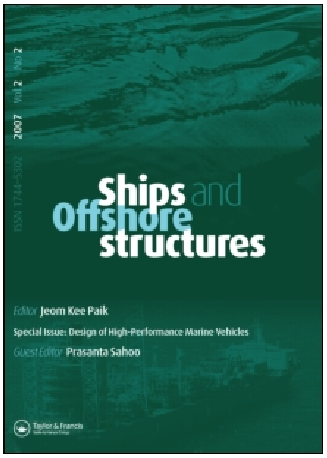


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# On improved analytical method for stress-strain relationship for plate elements under axial compressive load

Suhas Vhanmane [a]; Baidurya Bhattacharya [b]
[a] Research and Coordination Division, Indian Register of Shipping, Powai, Mumbai, India
[b] Department of Civil Engineering, Indian Institute of Technology Kharagpur, Kharagpur, India

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# On improved analytical method for stress–strain relationship for plate elements under axial compressive load

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Suhas Vhanmane[1] and Baidurya Bhattacharya

[1] *Research and Coordination Division, Indian Register of Shipping, 52-A, Adi Shankaracharya Marg, Opposite Powai Lake, Powai, Mumbai 400 072, India*

[2] *Department of Civil Engineering, Indian Institute of Technology Kharagpur, Kharagpur 721 302, India*

**Abstract:** Accurate prediction of load carrying capacity of the ship hull is very important for its structural safety. The stress–strain relationship of hull structural element plays a vital role in this. A simple analytical method to predict the average stress–average strain relationship for plating between stiffeners in ship structures is proposed, in which two different methods are combined: the membrane stress method by Paik *et al.* (2000) involving large elastic deformation theory and the rigid plastic collapse mechanism theory by Yao *et al.* (1991). The former governs the stress–strain relationship up to ultimate strength while the latter is used beyond ultimate strength. A MATLAB code is developed for this purpose. The plating between stiffeners is analysed under axial load for different aspect ratio values. The present results are compared with published FEA results. It is concluded that the present method is quite accurate for deriving the average stress–average strain relationship of ship hull plating.

**Key words:** Ultimate strength, compressive load, elastic large deformation, rigid plastic collapse mechanism, average stress–average strain relationship, ship hull.

The opinions expressed in this paper are those of the authors and do not necessarily reflect those of the Indian Register of Shipping.

## NOTATION

| | | |
|---|---|---|
| $a$ | = | Length of stiffened panel between transverse support frames |
| $b$ | = | Breadth of plating between longitudinal stiffeners |
| $D$ | = | Flexural rigidity $= Et^3/12(1-\upsilon^2)$ |
| $E$ | = | Modulus of elasticity of plate material |
| $m$ | = | Buckling half wave, taken as an integer satisfying $a/b \le \sqrt{m(m+1)}$ |
| $w_o$ | = | Initial deflection function for the plating |
| $\beta$ | = | Reduced slenderness ratio of plating between stiffeners |
| $\sigma_Y$ | = | Yield strength of plating |
| $\sigma_{rc}$ | = | Welding induced compressive residual stress in the plating |
| $\sigma_{rt}$ | = | Welding induced tensile residual stress in the plating |
| $\sigma_{xav}$ | = | Average value of axial compressive stresses |
| $\sigma_{\max}$ | = | Maximum value of axial compressive stresses |
| $\varepsilon_{xav}$ | = | Average value of strain due to axial compressive stresses |
| $\varepsilon_Y$ | = | Yield strain at yield strength of plating |

*Corresponding Author*:
Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Kharagpur 721 302
India
E-mail: baidurya@iitkgp.ac.in

## INTRODUCTION

A ship hull is a complex structure mainly consisting of unstiffened/stiffened plates, longitudinals, frames, transverses, etc. and is subject to longitudinal bending, transverse bending and torsion. A primary load effect is axial compression induced due to longitudinal bending. Behaviour of structural element under axial compressive load is very important.

