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Reliability of bridge deck subject to random vehicular and seismic loads through Subset Simulation

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ABSTRACT: This paper presents a reliability analysis of a simply supported bridge deck subjected to random moving and seismic loads. Vehicle structure interactions effects are taken into account, which are affected by vehicle speed, vehicle and bridge masses, deck surface irregularities and material and geometric properties of the bridge all of which could be random in nature. The random field modeling of the bridge surface roughness and the stochastic process modeling of non-stationary seismic loads result in a reliability problem of very high dimensions and typically low probability of failure. Subset simulation incorporating Markov Chain Monte Carlo moves is adopted in this study for the reliability analysis. The bridge is modeled as a single span simply supported Euler-Bernoulli beam and the vehicle is modeled as a SDOF system. Mid span deflection of the beam is computed using method of weighted residual and is used as the performance criteria.

1 INTRODUCTION

The study of bridges subjected to vehicle loads is a problem of interest to bridge engineers. A substantial body of research has looked into understanding the behavior of bridges under vehicular and train loads (Hino et al. 1985, Fryba 2001, Marur 2001 Garinei2006, Kim & Kawatani 2006, Zribi et al. 2006).The effects of dynamic stresses are generally taken into account in an empirical manner for the design of bridges (Wang & Huang 1992, Wiriyachai et al. 1982, Inbanathan & Weiland 1987, Palamas et al. 1995). Bridge codes specify impact factors that amplify the static responses of a structure and attempt to account for the additional stresses resulting from forces acting dynamically. The impact factors vary depending upon the type of bridge and the range of loading and are given by empirical formulas (Wiriyachai et al. 1982, Inbanathan & Weiland 1987).

The effect of surface roughness can be significant on the response of a bridge and studies have shown (Wang & Huang 1992, Wiriyachai et al. 1982, Inbanathan & Weiland 1987, Law & Zhu 2005) that on considering the effect of surface roughness values of impact factors increased two to three times the original value. The impact on the bridge also increases with vehicle velocity. Vehicle mass has a significant effect on the response of a bridge when either vehicle speed or the ratios of vehicular mass to total bridge mass is high (Palamas et al. 1995). A comprehensive study of the dynamics of railway bridges was undertaken by Fryba (2001). He studied resonance vibrations of a bridge, using the Euler-Bernoulli beam equation, and proposed relations between resonance amplitude and various geometrical and material properties of the bridge and the vehicle. Fryba's work on vibrations of solids helped form the basis of any studies related to vibration of bridges under vehicular loads.

This paper deals with the reliability analysis of a beam subjected to vehicle structure interactive forces and random earthquake loads. This stochastic dynamic system gives rise to a reliability problem of very high dimensions and very low failure probability. This poses a problem for analytical reliability analysis techniques and also the very robust brute force Monte Carlo simulation. Subset simulation is used in this study as a tool for estimation of reliability. This paper includes sensitivity studies of reliability of the bridge with respect to parameters assumed to be random. To reduce the computational effort these studies shed light on the necessities of the parameters to be taken as random. The necessity of random field modeling of surface irregularities is also looked at. The non linear interaction between vehicle structure interaction and seismic loading is demonstrated in terms of failure probability.

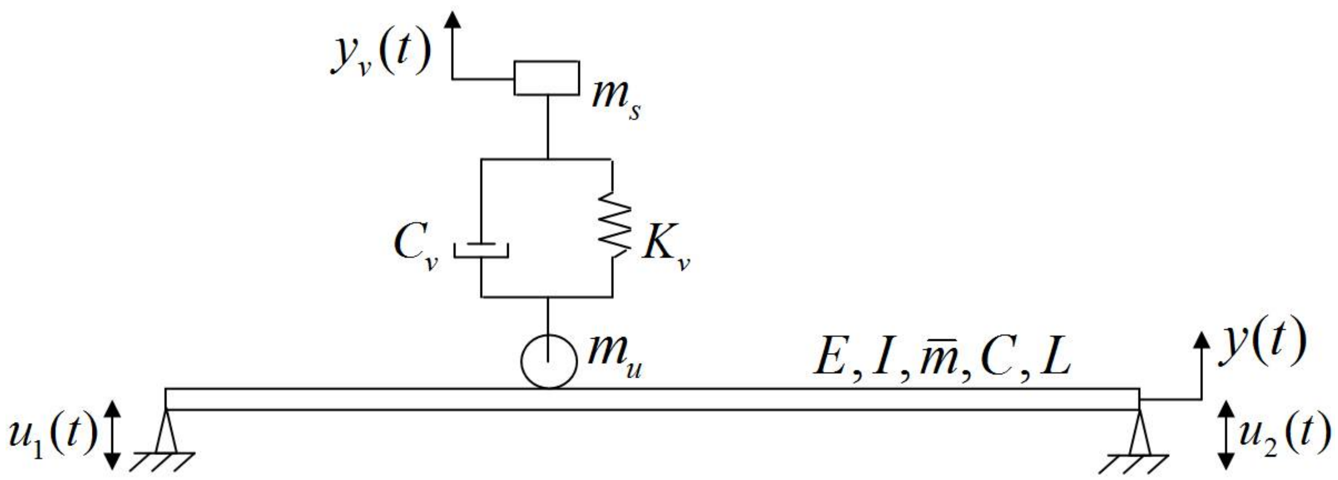

Figure 1: The model of bridge and vehicle system that is used in the current study (only one vehicle is shown here)

2 VEHICLE STRUCTURE INTERACTION

Vehicle structure interaction problems have governing equations that have time varying coefficients. The moving mass generates inertial forces that make the coefficients of the governing equation time dependent (Fryba 2001, Nasrellah & Manohar 2010).In such cases the notion of normal modes and natural frequencies are invalid. In addition to this closed form solutions cannot be obtained.

The bridge is assumed to be a single span simply supported homogeneous Euler- Bernoulli beam of uniform cross section and mass. The vehicles are assumed to be SDOF systems. The moving oscillator mass is divided into two parts- the sprung mass and the un-sprung mass. It is also assumed that the moving oscillator never lose contact with the beam. The beam oscillator system that is used for the current study is shown in Figure 1. It is assumed that the ground movement produces motion in the vertical direction only and acts only at the supports. It is also assumed that there is no phase difference between the vibrations produced at the two supports. This is a safe assumption as the length of the beam is quite small and hence it can be assumed that the travelling waves reach the supports at the same time.

Let $y(x,t)$ be the beam displacement in the vertical direction at location x and time t, and let $y_v(t)$ be the displacement of the sprung mass in the vertical direction at its current location. Let $r(x)$ be the surface roughness of the beam. The motion of the vehicles in the vertical direction is given by:

$$\begin{aligned}&m_s^{(n)}\ddot{y}_v^{(n)}(t)+C_v^{(n)}\times\\&\left[\dot{y}_v^{(n)}(t)-\frac{D}{Dt}\left(y\left(v\left(t-t^{(n)}\right),t\right)+r\left(v\left(t-t^{(n)}\right)\right)\right)\right]\\&+K_v^{(n)}\times\\&\left[y_v^{(n)}(t)-\left(y\left(v\left(t-t^{(n)}\right),t\right)+r\left(v\left(t-t^{(n)}\right)\right)\right)\right]=0\end{aligned} \tag{1}$$

Where, $C_v^{(n)}$ and $K_v^{(n)}$ are the coefficient of damping and the spring constant of the oscillators respectively (‘n’ represents oscillator number). $t^{(n)}$ represents the time between entry time of current oscillator and of the first oscillator on the beam ($t^{(n)}=0$ for n=1). The governing equation of the beam vibration is given by:

$$\begin{aligned}&EIy^{IV}(x,t)+\bar{m}\ddot{y}(x,t)+C\dot{y}(x,t)=\\&\sum_{(n)}R^{(n)}(x,t)\delta\left(x-v\left(t-t^{(n)}\right)\right)\end{aligned} \tag{2}$$

