A non-iterative structural damage identification methodology using state space eigenstructure assignment

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ABSTRACT

In this article, a novel approach to damage identification (location as well as intensity) is presented using eigenstructure assignment (ESA)-based finite-element model (FEM) updating. ESA is a control-based approach that utilises state or output feedback of a system to alter its eigenstructure. The proposed method identifies the system’s state transition matrix and its eigenstructure from the response time history. The identified eigenstructure is first mapped onto the physical space and then reconstructed in state space in the preferred orientation and order which, in turn, is used as the target for the ESA algorithm to uniquely update the system matrices of the baseline FEM. Comparing the updated stiffness matrix with the baseline, the location and intensity of damage are estimated. Numerical validation of the method is performed on a shear frame, a Euler-Bernoulli beam, and an aluminium plate. A parametric study involving different levels of noise in the simulated response histories is undertaken. The algorithm is then tested with actual response histories from a damaged (notched) two span continuous steel beam and a damaged (indented) aluminium plate in the laboratory. The accuracy of the method in identifying the location and extent of damage is found satisfactory. Being eigenstructure based, the proposed methodology is restricted to linear time-invariant systems.

1. Introduction

As structural systems become larger and more complex, the task of maintaining them at adequate levels of safety and serviceability becomes more challenging. Periodic, or in some cases continuous, health monitoring through a network of sensors has gained wide acceptance in recent years for structural systems as varied as buildings, aircraft, space and ship structures. Available approaches for assessing the health of a structural system can be variously classified as static vs. dynamic, statistical vs. deterministic, or online vs. offline, etc.

For dynamic property-based methods, the current dynamic properties of the structure obtained from response histories measured using a set of sensors placed at critical locations on the structure are compared with the baseline dynamic property representing the undamaged state of the same structure. In addition to global health monitoring, information about the local deterioration of structural members caused by a flaw, weak material zones or lumped impurities is also required to maintain the necessary safety level.

Damage identification methods (encompassing the four stages of damage detection discussed by Rytter and Kirkegaard [1994]) can broadly be classified into two categories: model independent and model based. Model-independent methods mostly include non-destructive testing techniques to identify damage, such as ultrasonic (Jian & Dixon, 2007; Jian, Dixon, Quirk, & Grattan, 2008), acoustic emission (Hill, Geng, Cowking, & Mckersie, 1993; Geng, 2006) and vibration measurements (Fan & Qiao, 2011). Farrar, Doebbling, and Nix (2001) presented an extensive review on vibration-based damage identification which was later updated by Sohn et al. (2004).

Early works in vibration-based damage identification tried to identify changes in the system through shifts in modal frequency or mode shape. Brincker, Kirkegaard, Andersen, and Martinez (1994) and Brincker, Andersen, Kirkegaard, and Ulfkjaer (1994) demonstrated statistical techniques to identify damage from the shift in structural frequency considering the first two moments of frequency. Several other researchers (Biswas, Pandey, Bluni, & Samman, 1994; Mayes, 1991; Pandey, Biswas, & Samman, 1991; Yuen, 1985) have also tried to exploit the local information stored in identified mode shape to detect and locate the damage.

However, damage localisation based on the changes in dynamic properties may not be authentic (as discussed by Farrar et al. [1994]) due to the global nature of the dynamic response of the system. Also, damage detection is not guaranteed with modal change-based methods, especially for small and localised damage. These shortcomings of model-independent methods led researchers to focus on model-based approaches discussed next.

Model-based identification optimises stiffness or flexibility matrices of a primary finite-element model (FEM) to match the experimentally obtained modal data, and the damage is interpreted as changes in the updated system (Doebbling, Hemez, Barlow, Peterson, & Farhat, 1993; Hemez & Farhat, 1995; Li & Smith, 1995; Mottershead & Friswell, 1993). Sensitivity and optimisation-based updating algorithms are two major classes of model-based identification method. Sensitivity-based algorithm tries to minimise an error function based on its first-order
sensitivity with respect to perturbation of matrix elements. Solutions are then achieved using linear or nonlinear constrained optimisation techniques. An extensive review of these types of techniques is presented in the doctoral thesis of Hemez (1993).