The ultimate bending strength gives the maximum bending that a ship hull girder can sustain beyond which the hull can deform but carry progressively lesser loads. In determining the ultimate capacity, it is important that the reduction in load carrying capacity of each structural

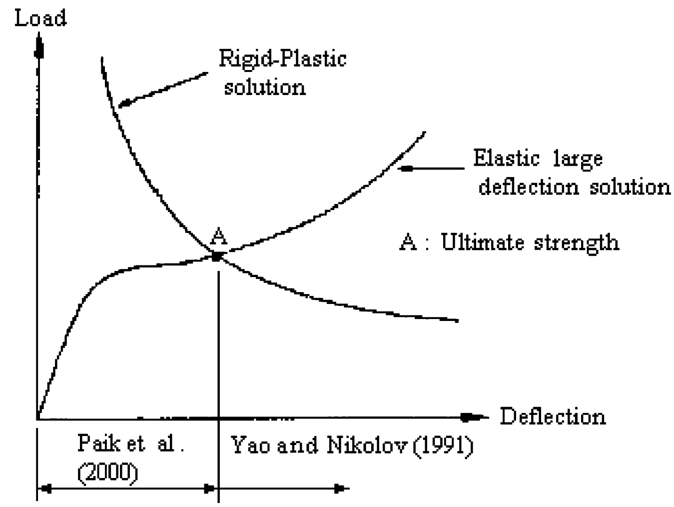


*Figure 1* Combined elastic large deflection and rigid plastic analysis.

element, after it attains ultimate strength, should be considered; thus, 'progressive collapse analysis' approach is the most preferred method to determine ultimate strength of the ship hull. The accuracy of progressive collapse analysis is governed in turn by the accuracy of the stress–strain relationship and ultimate strength of the structural elements. Although the finite element analysis (FEA) on a ship's hull incorporating geometrical as well as material nonlinearities can be used to accurately derive the hull's ultimate bending capacity, this type of analysis may require a large amount of manpower and computational resources. A simple analytical method, though less accurate, may be preferred in such cases instead of a detailed FEA, at least for the first cut.

Smith (1977) proposed a simple method for progressive collapse analysis of box girder structures under longitudinal bending by considering progressive stiffness loss due to buckling and yielding of the structural element, which was an extension of the method proposed by Caldwell (1965). Smith suggested elasto-plastic large deflection analysis by finite element method (FEM) to obtain stress–strain relationship with progressive stiffness loss arising from bucking and yielding of the structural element. Such analysis is expensive and time consuming in the case of large number of elements such as in a ship hull.

Yao and Nikolov (1991) developed a simple analytical method to simulate the progressive collapse behaviour of a ship's hull girder subjected to longitudinal bending. In this, buckling and yielding have been considered in both stiffener and plate elements. The relationship between average compressive stress and deflection has been derived from elastic and plastic mechanism analyses such that their intersection point gives the compressive ultimate strength. The stress–strain relationship of an isolated plate has been derived analytically based on the results of elastic large deflection analysis and plastic mechanism analysis assuming rigid- perfectly plastic material. The elastic analysis takes into account the influences of initial imperfections, initial deflection and welding residual stresses.

A simplified method to predict the ultimate compressive strength (and not the full stress–strain relationship) of plating was proposed by Paik and Pedersen (1996). They used three pre-defined plastic collapse modes whereas Yao and Nikolov (1991) had used only two such modes. The point of ultimate strength is obtained as the intersection of the load–deformation curve from elastic large deformation theory and that from rigid plastic mechanism theory. The authors then compared ultimate strength results with FEA.

Paik *et al.* (2000) formulated a simple closed form expression without plastic collapse mechanism to calculate ultimate strength of ship plating subjected to a combination of longitudinal axial load, lateral pressure and edge shear. The governing equilibrium and compatibility differential equations of large deflection plate theory are solved and membrane stresses inside the plating are calculated. The ultimate strength formula is derived assuming that the plate edges yield in ultimate limit state. The validity of formulation of Paik *et al.* (2000) is confirmed by comparing with experimental results and FEA. The emphasis, again, was on the formulation of ultimate strength and not on the stress–strain relationship.