E, I, $\bar{m}$ and C are the modulus of elasticity, the moment of inertia, mass per unit length and the coefficient of damping per unit length respectively. The RHS defines the reaction force per unit length acting on the beam due to the oscillators moving over it.

$$\begin{aligned}&R^{(n)}(x,t)=(m_s^{(n)}+m_u^{(n)})g-m_u^{(n)}\frac{D^2}{Dt^2}\left[y(x,t)+r(x)\right]\\&+C_v^{(n)}\left[\dot{y}_v^{(n)}(t)-\frac{D}{Dt}(y(x,t)+r(x))\right]\\&+K_v^{(n)}\left[y_v^{(n)}(t)-(y(x,t)+r(x))\right]\end{aligned} \tag{3}$$

The boundary conditions and initial conditions for the above system are as follows:

Boundary Conditions:

$y(0,t) = u_1(t)$ and $EIy''(0,t) = 0$

$y(L,t) = u_2(t)$ and $EIy''(L,t) = 0$ (4)

Initial Conditions:

$y(x,0) = 0$ and $\dot{y}(x,0) = 0$

Let the solution to the beam deflection be of the form:

$$y(x,t) = \sum_{i=1}^{N} \phi_i(x) a_i(t) + \left(1 - \frac{x}{L}\right) u_1(t) + \frac{x}{L} u_2(t) \quad (5)$$

$\phi_i(x)$ are the mode shapes of vibration of the beam. $a_i(t)$ are the time dependent amplitude. $u_1(t)$ and $u_2(t)$ are the displacement due to earthquake at the supports. These terms are included in the solution so that boundary conditions for a simply supported condition can be incorporated. For simplicity, in the current study we have assumed $u_1(t) = u_2(t) = u(t)$.

Putting in the boundary conditions associated with a simply supported beam, the above system of equation reduces to

$$\begin{bmatrix} M & 0 & \cdots & 0 \\ 0 & m_s^{(1)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_s^{(N_v)} \end{bmatrix} \begin{pmatrix} \ddot{A}(t) \\ \ddot{y}_v^{(1)}(t) \\ \vdots \\ \ddot{y}_v^{(N_v)}(t) \end{pmatrix} + \begin{bmatrix} C & \{c^{(1)}\}^T & \cdots & \{c^{(N_v)}\}^T \\ c^{(1)} & C_v^{(1)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c^{(N_v)} & 0 & \cdots & C_v^{(N_v)} \end{bmatrix} \begin{pmatrix} \dot{A}(t) \\ \dot{y}_v^{(1)}(t) \\ \vdots \\ \dot{y}_v^{(N_v)}(t) \end{pmatrix} + \begin{bmatrix} K & \{k^{(1)}\}^T & \cdots & \{k^{(N_v)}\}^T \\ k^{(1)} + \tilde{k}^{(1)} & K_v^{(1)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ k^{(N_v)} + \tilde{k}^{(N_v)} & 0 & \cdots & K_v^{(N_v)} \end{bmatrix} \begin{pmatrix} A(t) \\ y_v^{(1)}(t) \\ \vdots \\ y_v^{(N_v)}(t) \end{pmatrix} = \begin{pmatrix} F \\ F^{(1)} \\ \vdots \\ F^{(N_v)} \end{pmatrix} \quad (6)$$

