Effect of relative failure consequences in reliability based dual performance design

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Abstract: During its design life, a structure has to satisfy several performance requirements. It is now common to express all safety requirements, as well as some functionality requirements, in reliability based formats. Failure consequences (hence target reliabilities), structural behavior, capacity, loads and modeling uncertainties and thus the limit states themselves are different at these various performance levels, and the design must take these aspects into account. This paper develops optimal partial safety factors over a range of structural configurations for the design of partially prestressed sections in flexure corresponding to two performance levels — cracking and collapse — satisfying respective target reliabilities. Detailed numerical examples and derivations are presented. The role of relative failure consequences at the two performance levels in determining the governing limit state, the "balance point" where both limit states are active, and the possible implication on maintenance strategies, measured in terms of average rating factors for a prestressed section satisfying both performance requirements, is highlighted.

Key words: performance based design, structural reliability, partial safety factor, prestressed beams, ultimate limit state, serviceability limit state.

Résumé : Durant sa vie utile, une structure doit satisfaire à plusieurs exigences de comportement. Il est maintenant fréquent d'exprimer toutes les exigences de sécurité ainsi que certaines exigences de fonctionnalité, en termes de fiabilité. Les conséquences des défaillances (donc des fiabilités ciblées), le comportement structural, la capacité, les charges et les incertitudes en modélisation, et donc les états limites eux-mêmes, sont différents à ces divers niveaux de rendement et la conception doit tenir compte de ces aspects. Cet article développe des coefficients de sécurité partiels optimaux sur une variété de configurations structurales pour la conception des sections partiellement précontraintes en flexion correspondant à deux niveaux de comportement — fissuration et effondrement — répondant aux fiabilités respectives ciblées. Des exemples numériques détaillés et des dérivations sont présentés. Le rôle des conséquences relatives des défaillances aux deux niveaux de comportement pour déterminer l'état limite directeur, le point d'équilibre où les deux états limites sont actifs, et l'implication possible sur les stratégies de maintenance, mesurées en termes de coefficient de rendement d'une section précontrainte répondant aux deux exigences de comportement, est souligné.

Mots-clés : conception basée sur le rendement, fiabilité structurale, coefficient de sécurité partiel, poutres précontraintes, état-limite ultime, état-limite de fonctionnalité.

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1. Introduction

A structural system has to fulfill several performance requirements set by the owner and regulators depending on the type of the structure, its design life, location, history (if any), and the societal context (Galambos 1992; Augusti and Ciampoli 2008). The aim of structural design is to ensure that throughout its design life the structure satisfies all its relevant performance requirements with adequate assurance. Performance requirements can be classified into safety, functionality, environment, and other related groups. Although cost could in principle be a performance requirement (e.g., with an upper limit defined by available budget, in which case some performance objective(s) is/are chosen to be maximized instead), it is commonly treated as an objective that is minimized subject to relevant performance requirements. Aesthetic and historical considerations, if any, may put additional constraints. The performance requirements are not necessarily unconnected — they may overlap, and can sometimes reinforce and at others counteract each other. It is also possible that for a given structure only one of the requirements may end up governing its design.

It has become increasingly common to express safety requirements, as well as some functionality requirements, in reliability based formats. A reliability based approach to design, by accounting for randomness in the different design variables and uncertainties in the mathematical models, provides tools for ensuring that the performance requirements are violated as rarely as considered acceptable.

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Fig. 1. Design at two performance levels.

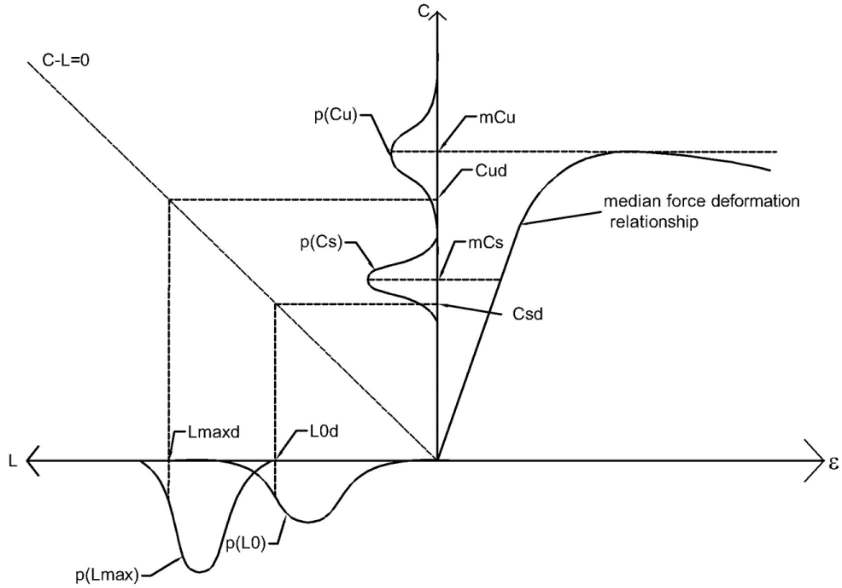


Figure 1 gives a possible schematic of reliability based design at two performance levels when capacities and loads are random in nature. A typical plot of structural response, ε, under incremental loading is given on the right. The median force deformation relation is linear at first: the median serviceability capacity, mC_s, corresponds to the onset of plasticity at some critical location in the structure. The response becomes increasing non-linear and the median ultimate capacity, mC_u, is obtained from the peak of the response curve. The probability density functions (PDFs) of the serviceability and the ultimate capacities, $p(C_s)$ and $p(C_u)$, are defined around the respective medians by accounting for appropriate uncertainties – both epistemic and aleatory. The left half of the figure shows the load-capacity space (expressed in same units). The 45° line is the limit state equation, $C - L = 0$. We define two loads — working (L_0) and extreme (L_{max}) whose PDFs are shown on the L axis.

The two performance requirements for the structure are (*i*) not to yield under working loads with target reliability β_{Ts} and (*ii*) not to collapse under extreme loads with target reliability β_{Tu}. Thus, two sets of design points can be found, one for serviceability and one for collapse — giving rise to the design loads L_{0d} and L_{maxd}, and the design capacities C_{sd} and C_{ud} with available reliabilities β_s and β_u, respectively. Of course, if the design values are different from the respective nominal values, appropriate partial safety factors (PSFs, discussed later) can be used. Designing the structure to these loads and capacities will ensure that the two performance levels are satisfied at the available reliability levels β_s and β_u.

How these available reliabilities differ from their respective targets, β_{Ts} and β_{Tu}, will determine the need and extent of redesigning the structure. Nevertheless, it may seldom be possible to satisfy both targets exactly and simultaneously, and it is more likely that one of the two requirements will end up governing the design.

2. Target reliabilities at various performance levels

The cause and consequences of violation of different performance requirements may vary, the service or exposure lives may be different, and if a reliability approach is taken, the target reliability in each performance requirement must take such difference into account (ISO 1998; Bhattacharya et al. 2001; JCSS 2001; Wen 2001). If the structure gives appropriate warning before collapse, the failure consequences reduce and that in turn can reduce the target reliability for that mode (DNV 1992; JCSS 2001). Functionality target reliabilities may be developed exclusively from economic considerations. The safety target reliability levels required of a structure, on the other hand, cannot be left solely to the discretion of the owner, or be derived solely from a minimum total expected cost consideration, since structural collapse causing a large loss of human life and (or) property may not be acceptable either to the society or the regulators. Design codes, therefore often place a lower limit on the reliability of safety related limit states (Galambos 1992; Bhattacharya et al. 2001). For optimizing a structure with multiple performance requirements, Wen et al. (1996) suggested minimizing the weighted sum of the squared difference of the target and actual reliabilities.

ISO 2394 (ISO 1998), and later JCSS (2001), proposed three levels of requirements with appropriate degrees of reliability: (*i*) serviceability (adequate performance under all expected actions), (*ii*) ultimate (ability to withstand extreme and (or) frequently repeated actions during construction and anticipated use), and (*iii*) structural integrity (i.e., progressive collapse in ISO 2394 and robustness in JCSS). Target reliability values were suggested based on the consequences of failure for ultimate limit states and relative cost of safety measures for serviceability limit states. The Canadian Standards Association (CSA 1992) defines two safety classes and one serviceability class (and corresponding annual target reliabilities) for the verification of the safety of offshore structures. Det Norske Veritas (DNV 1992) specifies three types of structural failures for offshore structures and target reliabilities for each corresponding to the seriousness of the consequences of failure. The American Bureau of Shipping (ABS 1999) identified four levels of failure consequences for various combinations of limit states and component class for the concept Mobile Offshore Base and assigned target reliabilities for each. Ghosn and Moses (1998) suggest three levels of performance to ensure adequate redundancy of bridge structures corresponding to functionality, ultimate and damaged condition limit states, while Nowak et al. (1997) recommend two different reliability levels for bridge structures corresponding to ultimate and serviceability limit states. Nuclear power plant containment structures are designed for earthquakes at two different levels of intensity and correspondingly to two different criteria for failure (CSE-3 2007; E.D.F. 1988; USNRC 1973). Damage, if any, caused by the operating basis earthquake (OBE) must not lead to loss of functionality of the nuclear power plant; whereas the safe shutdown earthquake (SSE) that has a higher intensity and longer recurrence interval than OBE, is allowed to cause the power plant to shutdown but must not cause any radioactive leakage to the environment or loss of structural integrity.

Performance based design perhaps has most enthusiastically been espoused in the seismic engineering community, as evident in SEAOC (1995), ATC-40 (1996), and FEMA 273/274 (FEMA 1996). Performance levels for seismic design are commonly defined in terms of increasing severities, e.g., (*i*) immediate occupancy (IO), the state of damage at which the building is safe to occupy without any significant repairs; (*ii*) structural damage (SD), an intermediate level of damage in which significant structural and non-structural damage has occurred without loss of global stability; and (*iii*) collapse prevention (CP), representing extensive structural damage that causes global instability (Kinali and Ellingwood 2007). A comparison of the performance of structures designed to one ultimate design earthquake vs. those designed to dual level performance levels indicated that the latter produces relatively stronger structures (Wen et al. 1996; Ghobarah 2001).

This paper looks at reliability based design of flexural members at cracking (i.e., serviceability) and collapse (i.e., ultimate) limit states. The methodology, based on FORM, to determine partial safety factors (PSFs) optimized over a range of structural configurations for a given limit state and specified target reliability is presented in detail. The role of relative failure consequences at the two performance levels in bringing out the governing limit state and the "balance point" where both limit states are active is highlighted. The concepts and methodology are demonstrated with the help of numerical examples involving partially prestressed sections in flexure.

3. Partially prestressed concrete

Prestressed concrete (for shells, slabs, girders etc.) is often adopted when in addition to satisfying strength requirements, the member is also required to be slender (e.g., from aesthetic or weight considerations) and (or) to limit cracking (e. g., to satisfy leak-tightness). In ordinary reinforced concrete, the reinforcing steel is used to carry the tensile stresses, and the concrete in the tensile zone may crack. Prestressing is intended to artificially induce compressive stresses in the concrete to counteract the tensile stresses caused by external loads, such that the loaded section remains mostly if not entirely in compression. Prestressed concrete members are relatively lightweight, more resistant to shear, and can recover from effects of overloading. Prestressed sections usually have a minimum amount of ordinary reinforcement, and may fail in several possible ways including a combination of flexure, shear, torsion, excessive deflections etc. Although loss of prestress with time is built into the design, unintended loss of prestress arising from corrosion of the tendons, slippage, bursting of end blocks, anchorage or connection failures etc. can have catastrophic consequences (Raju 2007; Nawy 2010 etc.).

Al-Harthy and Frangopol (1997) looked at three limit states (ultimate strength in flexure, cracking in flexure, and permissible stresses at initial and final stages of prestressing) on 73 prestressed beams designed to ACI 318 (1989) and concluded that the reliability indices implied by that standard are non-uniform over various ranges of loads, span lengths and limit states. The limit state of permissible tension in the final stage was found to be critical in most cases. Hwang et al. (1985) adopted an octagonal limit state surface (corresponding to ultimate strength of concrete) in the 2-D space of membrane stress and bending moment while developing a load and resistance factor design (LRFD) based approach for nuclear power plant concrete containment structures. Yielding of reinforcements was permitted. Working also on the reliability of concrete containments, Pandey (1997) on the other hand took the limit state as tensile cracking of concrete to represent the failure mode of through-thickness cracking. Varpasuo (1996) focused on seismic reliability of a VVER-1000 containment structure and took cracking of concrete after yielding of reinforcement as the limit state. Both Pandey's and Varpasuo's limit states form sides of the octagonal limit state considered by Hwang et al. (1985) along with failure corresponding to simultaneous yielding of reinforcement and cracking of concrete.

In this paper, we look only at cracking and collapse limit states of partially prestressed sections in flexure. The cracking limit state corresponds to the depth of cracking exceeding the cover depth (similar to type 2 prestressed concrete as defined by the Indian Standard IS 1343 (BIS 2003)) and the collapse limit state corresponds to crushing of concrete in compression (reinforcements may yield).

The moment capacity of a partially prestressed concrete section, given the amount of prestressing force and the geometric and material properties can be obtained in the form of an interaction diagram using strain compatibility equations and force balance. Interaction diagrams are plots of normalized compressive force, $P' = P/f_{ck}bD$, and normalized moment capacity, $M' = M/f_{ck}bD^2$, where b and D are the width and the depth of the section, respectively. The prestressing force, P, is a function of several factors such as the area of the prestressing cable, the yield and ultimate strengths of the prestressing steel, the modulus of elasticity of prestressing steel, the stress–strain behavior of the prestressing steel etc.

For a given amount of prestress the position of the neutral axis is determined iteratively by balancing the tensile and compressive forces on the section. The moment capacity can then be found by taking the moment of the forces about any convenient point. In determining the collapse moment capacity, two cases are possible (Fig. 2*a*): the neutral axis (NA) outside and the NA inside the section. In the former, the entire section is in compression and in the latter, concrete has cracked and is assumed not to carry any load in the tensile zone.

The cracking limit state is reached when the tensile strain in concrete at depth equal to the cover exceeds $\varepsilon_{\mathrm{tmax}}$, while the maximum compressive strain ε_{c} on the opposite edge can lie anywhere between 0 and 0.0035 (Fig. 2*b*), which is determined iteratively, from which the cracking moment capacity is determined.

4. Reliability based design equation

A performance function, $g(\boldsymbol{X})$, may be defined for a structural component in terms of the basic variables, $\boldsymbol{X}$, such that $g(\boldsymbol{X}) \leq 0$ denotes failure, $g(\boldsymbol{X}) > 0$ denotes satisfactory performance, and the surface given by $g(\boldsymbol{X}) = 0$ is called the limit state equation or limit state surface. The basic variables include quantities like material properties, loads or load-effects, environmental parameters, geometric quantities, modeling uncertainties, etc. and are modeled as random variables. The general expression of failure probability is

$$[1] \qquad P_{\mathrm{f}} = P\Big(g(\boldsymbol{X}) \leq 0\Big) = \int_{g(\boldsymbol{x})\leq 0} f_{\boldsymbol{X}}(\boldsymbol{x})\mathrm{d}\boldsymbol{x}$$

where $f_{\boldsymbol{X}}(\boldsymbol{x})$ is the joint probability density function for $\boldsymbol{X}$. The reliability of the structure would then be defined as $\mathrm{Rel} = 1 - P_{\mathrm{f}}$. The forward problem in structural reliability involves finding P_{f} given the limit state and the joint distribution of $\boldsymbol{X}$; while the inverse problem requires choosing distribution parameters (e.g., the mean or nominal values) of a few select members of the vector $\boldsymbol{X}$ — typically the strength and (or) geometric variables — such that the target failure probability is satisfied. It is the inverse problem, performed directly or indirectly, that qualifies as reliability based structural design.

Closed-form solutions to eq. [1] are generally unavailable. Two different approaches are widely in use: (*i*) analytic and (*ii*) simulation based algorithms. The first kind, grouped under first order reliability methods (or FORM), holds a distinct advantage over the simulation based methods in that the design point(s) and the sensitivity of each basic variable can be explicitly determined, and is adopted in this paper for developing the partial safety factors.

4.1. First order reliability method

FORM calculates the reliability of a structure by mapping the limit state surface from $\boldsymbol{X}$ onto the standard normal space $\boldsymbol{Y}$ and then by approximating it with a tangent hyperplane at the design point (Shinozuka 1983). Several mappings algorithms are possible, (see., e.g., Melchers 1987); this paper uses the Rackwitz and Fiessler (1978) transformation that converts each X point-by-point into an equivalent normal U, and then the vector $\boldsymbol{U}$ into the independent standard normal vector $\boldsymbol{Y}$. The intermediate $\boldsymbol{U}$ vector is generally dependent, and is mapped onto the space of independent standard normals, $\boldsymbol{Y}$, through Cholesky factorization of the correlation matrix, $\mathbf{R}$, of $\boldsymbol{X}$; the error on account of the nonlinear transformation between each X_i and Y_i, is generally slight and can be easily corrected (Der Kiureghian and Liu 1986). Following this mapping of $g(\boldsymbol{X})$ on to $g_1(\boldsymbol{Y})$, the point $\boldsymbol{y}^*$ closest to the origin is the solution of the optimization problem:

$$[2] \qquad \min F = \boldsymbol{y}^{\mathrm{T}}\boldsymbol{y} \ \text{ subject } \ \text{to} \quad G = g_1(\boldsymbol{y}) = 0$$

Let $\beta = \sqrt{\boldsymbol{y}^{*\mathrm{T}}\boldsymbol{y}^{*}}$ be the distance of this optimal point from the origin. The approximate probability of failure is then

$$[3] \qquad P_{\mathrm{f}} = \Phi\big(-\beta\,\mathrm{sgn}\,[g_1(\boldsymbol{0})]\big)$$

The signum function determines whether the origin is in the safe domain or not. To solve eq. [2] we have used the gradient projection method originally developed by Rosen (1961) and described in detail in Liu and Der Kiureghian (1986).

The optimal point $\boldsymbol{y}^*$ can be transformed back into the basic variable space, yielding the "design" or "checking point", $\boldsymbol{x}^*$. If the structural element in question is designed using this combination $\boldsymbol{x}^*$, the reliability of the component would be β (within the approximations of FORM). This, in fact is the basis of partial safety factor design, discussed next.

4.2. Partial safety factors

Reliability based partial safety factor design is intended to ensure a nearly uniform level of reliability across a given category of structural components for a given class of limit state under a particular load combination (Ellingwood 2000). The design point, $\boldsymbol{x}^*$, obtained from a FORM analysis, satisfies

$$[4] \qquad g(\boldsymbol{x}^*) = 0$$

Since nominal or characteristic values of basic variables, instead of checking point values, are typically used in design, eq. [4] may be rewritten as

$$[5] \qquad g\left(\frac{X_{1,\mathrm{n}}}{\gamma_1}, \ldots, \frac{X_{k,\mathrm{n}}}{\gamma_k}, \gamma_{k+1}X_{k+1,\mathrm{n}}, \ldots, \gamma_m X_{m,\mathrm{n}}\right) \geq 0$$

where the subscript n indicates the nominal value of the variable. We have partitioned the vector of basic variables into k resistance type and $m - k$ action type quantities (ISO 1998). The partial safety factors, γ_i, are typically greater than one: for resistance type variables they divide the nominal values while for action type variables they multiply the nominal values to obtain the design point:

$$[6] \qquad \text{resistance PSFs}: \ \gamma_i = \frac{X_{i,\mathrm{n}}}{X_i^*}, i = 1, \ldots, k$$

$$[7] \qquad \text{action PSFs}: \ \gamma_i = \frac{X_i^*}{X_{i,\mathrm{n}}}, i = k+1, \ldots, m$$

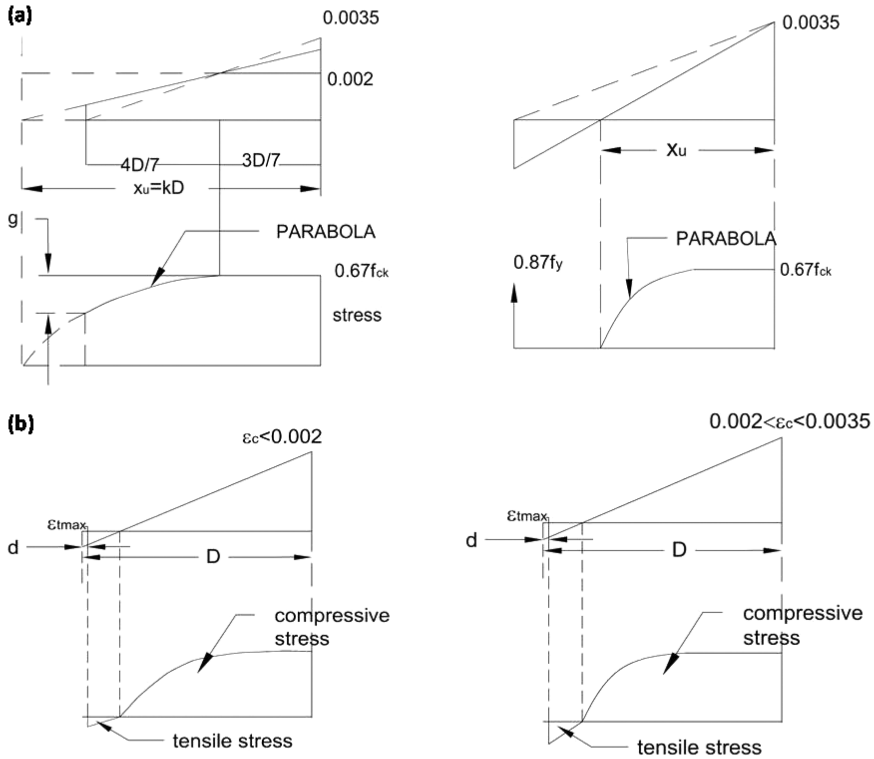
The failure probability of the component when eq. [5] is just satisfied will be $\Phi(-\beta)$.

If eq. [5] can be separated into a strength term and a sum of load-effect terms, the following format is adopted for design:

$$[8] \qquad R_{\mathrm{n}}\left(\frac{X_{i,\mathrm{n}}}{\gamma_i}, i = 1, \ldots, k\right) \geq l\left(\sum_{i=1}^{m-k}\gamma_i Q_{\mathrm{n}_i}\right)$$

where R_{n} is the nominal resistance and a function of factored

Fig. 2. Strain and stress distributions on section for (*a*) collapse limit state with neutral axis outside (left) and inside (right) the section, (*b*) cracking limit state for $\varepsilon_c < 0.002$ (left) and $\varepsilon_c > 0.002$ (right).



strength parameters, l is the load-effect function, γ_i is the ith load factor, and Q_{ni} is the nominal value of the ith load. There is no separate resistance factor multiplying the nominal resistance (as is done in LRFD i.e., load and resistance factor design, practised in North America) since material partial safety factors have already been incorporated in computing the strength.

Let the design equation be valid for n_r representative structural components, and let w_i be the weight (i.e., relative importance or relative frequency) assigned to the ith such component. These n_r representative components may differ from each other on account of different geometric dimensions, nominal loads, material grades etc. For a given set of PSFs, let the reliability of the ith component be β_i. Choosing a new set of PSFs gives us a new design, a new checking point, and consequently, a different reliability index. Let β_T be the target reliability index for all the n_r representative components in the given limit state. If there has to be one design equation, i.e., one set of PSFs, for all the n_r representative components, the deviations of all β_i's from β_T must in some sense be minimized. The design eq. [5], when using the optimal PSFs obtained this way, can ensure a nearly uniform reliability for the range of components. Several constraints may be introduced to the optimization problem to satisfy engineering and policy considerations (as listed in Agrawal and Bhattacharya 2010). