlation among them (ρ of 0.0, 0.2, 0.5, 0.8, and 1.0) for the tanker and the bulk carrier in sagging and hogging conditions. As expected, the bias is higher than unity because yield strength bias considered in this study is 1.1. The cov is seen increasing with increasing values of correlation while the bias (and hence the mean) is insensitive to the degree of correlation.

4.2 Effect of Correlated Random Initial Imperfections (Alone). The statistics of initial deflection amplitude A_0 of the plating between stiffeners and residual welding stresses σ_r has been summarized in Sec. 2. Here, we study the effect of randomness only in A_0 and σ_r on the hull girder ultimate strength; yield strength of each element is assumed to be deterministic and equal to its nominal value. For any two different plates, A_0 values are assumed to have correlation coefficient ρ; the same correlation coefficient is also assigned to σ_r of those same two plates. Different values of the correlation coefficient (ρ=0.0, 0.2, 0.5, 0.8, and 1.0) have been adopted for the sensitivity study. A_0 and σ_r are taken to be independent of each other whether they belong to the same plate or not. Table 6 lists the bias and cov of M_u for the two ships in sagging and hogging conditions. It is observed that the cov is higher in the sagging condition than in hogging in each case. As expected, due to the effect of the initial imperfections, the bias of ultimate strength in each case is less than unity and is not affected by correlation (Table 3). The uncertainty, however, goes up with increasing ρ.

4.3 Combined Effect of Correlated Random Yield Strength and Correlated Random Initial Imperfections. The random aspects studied in the previous sections (Secs. 4.1 and

Table 2 Dimensions of ships considered

Ship type	Length (m)	Beam (m)	Depth (m)
Bulk carrier	285.00	50.00	26.70
VLCC tanker	315.00	58.00	30.30

Table 3 Nominal ultimate strength

	Condition	
Type	Sagging (MN m)	Hogging (MN m)
VLCC tanker	25,500	29,600
Bulk carrier	15,400	19,000

Table 4 Randomness in yield strength

Source	Distribution type	Bias, B	cov, V (%)
DNV-classification notes 30.6 [21]	Lognormal	1.1	6.0
Guedes Soares and Kmiecik [22]	Lognormal	1.0	10.0
Paik and Frieze [23]	Normal	1.0	10.0
Harada and Shigemi [16]	Lognormal	1.1	6.0
IACS CSR [20]	Lognormal	1.15	8.0 (normal) 6.0 (HSS)

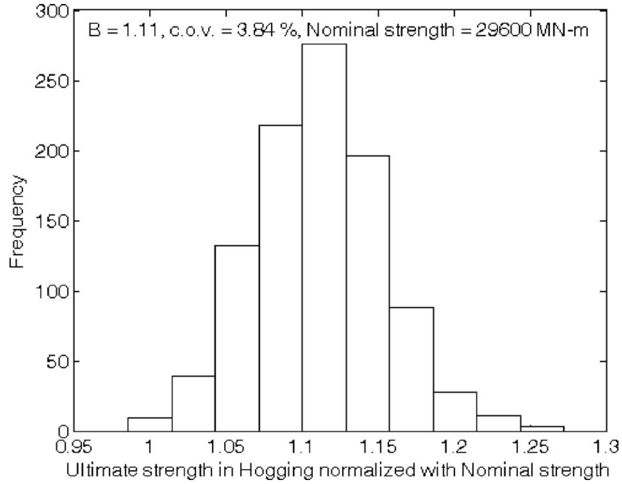

Fig. 8 Random yield strength (alone) for tanker-hogging (1000 cases, ρ=0.5)

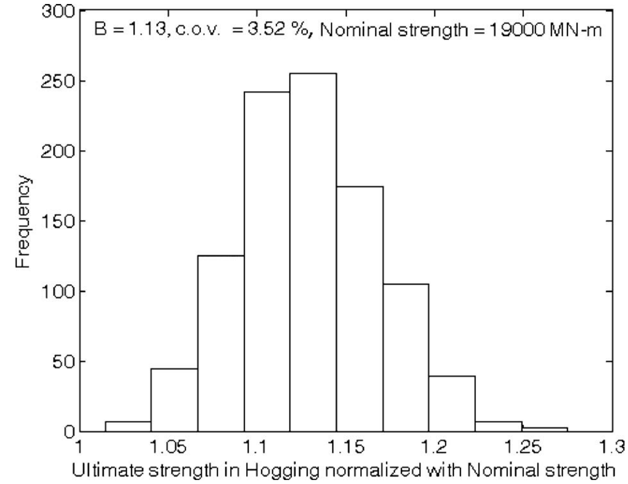

Fig. 10 Random yield strength (alone) for bulk carrier-hogging (1000 cases, ρ=0.5)

