Proceedings of ICTACEM 2010
International Conference on Theoretical, Applied, Computational and Experimental Mechanics
December 27-29, 2010, IIT Kharagpur, India
ICTACEM-2010/397

Identification of Bouc-Wen model parameters using Extended Kalman filter with adaptive process and measurement covariance matrices

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ABSTRACT

Modern day structural health monitoring involves prediction of structural health for possible future load cases for which structure may behave nonlinearly and thus rendering its simplistic linear predictor model obsolete. Among the existing nonlinear material models Bouc-Wen hysteresis model drew most of the attention in recent past due to its wide applicability for different material hysteresis types and ease of implementation. The accuracy of the response predicted by this model entirely depends on how correctly the model parameters have been selected. However due to the inherent nonlinearity and complexity in the model existing parameter identification algorithms are not always certain to produce exact parameter values using limited computational resources. In this article we demonstrate a new technique based on Extended Kalman filtering with adaptive selection of state and measurement covariance matrix to identify parameters of the nonlinear material model with the objective to reduce computational expense. Identification is performed using two different methods: first, in the "iterative" approach, in each iteration step the nonlinear model with estimated parameter is simulated and response is considered to be erroneous measurement which needs to be filtered, whereas, in the second ("sequential") approach, at each time step current estimate of parameter is used to simulate the model to predict response for next time step and subsequently filtering is performed in real time and parameters are updated at each time step i.e. online. Pros and cons for both these methods are discussed and a conclusive suggestion has been given based on its field of applicability.

Keywords: Bouc-Wen Model, Parameter identification, Nonlinear modeling, Extended Kalman filtering

1. INTRODUCTION

Research in structural health monitoring includes predicting the response of existing structures subjected to loading beyond their prescribed design limit. This prior capacity estimate helps to take decision whether the service life of existing and sometimes aging structures can further be prolonged under changed loading scenario without compromising required safety and serviceability restrictions. However, this additional load can cause nonlinear behavior in the structure for which response cannot be predicted by a simple linear model. In order to have a reliable nonlinear model researchers rely on nonlinear hysteretic material models and of them Bouc-Wen model is most accepted in recent research. The ease

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of implementation and broadband applicability on different material types helped Bouc-Wen model to be acknowledged as one of the most reliable approach to model hysteresis. The precision of this type of material model however depends entirely on the exactness of the model parameters which needs to be perfectly calibrated to obtain an accurate nonlinear predictor model. This problem of finding accurate model parameters can therefore be formulated as inverse problem where system parameters are identified using measured responses of an existing system. Gradient or Hessian based optimization or finding a global solution using evolutionary algorithms can be found as common approaches to solve these inverse problems. However because of the inherent nonlinearity within the system performance of gradient or hessian is not always certain. Although evolutionary algorithms are good approach to find global optimal solution for nonlinear problems, due to their demand for large number of model simulations they become impractical for complex expensive models owing to cost constraints. In this paper we have used filtering techniques to identify the parameters efficiently with an objective to keep model simulations as minimum as possible, in order to develop a computationally inexpensive parameter identification algorithm. Before going into details of the proposed algorithm, the nonlinear Bouc-Wen hysteretic model whose parameters are to be identified through its parameters is discussed in the next section.

2. THE BOUC-WEN NONLINEAR HYSTERETIC MODEL

The Bouc-Wen hysteretic material model was first introduced by Bouc [1, 2] and later developed by Wen [3]. This model is used extensively in charecterising magneto-rheological (MR) dampers found in vehicular vibration suppression systems because of its ability in reproducing hysteresis shapes [4, 5]. Apart from this there is proof in existing asserting capability in predicting hysteretic behavior of different materials. The governing differential equation of a nonlinear dynamic system using Bouc-Wen material model is given by:

$$m\ddot{x}(t) + c\dot{x}(t) + F(t) = u(t) \tag{1}$$

where, m is the mass of the system, c is the linear viscous damping parameter, $u(t)$ is the external forcing on the system and $F(t)$ is the nonlinear restoring force which is further described as:

$$F(t) = \alpha k_i x(t) + (1-\alpha) k_i z(t) \tag{2}$$

α is the ratio of post yield to the pre yield stiffness i.e. $\alpha = k_f / k_i$ where k_f is post yield stiffness and k_i is initial linear stiffness which can be obtained by dividing the yield force by

the yield displacement. *z(t)* is the hysteretic displacement which can be obtained as solution of the following equation:

$$\dot{z}(t) = A\dot{x}(t) - \beta\dot{x}(t)|z(t)|^n - \gamma|\dot{x}(t)||z(t)|^{n-1} z(t) \quad (3)$$

From the above discussion we can identify that there are five principal model parameters defining a nonlinear hysteretic system completely which are *A, α, β, γ, n*. Of them α is termed as the rigidity ratio and takes a value in the range $0 \leq \alpha \leq 1$. The values at the boundaries i.e. 0 and 1 signify the material behavior to be completely nonlinear or completely linear respectively. β, γ are the parameters that define the shape of the hysteretic cycle. *n* determines the order of transition from linear to nonlinear zone and always takes a value greater than 1. *A* is the parameter responsible for amplitude of hysteresis and usually takes a value of 1. The same system can be described in state space form as:

$$\begin{Bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{Bmatrix} = \begin{bmatrix} x_2(t) \\ m^{-1}[u(t) - cx_2(t) - \alpha k_i x_1(t) - (1-\alpha)k_i x_3(t)] \\ Ax_2(t) - \beta x_2|x_3|^n - \gamma|x_2||x_3|^{n-1} x_3 \end{bmatrix} \quad (4)$$

3. PARAMETER IDENTIFICATION

In order to predict the response of any hysteretic system through its nonlinear model we have to assign appropriate values for all the model parameters. For the case of Bouc-Wen material model parameter *A* is generally considered to be 1 and the other values are identified through optimization by minimizing the error in suitable selected objective function. Thus the parameter identification problem here is a problem in optimization defined as:

$$\begin{aligned} &\text{minimize } (Y - f(\Theta)) \\ &\text{subjected to } \Theta \in \theta \end{aligned} \quad (5)$$

where Θ is the parameter of the nonlinear model. In literature this optimization problem has been solved using various algorithms including least square methods, evolutionary algorithms [6, 8, 9] or hybrid techniques [7]. But because of the inherent nonlinearity in the system gradient or hessian based algorithms are not assured to converge. With the development in computation power researchers tried to use evolutionary algorithms to solve these types of nonlinear problems. For example Ye and Wang [8] used particle swarm optimization algorithm to solve for the model parameters, while Zhang and Huang [9] use GA to solve the same as an unconstrained optimization problem. However, no algorithm has emerged till date which can ensure finding the global minimum. Another problem related to evolutionary algorithm is its computational expense which restricts their use for complicated and

expensive models, efficiently. Among other approaches application of filtering techniques e.g. extended Kalman filtering [10, 11], unscented filtering or particle filtering to identify model parameters has also been explored. In these approaches the parameters have been appended as additional time invariant states which were then identified online along with response quantities.

In this study we use the same first order extended Kalman filter (EKF) to address the problem of parameter identification. To handle the nonlinearity in process and measurement equations The EKF uses the Taylor series expansion upto first order to locally linearize the nonlinear equations. Details of these filtering techniques can be found in much celebrated articles of Julier and Uhlman [11] and Welch and Bishop [12]. In this study unlike existing filtering based techniques we defined the process equation as the time evolution of the model parameters only. The states in the process equation therefore are the parameters of the model while the measurement equation deals with the mismatch between measured response and the estimated response. The process and measurement equations of the system therefore can be defined as:

$$\begin{aligned} x_{k+1} &= x_k + \varepsilon_k \\ y_{obsereved} &= FEM(x_k) + v_k \end{aligned} \qquad 6$$

where ε_k and v_k are process and measurement noise with covariance matrices Q and R respectively. *FEM(*)* is the nonlinear finite element model which uses Bouc-Wen model to describe its material and $y_{observed}$ is the response measured on the structure. Therefore the identification problem is cast as a problem of filtering to obtain an optimum x_k for which the error is minimum. We here propose two approaches for the solution, namely the iterative and the sequential which have been discussed in detail in following sections.