Optimisation-based algorithms optimise an objective function constructed using target modal information and then update local parameters to identify damage (Bagchi, 2005; Fan & Qiao, 2011; Marwala, 2010). Although mode shapes contain local information that can be incorporated into the system matrices, the sensitivity largely depends on the selection of a proper objective function. Thus, in most cases, these algorithms are useful only for high-intensity damage. Systems with small damage intensity, therefore, demand a better method for damage identification. Apart from that, optimisation-based methods are generally iterative hence computationally expensive which causes the cost of damage detection to go up. Such high costs can potentially restrict their use in real-time health monitoring of structures. In this paper, a non-iterative damage identification technique is developed which can be integrated with a real-time damage detection algorithm.

1.1. Proposed approach

In this work, Eigenstructure Assignment (ESA)-based FEM updating technique is adopted to identify structural damage. ESA is a control-based technique that assigns desired eigenvalues and eigenvectors of a system in their proper locations by a gain matrix derived using state feedback (Andry, Shapiro, & Chung, 1983). ESA is used for FEM updating as well as for damage detection by several researchers (Bai, Datta, & Wang, 2010; Lim & Kashangaki, 1994; Sen & Bhattacharya, 2015; Yuan & Liu, 2014). Zimmerman and Kaouk (1992) presented an iterative approach to update system matrix using ESA technique. Constrained ESA approach was adopted by Lim (1995), whereas Schulz, Pai, and Abdelnaser (1996) used similar frequency response function assignment technique using frequency response spectra to improve accuracy.

Existing applications of ESA for damage identification are mostly formulated in physical space, and assignment is achieved through iterative algorithms. However, state space models of mechanical systems offer greater flexibility to identify damage through simultaneous updating of stiffness and damping matrices, which to our knowledge has not been attempted thus far in structural damage identification. The proposed method presents a one-step damage identification scheme through embedding identified eigenstructure in state space.

In modal comparison-based identification, modal information matching is only approximate and entirely depends on the selection of the objective function. In contrast, ESA is an analytical approach that embeds full information about the real system in the model. Table 1 highlights the salient differences between the proposed algorithm and modal comparison-based damage identification techniques. Similar to modal comparison-based methods, the proposed ESA-based algorithm is restricted to locally linear time-invariant systems. Hence, the methodology applies to damage scenarios that do not drive the structure into non-linearity under ambient or induced excitations, but rather, put their signature on the linear response of the structure.

To define the identification problem in state space, state space identification is performed on the response measured from the real system and a primary state space model of the system is prepared. The eigenstructure of the identified state transition matrix is used to update the primary state space model of the system (see Table 1).

The necessary condition for ESA is that the assignable eigenstructure must be compatible in the sense of order and orientation with the system model on which it is supposed to be assigned. However, in reality, the identified state transition matrix, as well as its eigenstructure, is very likely to be obtained in different orientation transformed by an unknown transposition matrix. Thus, to achieve the necessary compatibility, the identified eigenstructure must be mapped onto the desired direction.

Being coordinate independent, eigenvalues can be identified easily. However, to map the identified eigenvectors onto assignable orientations without knowing the required transposition matrix is difficult. To avoid this problem, the eigenstructure of the identified state transition matrix is first mapped from state space onto physical space in this paper. The eigenstructure in desired orientation is then reconstructed using this modal information which is subsequently used for ESA-based damage identification.