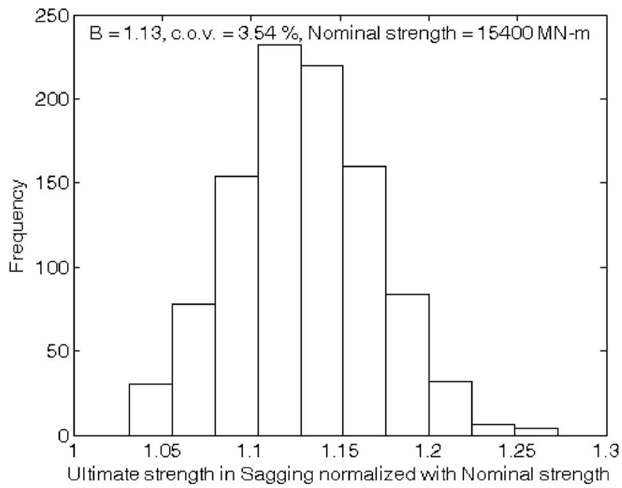

Fig. 9 Random yield strength (alone) for bulk carrier-sagging (1000 cases, ρ=0.5)

4.2) are now combined, and the results are listed in Table 7. As can be expected, the degree of statistical correlation does not have any effect on the bias factor, i.e., the mean value of ultimate strength but the cov of ultimate strength increases with the increase in correlation (Figs. 11 and 12). The bias factor at 0.89 is still lower than unity but is higher than 0.85 when uncertainty is considered only in initial imperfections. At any fixed value of ρ, the strength cov is lower when both uncertainties are considered than when uncertainties are considered only in yield or in initial imperfections.

5 Conclusion

In this work, the simplified incremental-iterative method given in IACS CSR to evaluate the ship's ultimate strength has been used to find the hull girder strength under random material (yield strength) and random geometric properties (initial deflections and residual stresses). The average stress-strain relationships for stiffened plate given in IACS CSR have been modified to incorporate the effect of random initial imperfections for plating between two stiffeners and random material yield stress of the structural members in the section. Different degrees of statistical dependence among yield strengths and initial imperfections of different structural elements have been studied. Two ship structures, a VLCC tanker and a cape size bulk carrier, have been analyzed and the statistics of their ultimate hull girder moment capacities have been obtained through Monte Carlo simulations. As may be expected, there is no effect of correlation on the mean value of ultimate strength but the uncertainty (measured by cov) increases significantly with higher correlation. Further, the cov of the hull girder strength is lower when both sources of uncertainties (yield strength and initial imperfections) are considered than when uncertainties are considered only in yield or in initial imperfections.

These results can be used for the probabilistic assessment of the hull girder ultimate strength in the early stages of its design for

Table 5 Effect of correlated random yield strength (alone)

Ship type		Tanker		Bulk carrier	
Condition		Sagging	Hogging	Sagging	Hogging
ρ=0.0	B	1.13	1.11	1.13	1.13
	V (%)	0.53	0.50	0.93	0.68
ρ=0.2	B	1.13	1.11	1.13	1.13
	V (%)	2.20	2.46	2.31	2.25
ρ=0.5	B	1.13	1.11	1.13	1.13
	V (%)	3.41	3.84	3.62	3.52
ρ=0.8	B	1.13	1.11	1.13	1.13
	V (%)	4.29	4.84	4.55	4.54
ρ=1.0	B	1.13	1.11	1.13	1.13
	V (%)	4.74	5.15	4.99	5.03

B=bias, V=cov, Nominal values (MN m): tanker (Sagg=25,500, Hogg=29,600) and bulk carrier (Sagg=15,400, Hogg =19,000).

Table 6 Effect of correlated random initial imperfections (alone)

Ship type		Tanker		Bulk carrier	
Condition		Sagging	Hogging	Sagging	Hogging
ρ=0.0	B	0.85	0.89	0.85	0.87
	V (%)	0.58	0.34	0.97	0.48
ρ=0.2	B	0.85	0.89	0.85	0.87
	V (%)	2.03	1.56	1.99	1.50
ρ=0.5	B	0.85	0.89	0.85	0.87
	V (%)	3.36	2.56	3.20	2.56
ρ=0.8	B	0.85	0.89	0.85	0.87
	V (%)	4.52	3.40	4.46	3.58
ρ=1.0	B	0.85	0.89	0.85	0.87
	V (%)	5.33	4.34	5.21	4.36

B=bias, V=cov, Nominal values (MN m): tanker (Sagg=25,500, Hogg=29,600) and bulk carrier (Sagg=15,400, Hogg =19,000).

Table 7 Effect of combined correlated random yield strength and correlated random initial imperfections

Ship type		Tanker		Bulk carrier	
Condition		Sagging	Hogging	Sagging	Hogging
ρ=0.0	B	0.89	0.95	0.90	0.93
	V (%)	0.48	0.31	0.73	0.52
ρ=0.2	B	0.89	0.95	0.90	0.93
	V (%)	0.91	0.74	1.01	0.84
ρ=0.5	B	0.89	0.95	0.90	0.93
	V (%)	2.00	1.56	2.01	1.74
ρ=0.8	B	0.89	0.95	0.90	0.94
	V (%)	3.21	2.46	2.97	2.77
ρ=1.0	B	0.89	0.95	0.90	0.94
	V (%)	4.14	3.31	3.59	3.10

B=bias, V=cov, nominal values (MN m): tanker (Sagg=25,500, Hogg=29,600) and bulk carrier (Sagg=15,400, Hogg =19,000).

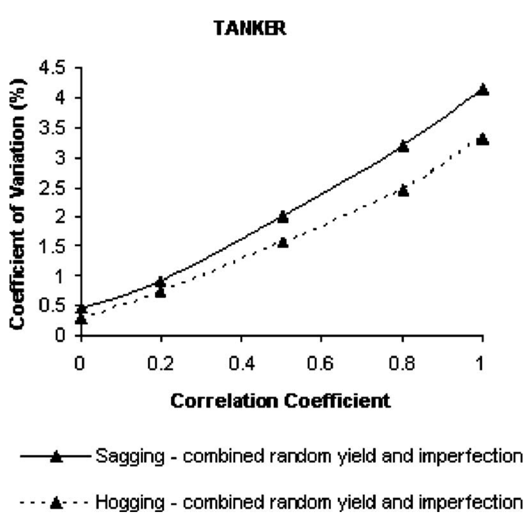

Fig. 11 Ultimate strength variation with correlation coefficient in tanker

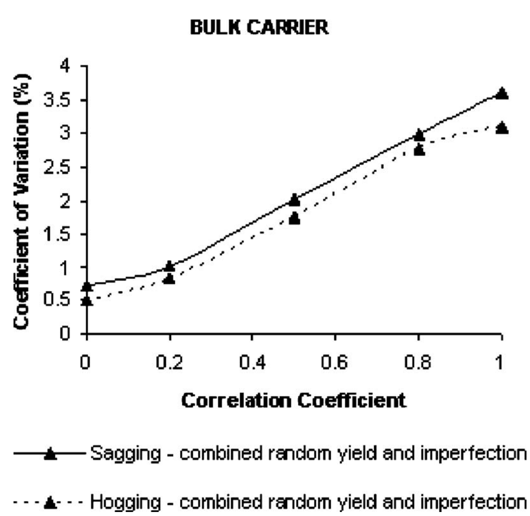

Fig. 12 Ultimate strength variation with correlation coefficient in bulk carrier

new and novel ship structures. This additional uncertainty can also be incorporated in the design equations and rules through "imperfection factors." More realistic statistical dependence models among yield strength, initial imperfection, thickness, and corrosion of the structural members will form part of our future work.

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