PROBABILISTIC FAILURE PREDICTION OF FILAMENT-WOUND GLASS-FIBER REINFORCED COMPOSITE TUBES UNDER BIAXIAL LOADING

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Abstract

Filament-wound, fiber-reinforced polymer-matrix composite laminate tubes have been used for a wide range of engineering applications, owing to their high specific stiffness, strength and superior corrosion resistance. Leakage failure of composite tubes under multiaxial loading has been an important design concern. Extensive analytical and experimental investigations have recently been reported on leakage failure of composite tubes. These investigations developed deterministic estimates of leakage failure for different combinations of axial load and internal pressure. Effects of uncertainties in material properties and laminate construction on leakage failure need to be studied in detail. In this study; probabilistic analyses are performed to predict first-ply failure of angle-ply composite laminate tubes under combined internal pressure and axial loading. Randomness is considered in (1) elastic moduli along principal lamina directions; (2) uniaxial matrix-dominated strengths of each lamina; and (3) local thickness variation of individual plies. The probabilistic finite element analysis is performed using the ANSYS™ program and Latin hypercube sampling. Influence of each basic variable on first-ply failure is evaluated. These are compared with experimental results available in the literature.

Introduction

Fiber-reinforced composite laminate tubes have been extensively used in the aerospace, automotive, and petroleum industries. One of the concerns in the use of these tubes is leakage of contents when the tubes are subjected to various combinations of internal pressure and axial tension. Leakage failure is primarily attributed to matrix-dominated cracking in individual plies of the composite laminate tubes. A complex process of initiation and growth of these cracks through the tube wall provides a passage for fluid to leak through the wall. Stresses in individual plies of the composite laminate tube are a function of the stiffness of the fibers and matrix, volume fraction of fibers, ply angles and ply thicknesses, laminate lay-up, and applied loads among others. Analytical and experimental studies on leakage failure of composite laminate tubes have been performed by Soden et al. (1993), Wang et al. (1997) etc. These studies focused primarily on deterministic mechanics and on phenomenological development of composite tube leakage failure methods. Accurate prediction of leakage failure of composite laminate tubes, however, should involve rigorous analytical models coupled with a detailed appreciation of the random aspects of the structure.

The stochastic nature of response of composite laminates due to inherent uncertainties in mechanical properties at the fiber/matrix scale, ply scale, and the structural scale is well recognized (e.g., Chamis and Shiao, 1993). Probabilistic estimates of initial and system failure of composite laminate plates and tubes (Mahadevan et al., 1997, Yushanov and Bogdanovich, 1998, and Frangopol and Recek, 2003) have been obtained. Probabilistic analyses of fiber-reinforced composite laminates using the stochastic finite element method (SFEM) have been used to obtain reliability estimates of structural response, system failure (Hong et al., 1995) and buckling and first ply failure (Lin et al., 1998). Comparisons of reliability estimates and computed failure probabilities with experiments on composite laminates are limited in the literature. Furthermore, the studies do not consider local uncertainties in ply thickness in the plane of the composite laminate. While these may not be as prevalent in laminates constructed using prepegs, filament wet winding could potentially induce such uncertainties in the ply and laminate thickness.

In this study the effect of uncertainties in geometric and material parameters in the manufacture of filament-wound composite laminate tubes on matrix-dominated first ply failure is examined. The basic variables include uncertainties in local variations of individual ply thicknesses, elastic moduli of a unidirectional ply, and failure strains. The basic variables are chosen from observations reported in the work by Srinivasan (1996). Uncertainties in ply fiber angles were not measured in that work and not considered in the study. Furthermore, material nonlinearity and damage mechanics are beyond the scope of the paper. Linear finite element analysis and Latin hypercube sampling (LHS) are used for the probabilistic calculations.

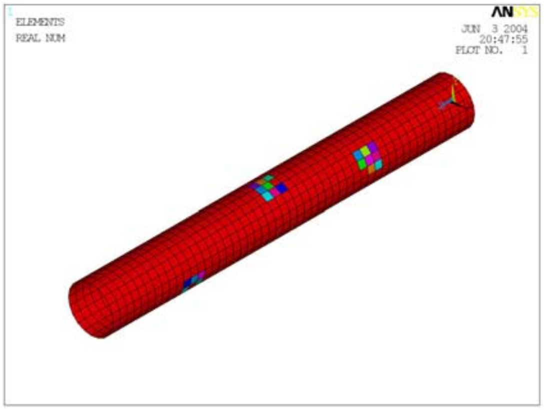
Method of Analysis

Composite tube lamination, geometry and loading

The composite laminate tube is made by filament wet winding of E-glass fibers and epoxy resin. The lay-up is $[\pm 55^{o}]_2$. The fiber angle is measured from the longitudinal axis of the tube. The nominal values of inner diameter and wall thickness of the tube are 29 mm and 2.03 mm, respectively. The tube is subjected to two combinations of internal pressure and axial tension. They are hoop to axial stress ratios of 0:1 (denoted as 0h:1a) and 1.86:1 (denoted as 1.86h:1a). One end of the tube is restrained and a uniform axial load is applied at the other end.

Finite element modeling

The probabilistic analysis of the fiber-composite laminate tube is performed using linear analysis and the ANSYS™ finite element program (ANSYS, 2003). Eight-node isoparametric shell elements (SHELL99) with five degrees of freedom per node, i.e., three displacements and two rotations, are used for structural discretization. The elements are C^{o} continuous, with Mindlin-type displacement field assumptions which include transverse shear deformations. The element is defined using corner layer thicknesses, layer material direction angles, and orthotropic material properties. The finite element model, load and constraints are shown in Figs. 1 and 2. Regions with local thickness variations are depicted with different colors in Fig. 1.



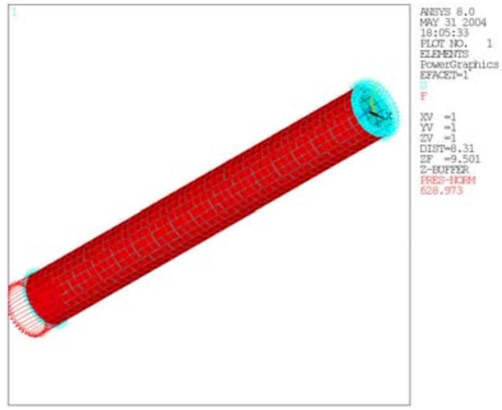


Fig. 1. Finite element model **Fig. 2. Loads and constraints**

Constitutive equations and failure criteria

The stress-strain expression for a fiber composite ply, in material coordinates (1,2) can be written as

$$\sigma'_i = Q'_{1ij} \ \varepsilon_j, \qquad \tau_{ti} = Q'_{2ij} \gamma_{tj} \tag{1}$$

where Q'_{1ij} is the in-plane stiffness matrix of the ply, Q'_{2ij} is the transverse shear stiffness matrix of the ply and $\{\sigma'\} = \{\sigma_1, \sigma_2, \tau_{12}\}^T$, $\{\tau_t'\} = \{\tau_{13}, \tau_{23}\}^T$, $\{\varepsilon'\} = \{\varepsilon_1, \varepsilon_2, \gamma_{12}\}^T$, and $\{\gamma_t'\} = \{\gamma_{13}, \gamma_{23}\}^T$ are the in-plane and out-of-plane stresses and strains in the material coordinate system. Here 1 and 2 indicate fiber direction and ply transverse directions, respectively.

The finite element laminate constitutive formulation follows the well-known Classical Lamination Theory (Jones, 1975) (CLT) with appropriate modifications for ply thickness variations. Further details are discussed in the ANSYS Theoretical Manual (ANSYS, 2003). In this study Hashin's (Hashin, 1980) criterion for matrix-dominated failure of unidirectional fiber-composites expressed in terms of strains is used. The performance index for matrix-dominated failure of a unidirectional fiber-composite due to a tensile strain transverse to the fiber is summarized as

$$F = \left(\frac{\varepsilon_2}{\varepsilon_2^f}\right)^2 + \left(\frac{\gamma_{12}}{\gamma_{12}^f}\right)^2, \tag{2}$$

where ε_2^f is the uniaxial failure strain transverse to the fiber and γ_{12}^f is the uniaxial in-plane shear failure strain. Failure in any ply of a composite laminate is said to occur when the performance index $F \geq 1$ in Eq. (2).

Probabilistic modeling and simulation procedure

Four sets of basic variables were chosen for evaluating first ply failure of the composite laminate tubes. They are individual ply and total tube wall thickness, axial and circumferential locations of local thickness variations in the tube, and elastic moduli and failure strains of a unidirectional E-glass/epoxy ply. As noted before, these were chosen based on ad-hoc observations in the work of Srinivasan (1996). The statistics of ply thicknesses, moduli and failure strain are shown in Table 1. The elastic moduli and failure strains basic variables do not vary spatially. Note that the plies are numbered sequentially from the inner most ply at the internal diameter (Ply 1) towards the outside of the tube (Ply 4). The total area exhibiting local thickness variations is assumed to be a constant 5% of the total surface area of the tube and always constitutes six regions of equal area. The centroids of these regions follow a 2D conditional Poisson process thinned so that no two regions overlap.

Random Var.	Distribution	Mean	Stdev
Ply 1 thickness	Trunc Gauss	0.51 mm	0.076 mm
Ply 2 thickness	Trunc Gauss	0.51 mm	0.1 mm
Ply 3 thickness	Trunc Gauss	0.51 mm	0.1 mm
Wall thickness	Trunc. Gauss	2.03 mm	0.20 mm
E_{11}	Gauss	45.0 GPa	2.25 GPa
E_{22}	Gauss	12.95 Gpa	1.3 Gpa
G_{12}	Gauss	5.85 Gpa	0.59 Gpa
ε_2^f	Weibull	Exponent 25	Char value 0.004 Min. value 0.003
γ_{12}^f	Weibull	Exponent 25	Char value 0.05 Min. value 0.044

Table 1. Probability distribution functions of input variables

Latin hypercube sampling implemented in ANSYS (ANSYS, 2003) is used to perform the probabilistic calculations. This is a cost effective procedure for computing probabilistic estimates of first ply failure of the tubes.