In the present study, the focus is on formulation of average stress–average strain relationship for plating between stiffeners. The ultimate strength is determined by the method of Paik *et al.* (2000). Further, the average stress–average strain relationship is derived analytically by the combination of Paik *et al.* (2000) and Yao and Nikolov (1991) as follows (Figure 1): Up to the point of ultimate compressive strength, the average stress–average strain relationship follows elastic large deflection analysis proposed by Paik *et al.* (2000). After the point of ultimate compressive strength, the average stress–average strain relationship follows rigid perfectly plastic mechanism proposed by Yao and Nikolov (1991).

The present idea is applied to a plate between stiffeners with initial imperfections (in the form of initial deflection

 

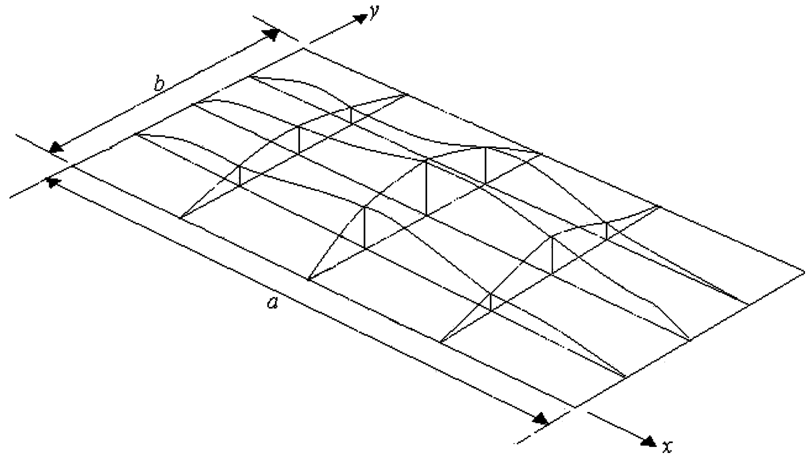


*Figure 2* Initial deflected shape of plating between the stiffeners.

and residual welding stresses). The average stress–average strain relationship is compared with FEA results taken from Yao and Nikolov (1991) who performed large elasto-plastic deformation analysis. The advantage of the present method is that it results in an analytical expression considering two different methods and gives results that are closer to the presumably accurate FEA solutions. The results show notable improvement over Yao and Nikolov's (1991) method.

The following two sections briefly review the membrane stress theory (which is based on the elastic large deformation plate theory) and the rigid plastic mechanism theory—both of these are central to the subject of this paper. As stated above, these two theories are combined in the present method to obtain the average stress–average strain relationship for plating between stiffeners. A set of numerical examples involving unstiffened rectangular plates with different aspect ratios is provided at the end.

## MEMBRANE STRESS THEORY

In this method, suggested by Paik *et al.* (2000), the membrane stresses inside the plate are determined by solving the well-known nonlinear governing differential equations of large deflection plate theory. The plate collapses when the maximum membrane stress reaches the yield stress. This section describes the solution to the governing compatibility differential equation of imperfect plating approach to obtain the membrane stress distribution inside the plate.

A simple expression for evaluating the membrane stresses is analytically given taking into account the influence of initial deflection and residual welding stresses. Figure 2 shows the initial deflected shape of plating due to imperfect manufacturing process. The plating between stiffeners is considered as simply supported at all edges. The initial deflection, $w_0$, of the plating is simplified and expressed as a Fourier series that includes only the buckling mode initial deflection

$$w_0 = A_0 \sin\frac{m\pi x}{a}\sin\frac{\pi y}{b} \tag{1}$$

where, $A_0$ = the buckling mode initial deflection amplitude; $m$ = buckling mode half wave number in the $x$ direction.

Similarly, the deflection due to axial compressive load is given by

$$w = A\sin\frac{m\pi x}{a}\sin\frac{\pi y}{b} \tag{2}$$

where $A$ = unknown amplitude of the added deflection function.

In the idealised welding induced residual stress distribution used in the present method, the tensile residual stresses of magnitude, $\sigma_{rt}$, are developed at the edges of the plating, that is, along the welding line; and the residual compressive stresses, $\sigma_{rc}$, are developed in the middle of part of the plating. The breadth of the tensile residual stress zone is obtained by equilibrium condition (Figure 3) as follows:

$$2b_t = \frac{\sigma_{rc}}{\sigma_{rc} - \sigma_{rt}} b \tag{3}$$

Hence, the residual distribution may be expressed by (Paik and Thayamballi 2003)

$$\begin{aligned}\sigma_r &= \sigma_{rt} \text{ for } 0 \le y < b_t\\ \sigma_r &= \sigma_{rc} \text{ for } b_t \le y < b - b_t\\ \sigma_r &= \sigma_{rt} \text{ for } b - b_t \le y \le b\end{aligned} \tag{4}$$

The compatibility differential equation for an initially deflected plate is given as

$$\begin{aligned}&\frac{\partial^4\phi}{\partial x^4} + 2\frac{\partial^4\phi}{\partial x^2\partial y^2} + \frac{\partial^4\phi}{\partial y^4} =\\ &E\left[\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2 - \left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right) - \left(\frac{\partial^2 w_0}{\partial x\partial y}\right)^2\right.\\ &\left.\quad + \left(\frac{\partial^2 w_0}{\partial x^2}\right)\left(\frac{\partial^2 w_0}{\partial y^2}\right)\right]\end{aligned} \tag{5}$$

where $w$ = Total deflection of the plating, $\phi$ = Airy stress function, $E$ = Modulus of elasticity of plate material.

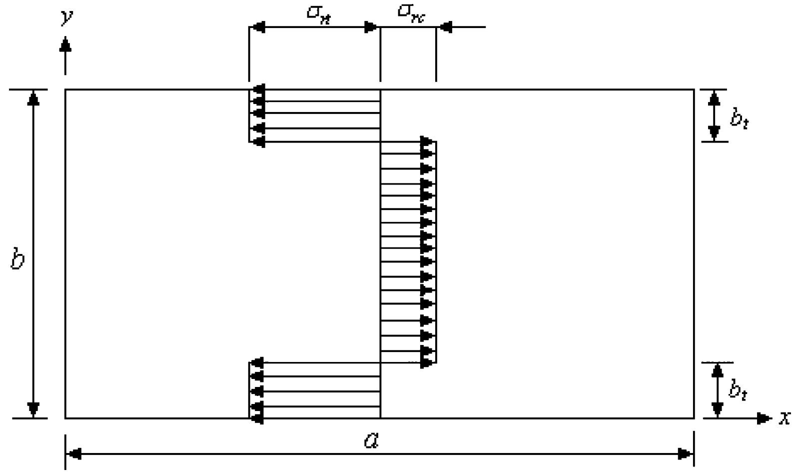


*Figure 3* Idealised welding residual stress distribution of plating between stiffeners.

After substituting Equations (1), (2), (4) into Equation (5) and simplifying the Airy stress function, the membrane stresses in the plating in $x$ and $y$ direction, respectively, are determined by

$$\sigma_x = \frac{\partial^2\phi}{\partial y^2} = \sigma_{xav} + \sigma_r - \frac{m^2\pi^2 E}{8a^2}\left(A^2 - A_0^2\right)\cos\frac{2\pi y}{b} \tag{6}$$

$$\sigma_y = \frac{\partial^2\phi}{\partial x^2} = -\frac{\pi^2 E}{8b^2}\left(A^2 - A_0^2\right)\cos\frac{2m\pi x}{a} \tag{7}$$