2. Eigenstructure assignment

ESA is a control-based technique commonly used to alter the eigenstructure of a system in order to achieve a target transient response. ESA uses feedback to define a controller or gain matrix to assign a target eigenstructure in to the system state matrix. This capability of ESA technique to change eigenstructure of any matrix through updating is exploited in this paper. Upon obtaining the actual eigenstructure from the real system through identification, ESA is adopted to assign this eigenstructure to a primary state space model of the system dynamics. Consider the discrete time description of the system dynamics as:

$$\bar{x}(k+1) = A\bar{x}(k) + Bu(k)$$
$$\bar{y}(k) = C\bar{x}(k)$$

(1)

where $\bar{x}(k)$ is the state vector at $k$th instant, $A$, $B$ and $C$ are state, input and output matrices, respectively, $\bar{u}(k)$ is the input vector (playing the role of a virtual control force), $\bar{y}(k)$ is the measured output vector. If an input sequence is selected in such a way that $\bar{u}(k) = K\bar{x}(k)$, then Equation (1) can be rewritten as:

$$\bar{x}(k+1) = A\bar{x}(k) + BK_x\bar{x}(k) = (A + BK_x)\bar{x}(k) = \hat{A}\bar{x}(k)$$

(2)

Thus, by a proper selection of the gain matrix, this method yields an altogether new system with $\hat{A}$ as its updated state transition matrix that has the same eigenstructure as desired by the designer. For a system described by Equation (1), Wonham (1967)’s theorem enables only eigenvalue embedding to alter the system and as such does not ensure an unique solution. Moore (1976) derived the necessary and sufficient conditions for a proper gain matrix $K_x$ for which the updated state matrix satisfies the uniqueness in the solution while assigning the desired eigenstructure in the system. The ESA algorithm used in this article is due to Moore (1976):
3. System identification

3.1. Discrete time stochastic subspace identification (SSI)

The system considered here is identified using its measured response time history in its state space. For any stochastic mechanical system, its $n^{th}$ order governing differential equation can be rewritten using $2n$ coupled first-order equations describing the continuous time dynamics of the stochastic system in state space as:

$$\mathbf{x}_t = \begin{bmatrix} \mathbf{q}_t \\ \dot{\mathbf{q}}_t \end{bmatrix}$$

(3)

$$\dot{\mathbf{x}}_t = \mathbf{A}_c \mathbf{x}_t + \mathbf{w}_t$$

(4)

where $\mathbf{w}_t$ is the process noise and the state vector $\mathbf{x}_t$ contains the displacements ($\mathbf{q}_t$) and velocities ($\dot{\mathbf{q}}_t$). The continuous time state transition matrix $\mathbf{A}_c$ is given in terms of the mass $\mathbf{M}$, stiffness $\mathbf{K}$ and damping $\mathbf{D}$ matrices as:

$$\mathbf{A}_c = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{M}^{-1} \mathbf{K} & -\mathbf{M}^{-1} \mathbf{D} \end{bmatrix}$$

(5)

The output vector which can be described as a function of measurable quantities like accelerations, velocities or displacements as:

$$\mathbf{y}_t = \mathbf{C}_a \dot{\mathbf{q}}_t + \mathbf{C}_v \mathbf{q}_t + \mathbf{C}_d \mathbf{q}_t$$

can be rewritten as:

$$\mathbf{y}_t = \mathbf{C}_c \mathbf{x}_t + \mathbf{v}_t$$

(6)

in terms of the measurement noise $\mathbf{v}_t$ and the output matrix $\mathbf{C}_c$ where

$$\mathbf{C}_c = \begin{bmatrix} \mathbf{C}_{d_b} - \mathbf{C}_a \mathbf{M}^{-1} \mathbf{K} \\ \mathbf{C}_v - \mathbf{C}_a \mathbf{M}^{-1} \mathbf{D} \end{bmatrix}$$

(7)

$\mathbf{C}_{d_b}$, $\mathbf{C}_v$, $\mathbf{C}_d$ are the output matrices for acceleration, velocity and displacement, respectively. This continuous time system can be sampled in discrete time to yield:

$$\mathbf{x}(k+1) = \mathbf{A}_d \mathbf{x}(k) + \mathbf{w}(k)$$

$$\mathbf{y}(k) = \mathbf{C}_d \mathbf{x}(k) + \mathbf{v}(k)$$

(8)
where $A_d$ and $C_d$ are the discrete time counterparts of $A_e$ and $C_e$ above. Using Kalman (1960)’s forward innovation technique, the same system can be described in discrete time domain as:

$$x(k+1) = A_d x(k) + K_e e(k)$$
$$y(k) = C_d x(k) + e(k)$$

(9)

where $e(k)$ is the innovation vector and $K_e$ is the Kalman gain matrix.

Discrete time SSI algorithm given by Van Overschee and De Moor (1996), (see page no: 61 in the book titled ‘Subspace identification for linear systems: Theory-Implementation-Applications’ [Van Overschee & De Moor, 1996]) is employed in this article to identify the system in its state space. This is a non-iterative approach to state space identification of any system to identify the underlying discrete time system model $(A_d, C_d$ and $K_e)$ and corresponding eigenstructure from the measurement vector $y(k)$ (commonly the acceleration time history).

### 3.2. Identification of assignable eigenstructure

To identify a set of stable roots of the system, SSI is performed with different model orders, and the corresponding eigenstructures are obtained in each case. However, one has to select the most likely eigenstructure from all these identified sets and use it as the assignable eigenstructure for ESA-based updating. ESA-based updating requires that the identified eigenstructure be of the same order and be in the same orientation as that of the baseline model. Additionally, it has to be ascertained that the assignable eigenstructure is achievable and feasible.

The continuous time state space model (Equations (4) and (6)) has the following eigenstructure:

- Eigenvectors : $\Phi_n = \begin{bmatrix} \lambda_n^{-1}\phi_n \\
\phi_n \end{bmatrix}$
- Eigenvalues : $\lambda_n = -\xi_n \omega_n \pm i\omega_n \sqrt{1 - \xi_n^2}$

(10)

where $\omega_n$, $\xi_n$ are natural frequencies and damping ratios in physical space for $n^{th}$ mode of the structure.

However, since the SSI is performed on a sampled time signal, the identified state matrices are in discrete time domain and does not conform to this particular structure. Therefore, the identified discrete time system $(A_d, C_d$ and $K_e)$ is first transformed into continuous time domain using zero-order-hold (ZOH) technique yielding $A_e$ and $C_e$. Pre-multiplying the eigenvector of the continuous time state matrix $A_e$ with the output matrix $C_e$ yields the mode shape coordinates in physical space at the predefined sensor locations:

- Continuous time system $(A_e, C_e)$
  - $\lambda_n$ = eigenvalue($A_e$)
  - $V_n$ = eigenvector($A_e$)
  - Physical space eigenvector: $\phi_n = C_e \times V_n$

Roots of the identified state matrix ($A_e$) conform to the special structure (see Equation (10)) from which the associated modal frequency and the damping ratio can be obtained.

Based on this information, all those identified roots for which frequency and damping ratio are not feasible (e.g. negative eigenvalue, the damping ratio being greater than 30%) can be eliminated. For the remaining roots, corresponding eigenvectors are compared against analytically obtained undamaged eigenvectors using modal assurance criterion (MAC) and only roots with MAC value $\geq 0.9$ are considered for the next step. This assures that the identified eigenstructure will be achievable.

In the following, the imaginary part of all these eigenvalues, i.e. $\text{Im}(\lambda_n)$ are plotted against increasing model order in a stabilisation diagram. Among the roots, those having physical significance stabilise over increasing model order; whereas those that are mere computational poles scatter away. Each stabilised root is associated with a particular mode of the system defined by a particular set of natural frequency and damping ratio ($\omega_n$ and $\xi_n$). For each mode number, all the identified $[\omega_n, \xi_n]$ sets are usually a little scattered. These values are plotted in 2D histograms with $\omega_n$ and $\xi_n$ constituting the independent axis and the mode values of these histograms are selected as the most likely $[\omega_n, \xi_n]$ set. This frequency damping ratio pair along with corresponding physical space eigenvector ($\phi_n$) constitutes the physical space eigenstructure.

These identified physical space eigenstructures are then used to reconstruct the eigenstructure in state space using Equation (10) which is then assigned to the baseline state space model of the system using ESA technique. Because the sensors typically collect responses pertaining to only a subset of all the structural degrees of freedom (DOFs), the baseline model must also be reduced to this subset of DOFs to achieve compatibility with the assignable eigenstructure.

### 4. Damage detection

The ESA-based updating alters every element of the state matrix independently to achieve the desired eigenstructure in the updated model. Thus, the updating is local, and the updated state transition matrix can be compared to the baseline model to interpret the local changes in terms of damage. This comparison, however, requires that the structure of the two matrices be identical (i.e. null matrix in the upper left block and identity matrix in upper right block of the same order of the FEM), which can be ensured by controlled updating of the elements of baseline state matrix through suitable selection of $B$ matrix.

The choice of $B$ matrix in the ESA algorithm described above is arbitrary as long as: (i) it does not affect controllability which can be ensured by specifying $\text{Rank}([I_n - B A]) \geq n$, (ii) after updating the upper half of the updated state matrix does not change. We choose $B$ as:

$$B = \begin{bmatrix} z(n) \\
B_1 \\
B_2 \end{bmatrix} : A_{\text{baseline}} = \begin{bmatrix} z(n) & I(n) \\
I(n) \end{bmatrix}$$

(11)

$$A_{\text{updated}} = A_{\text{baseline}} + B \begin{bmatrix} k_1 & k_2 \\
k_3 & k_4 \end{bmatrix} = \begin{bmatrix} -z(n) & I(n) \\
A_1 + B_1 k_1 + B_2 k_3 & A_2 + B_1 k_2 + B_2 k_4 \end{bmatrix}$$

(12)

where $z(n)$ and $I(n)$ are the null matrix and the identity matrix, respectively, of order $n$. This updated matrix, $A_{\text{updated}}$, thus retains the desired structure and can be compared with $A_{\text{baseline}}$.
Identification of location and intensity of damage is performed in two consecutive steps. In the first, the changes in the elements of stiffness matrix are used to identify the affected nodes $N_d$ and subsequently the set of damaged elements $E_d$ are determined using the connectivity information of those nodes. Depending on the problem at hand, damage intensity may be variously defined as overall loss of stiffness, the length of the dominant crack, loss of section modulus, etc. To quantify the intensity of damage for each element $E_i$ in $E_d$, nodal stiffness changes have to be interpreted in terms of elemental damage (loss of stiffness, crack length, thickness, etc.). However, because of the reduction in the model to a few numbers of DOFs only, damage intensity does not get reflected precisely through the nodal changes in the reduced model.

Therefore, in the second step, the damage intensity is estimated indirectly by establishing a functional relationship between nodal and elemental damage measure for each element $E_i$ in $E_d$. This is achieved by first constructing the full-scale structural model with a known amount of damage in that particular element and then reducing the model using SEREP (OCallahan, Avitabile, & Riemer, 1989) algorithm to the same order as in the reduced order undamaged model. The process is repeated for different intensities of damage in $E_d$. Thereafter comparing all these reduced order damage models to the undamaged model, a functional relation between nodal stiffness change and damage in $E_i$ is obtained. Using this relationship, the actual damage intensity in $E_i$ is then identified from the observed change in nodal stiffness. In the following, the proposed method is validated using three numerical and two laboratory experiments.

5. Numerical validation

The first numerical experiment is performed on a simple four-DOF structure so that the initial and updated system matrices can be listed explicitly. Following this, two other numerical experiments are performed on larger structures (a concrete beam with 82 DOFs and an aluminium plate with 121 DOFs) to explore the proposed method’s noise sensitivity and precision in localising different damage scenarios.

5.1. Numerical example 1: shear frame building

Two four-DOF lumped mass FEMs corresponding to damaged and undamaged condition (see Figure 1) of a shear frame building is prepared. The undamaged model is described by stiffness $K_{ud}$, mass $M_{ud}$ with 0.1% Rayleigh damping.

\[
K_{ud} = \begin{bmatrix}
700 & -700 & 0 & 0 \\
-700 & 2100 & -1400 & 0 \\
0 & -1400 & 3500 & -2100 \\
0 & 0 & -2100 & 4900
\end{bmatrix} \text{ kN/m;} \\
M_{ud} = \begin{bmatrix}
1500 & 0 & 0 & 0 \\
0 & 3000 & 0 & 0 \\
0 & 0 & 3000 & 0 \\
0 & 0 & 0 & 4500
\end{bmatrix} \text{ kg;} 
\]

A 25% stiffness reduction of the third story is defined as damage in this problem. The resulting damaged stiffness matrix ($K_d$) is:

\[
K_d = \begin{bmatrix}
700 & -700 & 0 & 0 \\
-700 & 1750 & -1050 & 0 \\
0 & -1050 & 3150 & -2100 \\
0 & 0 & -2100 & 4900
\end{bmatrix} \text{ kN/m;} 
\]

The acceleration time history of the damaged model under ambient vibration excitation is simulated using Newmark-beta algorithm. No noise is added to the response signal in this example (noise sensitivity of the proposed method is explored in the next two numerical experiments). The eigenstructure is identified from this response which is subsequently transformed to ensure compatibility during assignment as described previously. The updated stiffness matrix ($K_{up}$) extracted from the
updated system state matrix is:

\[
K_{up} = \begin{bmatrix}
    708 & -715 & 2 & 1 \\
    -715 & 1743 & -1016 & 5 \\
    2 & -1016 & 3124 & -2140 \\
    1 & 5 & -2140 & 4928
\end{bmatrix} \text{kN/m};
\]

Figure 2. Schematic diagram for numerical experiment of concrete beam and aluminium plate.

### 5.2. Numerical example 2: concrete beam

The FEM of the 6 m long concrete beam is created with 40 Euler–Bernoulli beam elements (see Figure 2 and Table 2) and acceleration response is simulated under ambient vibration excitation. To demonstrate noise sensitivity of the proposed method, the simulated response signal is contaminated with noise of various signal-to-noise ratios (SNR) (2, 5, 10 and 20%) before using it in stochastic SSI algorithm. A series of case studies are performed on this beam for different levels of induced damage listed in Table 3.

Each element of the beam is instrumented to measure the vertical acceleration response. We choose the 9th and the 22nd elements as damaged: their stiffness values are reduced by 30 and 70%, respectively. The acceleration signal for each node is sampled at 500 Hz under white noise excitation which is then contaminated by noise. State space identification is performed on measured response signal for different model order, and stable and physical roots are identified manually based on reasonable constraints (e.g., damping value negative and must be < 30%, imaginary part must be stable with increasing model order, associated physical space eigenvector should give MAC value ≥ 0.9 when compared with its analytical counterpart obtained from the baseline model, etc.).

The primary undamaged FEM is first reduced to meet compatibility issues and subsequently standardised so that the reduced order model can represent the undamaged state of the system properly. This can be achieved by an additional updating step of primary FEM using the undamaged response history. The updated system can then be accepted as baseline model representing the undamaged state of the system. However, as the baseline model is exactly known in this numerical experiment, this additional updating step is not attempted. ESA-based damage identification is then performed on the baseline model to match its eigenstructure to the desired set.

The affected elements and the extent of their damage are then ascertained in terms of reduced stiffness. Figures 3 and 4 compare the identified damage vs. actual damage in the beam at different SNR. It is observed that the proposed method can successfully identify damage up to 10% SNR. However, at noise levels higher than 20% SNR, although the damaged elements and the extent of their damage are correctly identified, some undamaged members are also falsely suspected as damaged.

### 5.3. Numerical example 3: aluminium plate

The aluminium plate is modelled with 100 Mindlin–Reissner plate elements with up to four damaged elements (see Figure 2 and Table 4). Damage once again is defined as loss of elemental stiffness. The response history is sampled at 500 Hz and then contaminated with up to 40% noise. Vertical acceleration sensors are assumed to be placed at each node. To match the dimension of identified eigenvectors, the undamaged model is reduced to 110 DOFs. Details of the several case studies are listed in Table 5. ESA-based damage identification is performed as described earlier. Figure 5 demonstrates the location and extent of damage predicted by the proposed approach vis-a-vis the actual. However, it is observed that with high level of noise (40%), the proposed method fails to identify damage location as well as intensity properly.

After performing three sets of numerical experiments, it is evident that proposed method is fit to be used as a damage identification technique and also it exhibits a good amount of robustness against noise. We also performed two laboratory experiments to justify our conclusion based on numerical experiments.
6. Experimental validation 1

The first laboratory experiment is performed on a two span continuous beam of dimension $0.05 \text{ m} \times 0.01 \text{ m} \times 1.65 \text{ m}$. The experimental beam is fixed at one end and simply supported at a distance 0.8 m and 0.85 m from the fixed end (see Figure 6). Damage is induced by making a notch at a distance 1.1 m from the fixed end. Six B&K of type 4503 accelerometers are used at distances 0.2, 0.4, 0.6, 1.0, 1.2 and 1.4 m from the fixed end of the beam. DEWESoft high-speed data acquisition system is employed in this experiment for recording the ambient response from the accelerometers at 1000 Hz sampling frequency. A sample ambient response signal is presented in Figure 7.
Figure 5. Actual and identified damage for different case studies on aluminium plate. Subfigure (a), (b), (c), (d), (e), (f) describes actual and identified damage for six different case studies. For each subfigure, left figure presents the original damage location and the right figure presents identified location and intensity using heatmaps.

Table 4. Details of FEMs of aluminium plate.

<table>
<thead>
<tr>
<th>Structure</th>
<th>Aluminium plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity</td>
<td>3200 MPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.334</td>
</tr>
<tr>
<td>Dimension</td>
<td>0.25 m × 0.45 m × 0.006 m</td>
</tr>
<tr>
<td>Density</td>
<td>2796.9 Kg/m³</td>
</tr>
<tr>
<td>Element type</td>
<td>Mindlin–Reissner Plate element</td>
</tr>
<tr>
<td>Boundary condition</td>
<td>Cantilever (clamped-free-free-free)</td>
</tr>
<tr>
<td>Number of elements</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 5. Details of case studies for aluminium plate.

<table>
<thead>
<tr>
<th>Case</th>
<th>Damaged element</th>
<th>Damage percentage (%)</th>
<th>Noise level (% SNR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>44, 45, 54, 55</td>
<td>60 % each</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>44, 45, 54, 55</td>
<td>60 % each</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>44, 45, 54, 55</td>
<td>20 % each</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>44, 78</td>
<td>60 %, 20 %</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>44, 45, 54, 55</td>
<td>20 % each</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>44, 78</td>
<td>20 % each</td>
<td>40</td>
</tr>
</tbody>
</table>
Following the method described in the numerical example, the stabilised roots and corresponding eigenstructures are separated from all the roots identified thorough SSI algorithm using the stabilisation plot. Figure 8(a) presents the stabilisation plot for this study. Upon identifying the assignable eigenstructure and subsequently assigning this damaged eigenstructure in the primary standardised undamaged model of the system, the damage is localised at the sixth segment of the beam between fourth and fifth accelerometers. Figure 8(b) presents the result of damage localisation study for different damage location and magnitude from which the damage is identified to be 26.3% of the primary EI value of that segment.

### 7. Experimental validation 2

A cantilever aluminium plate of dimension 0.45 m × 0.25 m × 0.006 m is adopted for the second experimental validation of the proposed method (Figure 9). The undamaged plate is excited by the roving hammer method in which an impact hammer of type B&K 8206-002, 2.3 mV/N hits each of the 54 nodes one at a time and the acceleration signal for each is recorded at node 44 with a sampling frequency of 2048 HZ using the 3560C B&K portable pulse acquisition front end acquisition system (PULSE, Analyzers and Solutions, Release, 2006).

The primary FEM of the undamaged plate (5 × 10 Mindlin–Reissner elements with parameters as in Table 4) is updated using these signals and ESA algorithm to yield the undamaged baseline model. Damage is then introduced in the plate by indenting an area of 0.05 m × 0.05 m to an average depth of 0.003 m (as shown in Figure 9). In the FEMs of the plate, this area corresponds to the 34th element connected to nodes 40, 41, 46 and 47. The roving impact hammer test is repeated and the 54 acceleration signals from the damaged plate are recorded as before.

The FRF spectra for the plate in both damaged and undamaged conditions are presented in Figure 10 and the first four natural frequencies are listed in Table 6 (columns 2 and 4). Clearly, there is little difference between the natural frequencies in the damaged and undamaged states (4% or less) which shows that frequency-based damage identification may not be successful in the case of localised damage. Table 6 also lists the frequencies from the primary and the baseline models of the undamaged plate (columns 1 and 3). After updating, the baseline model has the same natural frequencies as the actual undamaged plate and, therefore, this model is accepted for subsequent damage identification.

Following the method outlined in the numerical example for the plate above, location and extent of damage (defined here as

### Table 6. Natural frequencies of primary model, undamaged plate, baseline model and damaged plate.

<table>
<thead>
<tr>
<th>Primary FEM of undamaged plate (Hz)</th>
<th>Actual undamaged plate (Hz)</th>
<th>Baseline FEM of undamaged plate (Hz)</th>
<th>Actual damaged plate (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.5</td>
<td>21.0</td>
<td>21.0</td>
<td>20.3</td>
</tr>
<tr>
<td>93.3</td>
<td>86.8</td>
<td>86.7</td>
<td>86.0</td>
</tr>
<tr>
<td>157</td>
<td>161</td>
<td>161</td>
<td>155</td>
</tr>
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<td>316</td>
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Figure 8. System identification and damage localisation. (a) Stabilisation plot for the beam experiment (b) Result of the damage localisation study.

Figure 9. Experimental set-up for aluminium plate. Subfigure (a) presents the location of damage element and subfigure (b) presents the location of sensor.

Figure 10. Frequency response plot for undamaged (left) and damaged (right) system.
the loss of thickness) are identified. Figure 11 demonstrates that the damaged area is contained by elements 29, 30, 34, 35, 39 and 40 with the most likely element being 34. The maximum extent of damage is identified as 42% reducing to zero over the length of one to two elements which compares favourably with the 50% damage inflicted on element 34 in this experiment.

8. Conclusions

This paper describes for the first time a non-iterative algorithm for damage identification (location as well as intensity) using control theory-based ESA that uses global system properties to effect element-wise updating of system matrices. To achieve a compatible target eigenstructure (both in order and orientation) for ESA algorithm, the identified eigenstructure is first mapped from state space to physical space which in turn is used to reconstruct the state space eigenstructure in the required order and orientation. During state space identification of the damaged structure, the non-physical roots are removed using stabilisation plots and relevant practical constraints which partially mitigates the effect of noise in the signal. Using limited structural response data, the methodology can handle multiple locations of damage and multiple measures of damage (loss of stiffness, loss of area, etc.).

The non-iterative nature of the algorithm reduces computational cost to a great extent which can also be exploited for real-time damage identification of systems. In comparison, most existing modal comparison-based damage identification algorithms are iterative in nature and not sensitive to low-intensity damage. Since these methods are modal comparison based, thus unless the modal sequence is explicitly known a priori, damage-induced modal switching may often result in completely infeasible solutions. In contrast, the proposed method does not require the information about modal sequence and thus robust against modal switching.

The adoption of ESA algorithm necessarily restricts the proposed methodology to linear time-invariant systems. Hence, the methodology applies to damage scenarios that do not drive the structure into non-linearity under ambient or induced excitations, but rather, put their signature on the linear response of the structure. Also, the identification is dependent on the quality of the primary FEM: without a good prior model of the undamaged system, spurious or closely spaced modes may not be resolved. Finally, the computation load increases quadratically as the number of sensor locations and hence for large structures, damage may be identified by: (i) first segmenting the system into substructures using limited instrumentation and then (ii) identifying the affected substructures(s) for which detailed identification will be undertaken.

Disclosure statement

No potential conflict of interest was reported by the authors.

References