Four different basic variable cases were considered: (BV1) The ply thickness variation occurs over the whole domain of the tube, (BV2) local ply thickness variation; all basic variables uncorrelated (BV3) local ply thickness variation; failure strains are 50% correlated, and (BV4) local ply thickness variation; thickness of plies in six regions are fully correlated, failure strains are 50% correlated.

Results and Discussion

Linear elastic finite element analyses were performed using ANSYS (ANSYS, 2003) for the four basic variable cases and two load conditions described in the previous sections. The number of simulations range from 40 for BV1 to a maximum number of 150 for BV2 and BV3. For each simulation, first ply failure is identified as a first passage of the performance index. Performance indices are computed at each element and the maximum performance index in each ply over the whole domain of the tube was identified. Probability density functions (PDF's) of first ply failure computed from the maximum performance index of all four plies for all cases are shown in Figs. 3 and 4. In Fig. 3, the dotted line indicates the failure axial stress determined from experiment (Wang et al., 1997). The experimental result falls in the tail of the predicted range of occurrence of first ply failure. For the load ratio of 1.86h:1a, tube failure occurred at an axial stress of 24 ksi (Wang et al., 1997). First ply failure was not identified in the experiments.

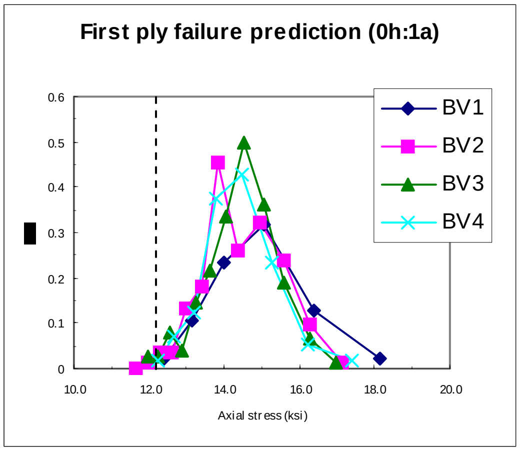


Fig. 3. PDF for 0h:1a load case

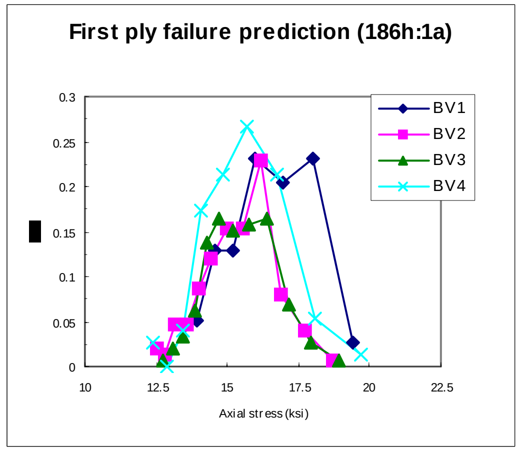


Fig. 4. PDF for 1.86h:1a load case

	Load Case 0h 1a				Load case 1.86h: 1 a			
Parameter	BV 1	BV2	BV3	BV4	BV 1	BV2	BV3	BV4
E_{11}	0.00	0.23	-0.08	0.15	-0.29	-0.33	-0.32	-0.35
E_{22}	-0.49	-0.65	-0.54	-0.55	-0.69	-0.70	-0.71	-0.77
G_{12}	-0.42	0.73	-0.68	-0.71	0.16	-0.12	0.14	0.27
Tlay (max)	-0.55	-0.36	0.42	0.16	0.61	-0.46	-0.56	0.37
Loc (max)	-	-0.27	-0.19	0.27	-	-0.20	-0.13	-0.250
ε_2^f	-0.17	-0.13	-0.30	-0.25	0.04	-0.12	-0.12	-0.31
γ_{12}^f	-0.03	-0.02	-0.22	-0.16	0.05	0.02	-0.03	0.00

Table 2 Correlation coefficients of basic variables with performance index

Sensitivities of the basic variables on first ply failure were determined in terms of correlation coefficients with the maximum performance index in a ply (Table 2). The influence of ply thickness is highest for case BV1. The quantities Tlay (max) and Loc (max) in the table indicate maximum of all correlation of layer thickness or location variables with the performance index.

Conclusions

Probabilistic finite element calculations of $[\pm 55^o]_2$ E-glass/epoxy composite laminate tubes were performed to determine uncertainties in predicting matrix-dominated failure. The failure predictions were compared with experiments. The basic variables considered were ply thickness, local ply thickness variations, elastic moduli, and failure strains. The results indicate that (a) the performance index for matrix-dominated failure is sensitive to ply thickness variations and ply transverse and shear elastic moduli (b) the performance index is not very sensitive to locations of ply thickness variations.

More rigorous probabilistic analysis that includes spatial variation of ply thicknesses, material nonlinearity, and damage mechanics may be required for predicting leakage failure of the tubes for load ratio 1.86h:1a.

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