3.1 Iterative method

In this method in each iteration measurement equation simulates the nonlinear model for a predefined time span and compares the response array for the whole time span with the actual observed response. Because of the fact that nonlinear systems are very much sensitive towards the initial condition the initial value for the model simulation is taken to be same as the initial value of the real response. After calculating the error measure algorithm calculates the gain matrix and then returns to the process equation to predict states for the next iteration step.

Algorithm 1:

1. *Initialize x_k which is the parameter estimate at iteration step k.*
2. *Go to: measurement equation*
   a. *Analyse the nonlinear model for time span 0 to t and obtain $y^k_{estimate}$.*
   b. *Calculate the error measure at k^{th} iteration step i.e. (y_{actual}- $y^k_{estimate}$).*
3. *Go to: Process equation*
   a. *Calculate gain.*
   b. *Predict parameters for next iteration step x_{k+1} by filtering.*
4. *Repeat steps 2 and 3 till tolerance is achieved.*

Thus in iterative algorithm filtering is done in pseudo-time where it has been represented as iteration steps. However as it analyses the system for complete duration in each iteration step this algorithms is computationally demanding. However the solution with acceptable tolerance can be achieved by increasing the number of iterations.

2.2 Sequential method

In the sequential approach parameter updating is done in real time. In each time step measurement equation simulates the nonlinear model to obtain model predicted response value for the next time step only. This response value is then compared with the observed response to obtain the error measure. Algorithm then returns to process equation to predict states for the next time step. This follows the sequential filtering procedure.

Algorithm 2:

1. *Initialize x_t which is parameter estimate at time instant t .*
2. *Go to: measurement equation*
   a. *Analyse the nonlinear model to obtain $y^{t+dt}_{estimate}$ at time instant (t+dt).*
   b. *Calculate the error measure (y^{t+dt}_{actual} - $y^{t+dt}_{estimate}$) at time instant (t+dt).*
3. *Go to: Process equation*
   a. *Calculate gain.*
   b. *Predict parameter estimate $x_{t+dt.}$*
4. *Repeat steps 2 and 3 till time upto which the measurement is available.*

The sequential approach is computationally less demanding because of the fact that it doesn't need to run the whole model for the entire time span and updating is performed only when new set of response data is available. However, in this algorithm the horizon to reach the optimum solution is limited by length of measured time signal and therefore it may or may not satisfy tolerance criteria before the signal ends.

2.3 Adaptive scheme for Q and R

In this paper performance of both these methods is compared based on their accuracy and computational demand. The problems related to both these methods are explored in this

endeavor in detail and sample results for a SDOF system are presented. The major short coming of these types of methods and possible break through is also discussed in this section. In this study the main problem that has been identified for both these methods are speed of convergence, required computation time and level of precision. It has been observed that fast convergence depends on the proper selection of the state and measurement error covariance matrices Q and R respectively. In the filtering techniques generally the process or measurement noises are modeled as zero mean gaussian noise sequence with constant covariance matrices whereas in the parameter identification problem the measurement noise accounts for both modeling (incorrect choice of parameter) and electronic noise in data acquisition. Thus covariance of this mismatch should not be represented by a constant entity and for the case of parameter identification we are free to choose this value. We can further observe that the gain is a function of covariance matrices Q and R and therefore theoretically we can actually control the gain if we exploit the freedom of choosing state and measurement noise covariances properly.

$$K_k = \frac{\left(F_{k-1} P_{k-1|k-1} F^T{}_{k-1} + Q_{k-1}\right) H_k^T}{\left(H_k P_{k|k-1} H_k^T + R_k\right)^{-1}} \qquad 7$$

It is evident from the above equation that higher values for Q or lower value for R will force larger updating in each step for the parameters but upon reaching near to the actual solution it may oscillate around the actual value. On the other hand smaller Q and higher R will cause small updating in each step rendering the method to be time taking and computationally demanding. It is experienced that at initial phase of updating generally larger updating of parameters is needed which should get damped as the parameter values converge towards their respective true values. Therefore instead of using constant Q and R value throughout the iteration we can actually use an adaptive scheme to define time varying values for Q and R which can ensure rapid yet smooth convergence. This is a trivial issue for complex time consuming nonlinear FE model updating as rapid along with smooth convergence ensures reduced number of required simulations to achieve the optimum solution. The philosophy behind the adaptive scheme for Q is that at the initial steps when prior states are less reliable we can assume a high value for Q causing high gain but as these states get filtered causing decrease in measurement error Q should get decreased as well. This can be achieved by defining a time varying scheme for process noise covariance matrix which gets reduced on convergence of solution or increases if the solution diverges from the actual value in each

consecutive step. Because of the deterministic assumption on the parameter values state covariance matrix P holds the information about the variance in error between true and estimated states. We therefore can use this information to define an adaptive scheme for Q, which forces Q to take a higher value when P is high and as P decreases it forces Q to decrease as well. This adaptive scheme is given in the following equation as:

$$Q_k = \delta_1 * Q_{k-1} + Q_{noise}$$
$$where \quad \delta_1 = \frac{diag(P_k)}{diag(P_{k-1})} \text{ and } Q_{noise} \text{ is assumed process noise.} \qquad 8$$

The constant matrix Q_{noise} accounts for process noise due to other modeling errors which only becomes significant when the error associated to parameters diminishes to some lower values than Q_{noise}.

It is already demonstrated that in this parameter identification problem, measurement equation accounts for the combined error due to incorrect parameter values and measurement noise. Both these errors act as noise in the measurement equation. Therefore it is quite practical to define measurement noise covariance matrix R as an inversely proportional function of measurement error itself. Therefore as the error decreases R increases reducing gain in each step. The adaptive scheme for R is described below:

$$R_k = \min(diag\left[\left|\delta_3 * e^{-\delta_4 \log_{10}(\varepsilon)}\right|\right], R_{noise}) \qquad 9$$

where δ_3 and δ_4 are scaling parameter defining the increase in the value of R as error converges and should be defined taking initial error measure and desired precision level in to consideration. The error invariant part of the equation R_{noise} describes the true measurement noise covariance which will be dominant when error dependent part becomes larger than R_{noise} allowing the algorithm to incur noise effects as well. A sample damping scheme for R is described in Fig. 1 with different values of δ_3 and δ_4 for which initial error measure is 1000 and desired precision level is 1×10^{-12}.

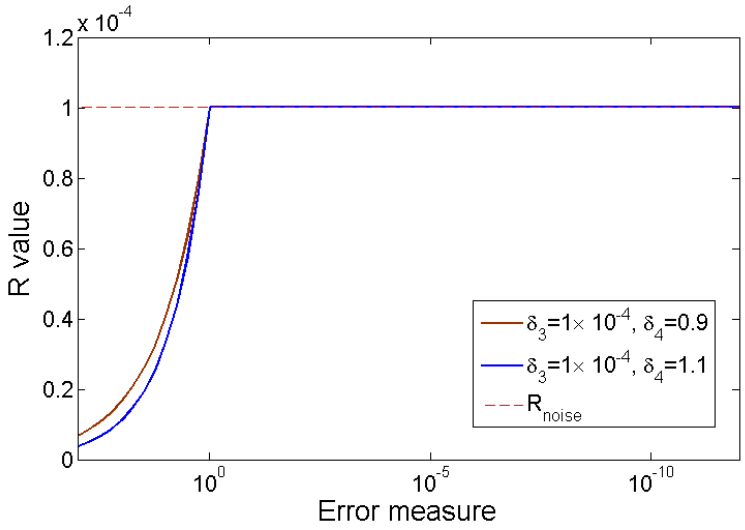

Figure 1: Adaptive scheme for R with different δ3 and δ4 value

This adaptive selection of Q and R helps to obtain rapid convergence and minimizes oscillation of the solution around its true value which is a major concern while using a constant Q and R value throughout the algorithm.