Amplitude of the total deflection $A$, can be obtained by the Galerkin method, satisfying the equilibrium condition

$$\int_0^a\int_0^b \left[D\left(\frac{\partial^4(w-w_0)}{\partial x^4} + 2\frac{\partial^4(w-w_0)}{\partial x^2\partial y^2} + \frac{\partial^4(w-w_0)}{\partial y^4}\right) - t\left(\frac{\partial^2\phi}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2\phi}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2\phi}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}\right)\right] \times \sin\frac{m\pi x}{a}\sin\frac{\pi y}{b}\,dx\,dy = 0 \tag{8}$$

The maximum compressive membrane stresses for plating with welding residual stresses are determined at $y = b_t$ or $y = b - b_t$ as

$$\sigma_{\max} = \sigma_{xav} + \sigma_r - \frac{m^2\pi^2 E}{8a_2}\left(A^2 - A_0^2\right)\cos\frac{2\pi b_t}{b} \tag{9}$$

The ultimate strength reaches when the maximum membrane stresses inside the plating at $y = b_t$ or $y = b - b_t$ equals the plate material yield strength, $\sigma_Y$. The corresponding $\sigma_{xav}$ gives the ultimate strength of the plating.

The average stress–average strain relationship up to ultimate strength is given by

$$\varepsilon_{xav} = \frac{\sigma_{xav}}{E} + \frac{m^2\pi^2}{8a^2}\left(A^2 - A_0^2\right) \tag{10}$$

## RIGID–PLASTIC MECHANISM THEORY

The relationship between stress and deflection is derived according to the plastic mechanism analysis assuming rigid-perfectly plastic material as proposed by Yao and Nikolov (1991). For the rigid–plastic deflection, two modes of collapse mechanism are considered in the load–deflection relation. The rigid parts (such as trapezoids and triangles shown in the Figure 4) cannot deform. However, they can move in (i) the original plane, and (ii) in the out-of-plane direction, which is perpendicular to the edges (assumed rigid). It is by these movements that the external forces do work according to the principle of virtual work. This external work is balanced by the virtual internal energy dissipation for an applicable, kinematically admissible collapse mechanism ($\alpha <= 1.0$ and $\alpha >= 1.0$ as shown in Figure 4). Depending on the aspect ratio, the two sets of plastic mechanism may exist as illustrated in Figure 3. For each mechanism, the following relationships are used as given in Yao and Nikolov (1991).

$$m_{45} + \left(\frac{1}{\alpha} - 1\right)m_{90}/2 = \left(\frac{2}{\alpha} - 1\right)\left(\frac{\sigma}{\sigma_Y}\right)A/t,\ \alpha \le 1.0 \tag{11}$$

$$m_{45} + (\alpha - 1)m_0/2 = \left(\frac{\sigma}{\sigma_Y}\right)A/t,\ \alpha \ge 1.0 \tag{12}$$

where,

$$\begin{aligned} \alpha &= a/bm \\ m_{90} &= 1 - \left(\frac{\sigma}{\sigma_Y}\right)^2 \\ m_0 &= 2m_{90}/\sqrt{1 + 3m_{90}} \\ m_{45} &= 4m_{90}/\sqrt{1 + 15m_{90}} \end{aligned} \tag{13}$$

The average stress–average strain relationship according to plastic mechanism analysis is expressed as

$$\varepsilon_{xav} = \frac{\sigma_{xav}}{E} + 2\frac{m^2}{a^2}\left(A^2 - A_0^2\right),\ \alpha \le 1.0 \tag{14}$$

 