Where the terms in the matrix are defined as follows:

$$M_{ij} = \frac{\overline{m}L}{2}\delta_{ij} + \sum_n m_u^{(n)} \phi_i^{(n)} \phi_j^{(n)}$$

$$C_{ij} = \frac{CL}{2}\delta_{ij} + \sum_n 2m_u^{(n)} \phi_i^{(n)} \phi_j'^{(n)} + C_v^{(n)} \phi_i^{(n)} \phi_j^{(n)}$$

$$K_{ij} = EI\left(\frac{i\pi}{L}\right)^4 \frac{L}{2}\delta_{ij} + \sum_n m_u^{(n)} \phi_i^{(n)} \phi_j''^{(n)} + C_v^{(n)} \phi_i^{(n)} \phi_j'^{(n)} + K_v^{(n)} \phi_i^{(n)} \phi_j^{(n)}$$

$$\begin{aligned} F_i &= \sum_n \overline{R}^{(n)} \phi_i^{(n)} - \left(m\ddot{u} + C\dot{u}\right)\frac{2L}{i\pi} && \text{for odd } i \\ &= \sum_n \overline{R}^{(n)} \phi_i^{(n)} && \text{for even i} \end{aligned}$$

$$\overline{R}^{(n)} = m_v^{(n)} g - m_u^{(n)}\left(v^2 r''^{(n)} + \ddot{u}\right) - C_v^{(n)}\left(v r'^{(n)} + \dot{u}\right) - K_v^{(n)}\left(r^{(n)} + u\right)$$

$$\begin{aligned} c_i^{(n)} &= -C_v^{(n)} \phi_i^{(n)} \\ k_i^{(n)} &= -K_v^{(n)} \phi_i^{(n)} \\ \tilde{k}_i^{(n)} &= -C_v^{(n)} \phi_i'^{(n)} \end{aligned}$$

$$F_v^{(n)} = C_v^{(n)}\left(\dot{u} + \dot{r}^{(n)}\right) + K_v^{(n)}\left(u + r^{(n)}\right)$$

$$A(t) = \begin{pmatrix} a_1(t) \\ \vdots \\ a_N(t) \end{pmatrix}, \phi_i^{(n)} = \phi_i\left(vt - vt^{(n)}\right), r^{(n)} = r\left(vt - vt^{(n)}\right)$$

N_v is the number of oscillators passing over the beam. When the beam vibrates there is an inertial effect of the mass in the vertical direction as well. This adds to the force acting in the vertical direction and hence gives rise to equation of time varying coefficients in Eq. (6). This type of a problem cannot be solved analytically and no closed form solution exists for the given set of equations. Numerical methods of integration are used to solve this system of differential equations.

3 TIME DEPENDENT STRUCTURAL RELIABILITY

In the present study we are dealing with a dynamics problem involving an extended structural system. Thus the general formulation of the limit state is infinite dimensional both in time and space and the reliability function is given by:

$$rel(t) = P\left[C\left(\underline{x},\tau\right) \geq D\left(\underline{x},\tau\right), \forall\, \tau \in (0,t), \forall \underline{x} \in \Omega\right] \quad (7)$$

Where, Ω is the spatial domain of the structure, C and D are capacity and demand at time instant τ and location x.

For solving such problems it is necessary to convert the infinite dimensional problem to a problem of finite dimensions. Discretization of time into small intervals helps in reducing the dimension of the problem to a certain extent. Another way of reducing the dimension is by simply removing the spatial dimension from the problem by focusing on the capacity and demand terms at certain critical locations only.

Simulation techniques like crude (or brute force) Monte Carlo Simulation are simple and robust. Even though the estimate of failure probability obtained from brute force Monte Carlo techniques is unbiased, maintaining a low coefficient of variation comes at the cost of increasing the number of simulations to an extent which makes the method inefficient. Various variance reduction techniques have been proposed to overcome the shortcomings of the Monte Carlo simulation technique, the most popular being importance sampling (Melchers 1999). For the method to be most effective one requires *a priori* knowledge of the behavior of the limit state which again is not always available.