3. NUMERICAL EXPERIMENTS

The proposed methodology is applied on a SDOF system modeled using Bouc-Wen material model. The SDOF system is described by a single spring mass damper system with mass M, stiffness K and damping C (M=1 Kg; K=3 N/m; C=0.9 N-sec/m). We set parameters *A*, *α, β, γ* as the parameters which needs to be identified from a measured response data corresponding to the actual system. The order of transition *n* has been selected as 1.4. A model made with *A=0.5, α=0.4, β=4* and *γ=2.1* is firstly considered as actual system which needs to be identified using proposed method. This actual system is excited with a forcing function defined as $F(t)=B\ sin\omega t$ with B=2 N and ω=1. This system is simulated using Runge-Kutta 4^{th} order algorithm with a sampling frequency of 100Hz for a time span of 5sec for iterative algorithm and 40 sec for sequential algorithm. The response signal is then contaminated with 2% noise to better represent field measurement scenario where noise contamination is obvious. Iterative and sequential algorithms are then applied on the noisy signal with a prior assumption on the parameter as A=1; α =0.8; β =2.2; γ =3. The required time to run the nonlinear model for a time span of 5 Sec is estimated to be 1.568×10^{-2} sec, whereas time taken to run for single time step is estimated to be 2.384×10^{-4} sec. Table 1 lists total time taken by each algorithm (iterative and sequential) with EKF and required iteration steps and corresponding time to reach two preset precision point (1×10^{-3} and 1×10^{-5}) denoted as P1 and P2. The proposed method is also compared with conventional filtering technique

using constant covariance matrices and comparison is demonstrated in Fig. 2. 7th column of Table 1 lists required run time for model only which calculates how much time the algorithm has spared to simulate the model. This data is important in the sense that as the nonlinear model becomes complicated and computationally expensive time required to run only the model which is denominated in this article as T_{model} increases significantly whereas remaining part termed here as T_{excess} depends only on the order of calculation and therefore does not increase in that extent. Thus in order to achieve faster convergence T_{model} should be minimized which is the main cause of higher computational demand.

Table 1: Comparison of required computation time between conventional filtering scheme using constant covariance matrix and using adaptive scheme for both iterative and sequential approach (ITR=Iterative, SEQ=Sequential, EKF=Extended Kalman filter, Const.=Constant covariance, Adapt= Adaptive selection of covariance)

Case SDOF	Total time	Iteration to reach P1	Time to reach P1	Iteration to reach P2	Time to reach P2	Computation time for model only to reach P2
ITR-EKF-Const	45.3512	132	20.6015	289	45.3512	21.6752
SEQ-EKF-Const	66.7525	1556	29.9430	Not achieved	Not applicable	Not applicable
ITR-EKF-Adapt	6.3988	29	4.5260	41	6.3988	3.0751
SEQ-EKF-Adapt	65.8152	458	7.5358	1571	25.8489	1.8066

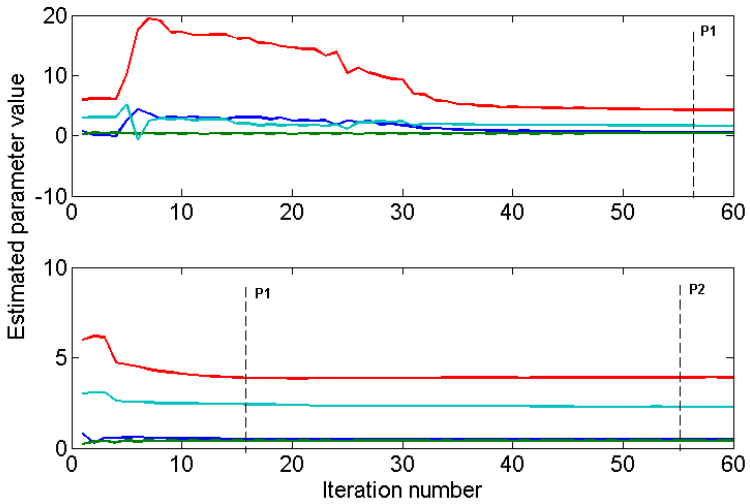

Figure 2: Comparison based on rate of convergence criteria for parameter identification between conventional filtering methods using constant covariance matrix (top) and using the adaptive scheme for covariance (bottom). P1 and P2 are the preset precision points. Four lines are due to four parameters α (Blue), β (Red), γ (Green) and n (cyan).

4. DISCUSSION

The primary goal of the proposed algorithm has been to achieve rapid yet stable convergence in order to maintain minimum requirement for model run. Fig. 2 shows sample convergence result where it has been shown that how the proposed algorithm achieves this goal by controlling gain in every step through adaptive Q and R. This adaptive scheme helped to reduce the oscillation of the solution around its true value which in turn reduced the demand for numbers of the model evaluation drastically. With this reduction the convergence is achieved much faster than for the cases with constant Q and R matrix.

From the result listed in Table 1 one can observe that for a simpler SDOF system adaptive scheme has reduced the time required to get converged result to a great extent. In both the cases (iterative and sequential) adaptive scheme has outperformed conventional filtering that uses constant covariance matrix. It can be further noticed that required time for convergence is much lesser for iterative algorithm compared to its sequential counterpart. Although upon considering the time taken for model run only i.e. T_{FEM}, we obtain a better understanding on the efficiency of these proposed algorithms. The total required time for both these algorithm to reach convergence is subdivided in to two parts, i.e., time taken for model run (T_{FEM}) and time required for other calculations within the algorithm ($T_{Identification}$). The first part depends on the computational demand of the model which is bound to increase as the considered model becomes more and more complex whereas the remaining part may not increase to the same extent with increasing complexity in the model and mostly depends on the number of parameters to be identified. This guides us to the conclusion that in order to achieve an inexpensive algorithm number of model evaluation should be maintained at its minimum possible level. While considering the model run time for both these algorithm we can observe that although sequential approach has consumed more time to reach convergence it actually ran the model for lesser time than its iterative counterpart. The iterative algorithm outperforms for only the reason that T_{FEM} is very small compared to $T_{Identification}$ for this SDOF model. This difference is supposed to be more and more prominent with the increasing time demand of the model.

However, if we consider the precision restriction for both these approach iterative scheme offers flexibility to choose the precision level whereas in the case of sequential algorithm required signal length has to be increased to obtain this flexibility. In the SDOF problem we achieved the required precision with a 5 sec. time span model simulation. Although to achieve that precision level the sequential algorithm needed response signal for 20 sec.

Therefore, this algorithm is not suitable for the cases when we have limited amount of data points.

In a nutshell, we observe that the adaptive scheme for Q and R helps the algorithm to enhance its time efficiency to achieve convergence. Since these methods are computationally inexpensive they are capable of handling complex models. Iterative algorithm is found to be most suitable for simpler system model with less time demand, whereas for system models with large computational cost, sequential algorithm is best suited as it does not require simulating the complete system in successive iterations. Furthermore, while the iterative algorithm demands availability of the complete signal prior to the commencement of the algorithm which restricts its online applications, sequential algorithm uses new sets of measurement only when they are available thus can easily be integrated in an online parameter identification scheme. Thus iterative algorithm is suitable for simpler model with limited length of signal but sequential algorithm holds promise as an efficient online identification algorithm.

5. CONCLUSION

In the above said method the focus was to update FE models for which system matrices are not available. This has been done by updating the parameters which control the response of the structure. Proposed Kalman filtering based updating basically considers the FE model as a black box which has to be identified using only the output response. As the FE models itself are computationally expensive we therefore need a method which requires less numbers of evaluation of the model. To obtain better convergence these methods use an adaptive scheme for Q and R which ensure smooth and faster convergence of solution. This study also compares the pros and cons of sequential and iterative approach and suggests their ideal situation for application to parameter identification problem. The iterative approach gives more flexibility on the choice of tolerance as the horizon for the algorithm is infinite and therefore more accurate than its sequential counterpart as its horizon is limited by the number of samples in the time history signal. However sequential approach is computationally less expensive because unlike iterative approach it doesn’t have to run the whole system for the entire time frame in every iteration step and therefore can be applied for large structures.

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