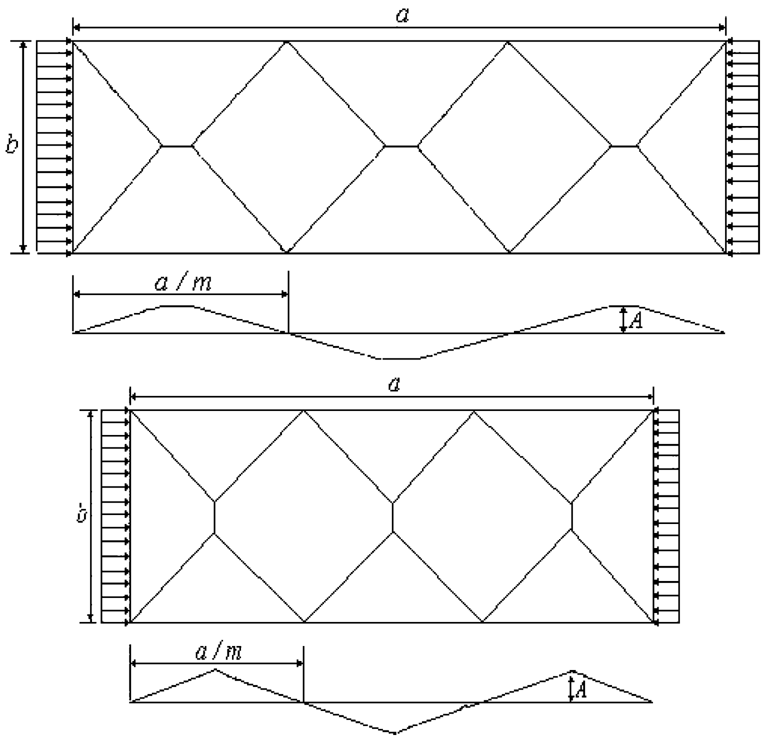


*Figure 4* Plastic collapse mechanism of the plating between stiffeners.

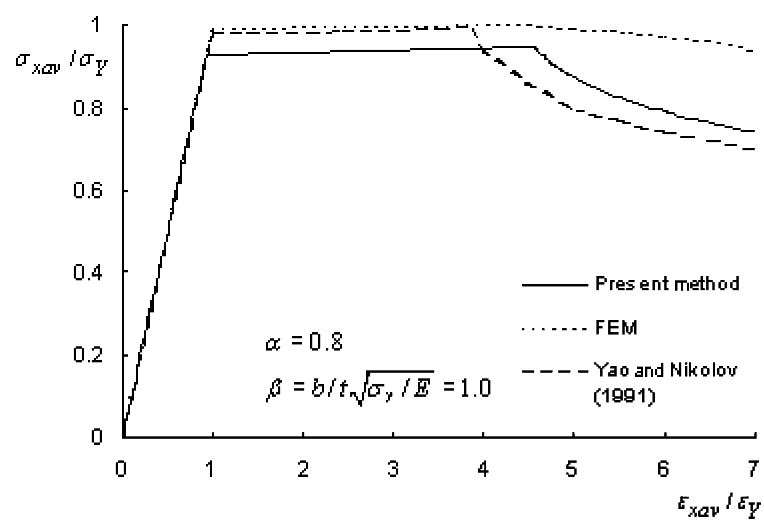


*Figure 5* Comparison of results, $\beta = 1.0$.

$$\varepsilon_{xav} = \frac{\sigma_{xav}}{E} + 2\frac{m^2}{ab}\left(A^2 - A_0^2\right), \alpha \geq 1.0 \qquad (15)$$

## NUMERICAL EXAMPLE

In the elastic region, the stress–strain relationship follows Equation (10). The strain at ultimate strength and then after is evaluated by Equation (11) and Equation (14) or Equation (12) and Equation (15) depending upon the aspect ratio of the plate. A MATLAB code is developed for this purpose incorporating the above stated equations. To check the effectiveness of the average stress–average strain relationship of the plates mentioned in the present method, the results are compared with the FEM results as well as those by Yao and Nikolov (1991) in Figures (5)–(9). The FEM analysis results used for comparison are obtained by Yao and Nikolov (1991) performing large elasto-plastic deformation analysis. Good correlations are observed between the results of the FEM analysis and the present improved method over Yao and Nikolov's (1991) method. The values of FEM and Yao and Nikolov (1991) results are taken from the plot for comparison.

From the Figures (5)–(9), it is clearly seen that the average stress–average strain relationship, evaluated by the present method, is an improvement over Yao and Nikolov

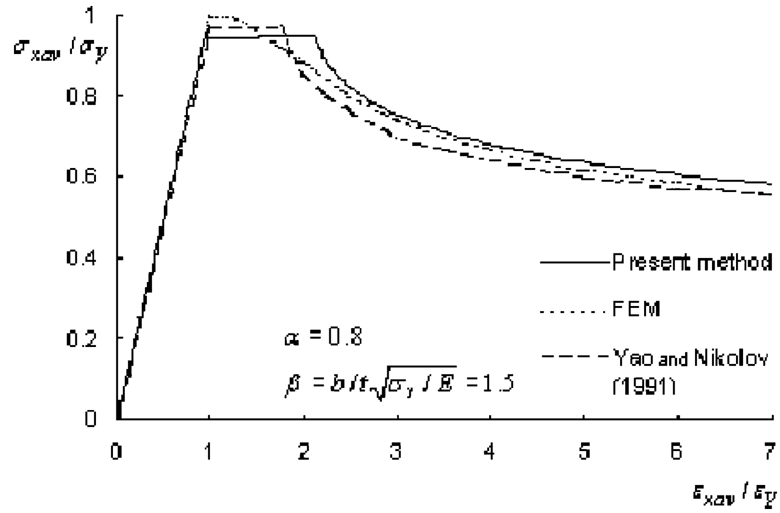


*Figure 6* Comparison of results, $\beta = 1.5$.

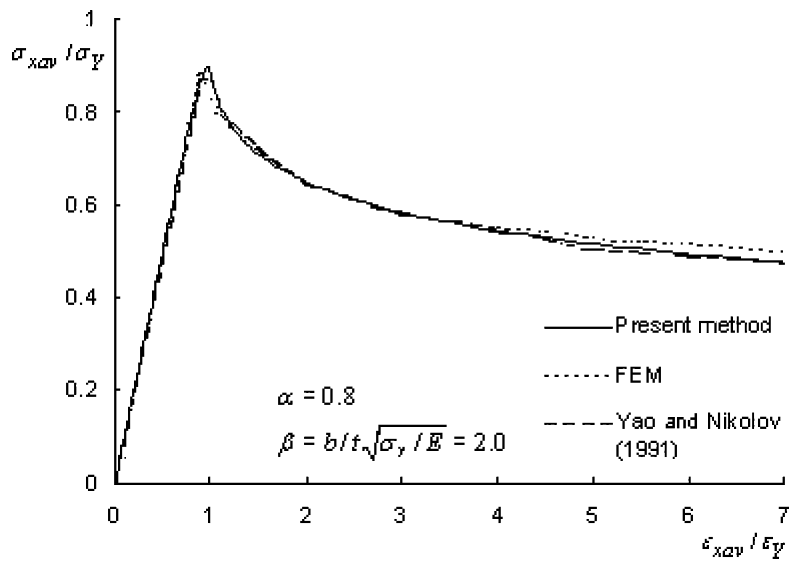


*Figure 7* Comparison of results, $\beta = 2.0$.

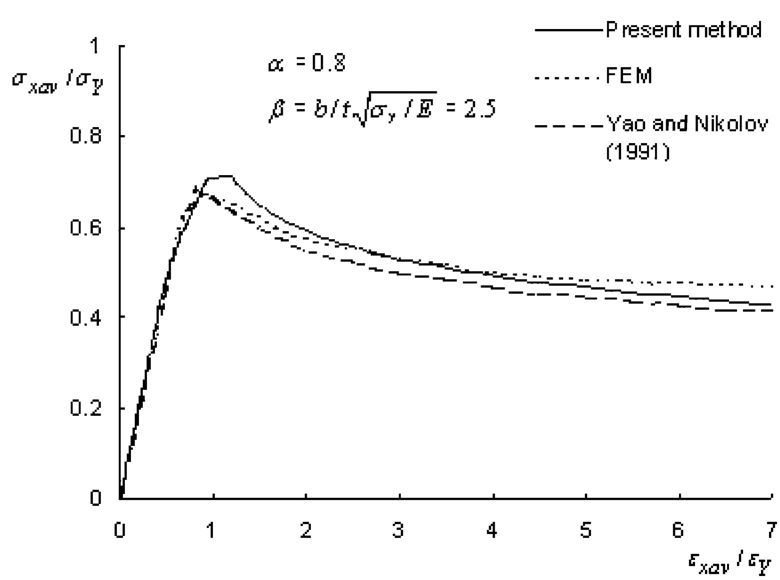


*Figure 8* Comparison of results, $\beta = 2.5$.

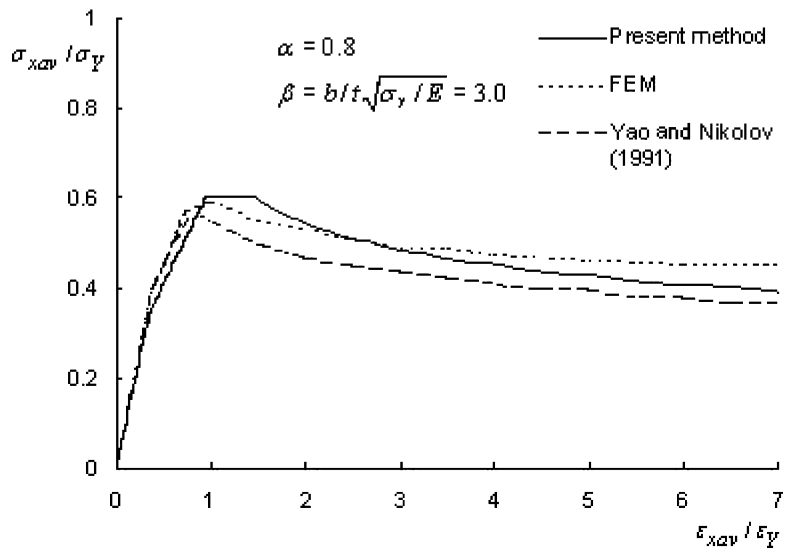


*Figure 9* Comparison of results, $\beta = 3.0$.

(1991) results, particularly in the post-ultimate region. Also, the improvement increases with plate slenderness ratio. Though the ultimate strength of stocky plates (lower aspect ratio, $\beta = 1.0$) is little underestimated, the post-ultimate stress–strain relationship is more accurate compared with the Yao and Nikolov (1991) results.

## CONCLUSION

Accurate stress–strain relationship plays an important role in the study of ultimate strength estimation. Particularly, the stress–strain relationship in the post-ultimate region is very important as it indicates the reduction in the capacity of the ship hull section. This behaviour strongly depends on the element behaviour of section, that is, whether the capacity reduction beyond their ultimate strength in the elements is correctly accounted for or not. Two different methods proposed by earlier authors are combined to obtain more accurate stress–strain relationship of the plating between stiffeners in the ship structures. The ultimate strength and the corresponding strain are evaluated by membrane stress theory within elastic region while rigid–plastic collapse mechanism is used to evaluate the strain beyond ultimate strength considering buckling and yielding. Thus a simple analytical method is improved for accurate determination of behaviour of plating under axial compressive load taking into account the effect of initial deflection and welding residual stresses. This idea gives encouraging results, particularly in the post-ultimate region. The comparison with the referred results shows very good agreement between the present method and FEA over Yao and Nikolov (1991) method. Therefore, the present stress–strain relationship can be used on ultimate strength estimation methodology of ship hull girder.

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