Au and Beck (2001) proposed subset simulation involving a series of nested limit states (for problems where such nesting is possible) that used a modified Metropolis Hastings algorithm (Au & Beck 2001,2003) for generating a sequence of conditional samples. This algorithm can drastically reduce the computational effort required for a problem (with respect to crude Monte Carlo Simulation) however the estimates produced have very high coefficients of variation (c.o.v). The basic underlying principle in subset simulation is the expression of the probability of any given event F as a product of a number of conditional probabilities involving nested sets $F_n, F_{n-1}, \ldots, F_3, F_2, F_1$. These sets are such that $F = F_n$ and $F_1 \supset F_2 \supset F_3 \supset \ldots \supset F_{n-1} \supset F_n$ Thus,

$$P(F_n) = P(F_1)P(F_2 \mid F_1)P(F_3 \mid F_2)\ldots\ldots P(F_n \mid F_{n-1}) \quad (8)$$

Sets $F_n, F_{n-1}, \ldots, F_3, F_2, F_1$ can be so chosen as to make values of $P(F_1)$, $P(F_2 \mid F_1)$,…, $P(F_n \mid F_{n-1})$ sufficiently large compared to $P(F_n)$, and their product yields the desired result which presumably is very small.

The probability of exceeding the first limit state, which can be computed using crude MCS as $P(F_1)$ will be quite high after an appropriate selection of number of intermediate levels. If the original pdf of the vector $\underline{x}$ is $f(\underline{x})$, then the conditional pdf from which the new random variables will be generated, to populate the intermediate failure levels, will be of the form $f(\underline{x} \mid F_i)$. Markov chain MCS is used for generation of samples at intermediate failure sets. The Markov chain Monte Carlo simulation is done using a modified Metropolis Hastings algorithm proposed by the authors (Au & Beck 2001, Au & Beck 2003, Au et al.2007). A sample having a conditional distribution $f(. \mid F_i)$ can be considered to be a state of a Markov chain. Using the algorithm we can then generate a new sample as the next sample which will be distributed as $f(. \mid F_i)$. This assumption was made by the authors based on the fact that the initial sample taken for generation of the Markov chains was part of the failure region upon which the target distribution is conditioned, hence the Markov chain follows the target distribution. Thus there are no issues associated with burn-in of the Markov Chain. A more pressing issue is that the length of the Markov chain generated is quite small (for longer Markov Chains larger number of samples is required which will defeat the purpose of using this technique) and hence sometimes may not sufficiently populate the variable space at the intermediate levels.

For the current study we have used subset simulation with Markov chain Monte Carlo simulation (Au & Beck 2001, Au & Beck 2003, Au et al.2007). The steps involved in the process are:

1. Generate N_0 samples using the distributions of each random variable
2. Get the number of samples that exceed the first intermediate limit state, n_1. Get $P(F_1) = n_1 / N$
3. Using these n_1 samples as seeds, generate N_1 samples using the MH Algorithm
4. Set $k = 2$ and repeat till the final (n^{th}) limit state is reached (that is, $k = n$):
   - Get n_k as the number of samples that exceed the k^{th} intermediate limit state
   - Get $P(F_k \mid F_{k-1}) = n_k / N_{k-1}$
   - Use n_k samples as seeds, generate N_k samples using the MH Algorithm
   - Set $k = k + 1$
5. Get $P(F_n) = P(F_1)P(F_2 \mid F_1)P(F_3 \mid F_2)\ldots . P(F_n \mid F_{n-1})$

In step 4 the Metropolis-Hastings algorithm can be further divided into the following steps. It involves generation of the $(k+1)^{\text{th}}$ state of the system using the k^{th} state (Au & Beck 2001, Au & Beck 2003, Au et al.2007):

a) Generate a trial sample ξ centered at $x^{(k)}$
b) For each member ξ_j of the n-dimensional vector ξ:

- Get, $r = f_x(\xi_j) \big/ f_x(x_j^{(k)})$, where f_x is the distribution of the random variable x
- Assign $x_j^{(k+1)} = x_j^{(k)}$ with the probability $\left(1-\min(r,1)\right)$
- Assign $x_j^{(k+1)} = \xi_j$ with the probability $\min(r,1)$

c) If ξ does not lie in F_k, assign $x^{(k+1)} = x^{(k)}$

d) Using, $x^{(k+1)}$ generate $x^{(k+2)}$ and so on, until the required number of samples are generated.

In order to generate the trial sample ξ centered at $x^{(k)}$ as mentioned in the first step of the algorithm one needs to use a proposal pdf which has the symmetry property expressed as $f\left(\xi \mid x^{(k)}\right) = f(x^{(k)} \mid \xi)$. This ensures that the transition probability is high and hence the target stationary distribution is attained faster.

4 GENERATION OF RANDOM SEISMIC LOAD SURFACE IRREGULARITIES

Synthetic earthquake records are generated for the current study using the power spectral density proposed by Kanai-Tajimi (Fan & Ahmadi 1990). The Kanai-Tajimi power spectral density (PSD) is:

$$S(\omega) = I\frac{(\omega_g^4 + 4\eta_g^2\omega_g^2\omega^2)}{(\omega^2-\omega_g^2)^2 + 4\eta_g^2\omega_g^2\omega^2} \tag{9}$$

η_g and ω_g are soil constants based on soil properties. I is the constant spectral density of the white noise excitation at the bedrock level.

The PSD is filtered by another function $H(\omega)$ to remove the low frequency content from the resulting stationary waves generated.

$$h(\omega) = \frac{\left(\omega/\omega_2\right)^2}{\left[1-\left(\omega/\omega_2\right)^2\right] - 2i\eta\omega/\omega_2} \tag{10}$$

$$H(\omega) = \left|h(\omega)\right|^2$$

From this PSD a stationary Gaussian stochastic process $X(t)$ with zero mean is generated as a Fourier series. The coefficients of the series are random variables with zero mean. The stationary wave is then filtered using an envelope function (Shinozuka & Sato 1967)

$$\ddot{u}(t) = e(t)X(t) \tag{11}$$

Where, $e(t) = A_0(e^{-\alpha t} - e^{-\beta t})$

The surface irregularities are also modeled as stochastic processes (in space). The PSD used in this case for generation of stationary Gaussian random process is given by Dodds and Robson (Palamas et al. 1995).

$$S(\lambda) = A_r\left(\frac{\lambda}{\lambda_0}\right)^{-2} \tag{12}$$

This power spectral density is the most widely used one for generation of surface roughness of roads as random field (Au et al. 2001, Silva 2004, Law and Zhu 2005). The road surface profile is generated using Fourier series in a similar way as the synthetic earthquake records are generated.

5 NUMERICAL STUDIES

The random variables associated with the problem are the oscillator mass, oscillator damping ratio, oscillator stiffness, modulus of elasticity and moment of inertia of the beam, and the time taken by the oscillator to traverse the beam. The time at which the first oscillator enters the beam is $t=0$. The time at which the earthquake starts with respect to the entry time of the first oscillator is also treated as random. This helps in generalizing the problem and ensures that we are not looking at any special case. The other random variables are the surface roughness coefficient and the intensity term of the Kanai-Tajimi PSD. The intensity term is responsible for the variation of magnitude of the earthquake. When two oscillators are considered the mass, stiffness and damping of that oscillator are taken to be random as well. It is assumed that the second oscillator travels with the same velocity as the first. The time difference between the two oscillators is taken to be random as well. The random variables associated with the problem and their definitions are given in Table 1.

Table 1: Basic variables

Random Variable	Description
Modulus of elasticity of bridge (E)	$\sim LN\left(2.87\times10^6 kPa, 5\%\right)$
Moment of inertia of bridge (I)	$\sim N\left(2.9m^4, 10\%\right)$
Vehicle mass (m_v)	$\sim LN\left(60t, 15\%\right)$
Time at which earthquake starts (t_e)	$\sim U\left(-3s, 3s\right)$

Time taken by vehicle to travel over the bridge (t_t)	$\sim Wald\left(2.5s, 1s\right)$
Vehicle stiffness (K_v)	$\sim N\left(1595kN/m, 10\%\right)$
Damping ratio of vehicle	$\sim U\left(0\%, 8\%\right)$
Surface Roughness coefficient (A_r)	$\sim U\left(5\times10^{-6}, 80\times10^{-6}\right)$
Intensity of Kanai-Tajimi PSD (I)	$\sim LN\left(0.005, 0.002\right)$
Time between two vehicles	$\sim U\left(0.5\text{s}, 1.5\text{s}\right)$

The uniform mass density of the beam is 2303 kg/m and is 25 m long. Coefficient of damping per unit length is assumed to be 3000 N-s/m-m. The ratio of the sprung mass to the un-sprung mass is $3:1$.

The constants used for the various constants involving the generation of random processes defining both acceleration due to ground motion and surface irregularity are given in Table 2

Table 2: Constants used in the current study

Constants	Values
Constants for Kanai-Tajimi PSD:	
η_g	0.6
ω_g	8π rad/s
Constants for correction of Kanai-Tajimi PSD:	
η	0.6
ω_2	5 rad/s
Constants for modulating function of the ground accelerations:	
A_0	4
α	0.1
β	0.4
Constants for road roughness PSD:	
λ_0 (Discontinuity frequency)	0.5π /m

Reliability analysis of the beam in the current study is based on a single time dependent limit state that the mid-span deflection under seismic and vehicular loads does not exceed a serviceable limit, Δ_0 This definition considerably reduces the dimensionality of the reliability problem by removing the spatial aspect so that one just has to deal with the temporal nature of the limit state.

As reviewed by Bhattacharya et al. (2009), a certain amount of subjectivity exists in the definition of deflection limits for bridge decks and girders – both in collapse and serviceability limit states. Roeder et al. (2002) concluded that deformations that cause bridge damage are relative deflections between adjacent girder members, local rotations and deformations. One of the earliest deformation based failure definition for bridges was based on avoiding the undesirable structural effects and undesirable psychological reaction (ASCE 1958) and a limit of span/800 for steel bridges (simple as well as continuous spans) under live load plus impact was suggested. Based on "the limit of visual observation", Galambos et al. (1993) proposed using a maximum permanent or residual deflection equal to span /300 as serviceability limit state in bridge inelastic rating. Ghosn and Moses (1998) considered span/100 as "dangerously high levels" of deformation. In this paper, the allowable deflection Δ_0 is assumed to be $L/350$ (where L is the effective span of the beam) since this study is limited to linear domain for both stresses and deflections.

For the subset simulation algorithm the intermediate conditional failure probability is taken as 0.1. The number of intermediate failure levels is generated adaptively based on this conditional failure probability (Au & Beck 2001). The final number of levels gets adjusted based on the estimate of failure probability. The proposal pdf for each basic variable was taken as Gaussian with standard deviation the same as in the original distribution. To begin with, we assumed only one oscillator passes over the beam. The number of samples generated at each intermediate failure level is 100. All coding has been done in MATLAB 7.6.0. A desktop PC (2.67 GHz, 1.98 GB RAM) was used for all the computations.

For studying the variance in the estimated failure probability, $\hat{P}_f$, we computed the failure probability estimate N=20 times. The mean, coefficient of variation (c.o.v.) and the range of the estimates are given in each case in Table 3

$$\hat{\mu} = \sum_{i=1}^{N} \frac{\hat{P}_{f,i}}{N}, \quad \hat{S} = \sqrt{\sum_{i=1}^{N} \frac{\left(\hat{P}_{f,i} - \hat{\mu}\right)^2}{N-1}}, \quad \text{c.o.v.} = \frac{\hat{\mu}}{\hat{S}} \quad (13)$$

The first row of Table 3 lists the mean, c.o.v and range of the 20 estimated failure probabilities when all 109 basic variables are assumed to be random. The average failure probability estimate $\hat{\mu}$ is 0.0105.

Table 3: Reliability estimates using 100 samples at each failure level of subset simulation

Randomness considered in	Mean Probability of failure, $\hat{\mu}$	c.o.v	min and max Pf	Ratio of $\hat{\mu}$ with respect to all random case	Range of the number of samples
All	0.0105	0.0493	0.0011,0.03	1	100-190
All except I	0.0091	0.1021	0.0009,0.02	1.15	100-280
All except Damping ratio	0.0135	0.4940	0.001,0.04	0.77	100-190
All except Ar	0.0129	0.5945	0.001,0.03	0.81	100-190
All except Road profile	0.0104	0.0376	0.0016,0.03	1.01	100-190
All except Kv	0.0101	2.0155	0.0004,0.03	1.04	100-280

Table 4: Effect of combination of seismic loads and vehicle loads on reliability

Combination considered	Mean probability of failure
Seismic load + one oscillator load	0.0105
Seismic load + two oscillator load	0.0120
Only two oscillators	0.0378
Only one oscillator load	5.63 e-4
Only Seismic load	3.997e -6

The subsequent rows of Table 3 correspond to the cases when one basic variable at a time (as indicated in the first column) is taken to be deterministic (at its mean value). The fifth row of Table 3 lists the case where the surface roughness is not taken as a random field. For this case the roughness coefficient is deterministic and equal to 42.5×10^{-6}. The last column of the table gives an idea of the sensitivity of the probability of failure to each basic variable. Clearly, earthquake intensity, damping ratio and surface roughness have the most influence on reliability. Conversely, when the beam surface profile is considered deterministic, the estimated mean probability of failure hardly changes from the all random case. Hence modeling the surface roughness as a random field may not be necessary: in this case the dimension of the problem will reduce by 20. When two oscillators are considered to be passing over the beam subjected to seismic loading the mean failure probability estimate is 0.0120 (Table 4). If the time difference between the two oscillators is made deterministic then the failure probability estimate goes down to 0.0093. This shows that it is important to model the time difference as a random quantity. Table 4 shows the values when five different cases are studied. It can clearly be seen that individually the mean probability of failure of the beam subjected to seismic loads and moving oscillator loads (only one oscillator) is very small. However when the combined effects (seismic load + one oscillator load) are studied the probability of failure increases drastically. Hence even though the two events (oscillator moving over the beam and earthquake occurring) are mutually exclusive events their combined probability of failure does not directly reflect that. The probability of failure when only two oscillators are passing over the beam is almost three times the failure estimate when seismic loads are combined with the two oscillators passing over the beam. One may conclude that the seismic load has a balancing effect when two oscillators pass over the beam. However this seems unlikely. This fact might be supported by the high c.o.v of the estimate obtained (almost 50%). There is a slight increase in the failure probability when two oscillators are taken in case of one oscillator. This can be explained based on the fact that the load acting on the beam has increased.

6 CONCLUSIONS

The current study demonstrated Subset Simulation with Markov Chain Monte Carlo moves as an efficient method for reliability analysis of a problem in high dimensions and low failure probability. A beam subjected to vehicle structure interaction (vehicle modeled as a SDOF oscillator) and seismic loads was adopted as the model system resulting in a reliability analysis problem with 109 (for single vehicle) and 113 (for two vehicles) random variables. Sensitivity studies showed that it is necessary to model the earthquake intensity, damping ratio and stiffness of the oscillator and the coefficient of roughness as random variables. Treating them as deterministic would affect the computed reliability of the bridge. Modeling the surface roughness as a random field may not be necessary as it had negligible effect on reliability and would help in reducing the dimension of the reliability problem, in this case by 20. The current study dealt with two types of load cases whose occurrence are mutually independent namely vehicle structure interaction forces and random seismic loads. The failure probabilities under each load case (seismic loading and single oscillator load) separately were quite low. However when these two loads acted simultaneously the failure probability was disproportionately high suggesting an amplification caused by vehicle earthquake interaction. The current study was confined to a linear structural analysis and future efforts should investigate the effects of nonlinearities (both geometric and material). Future efforts should also evaluate the efficiency of schemes such as Hybrid Subset Simulation for this problem.

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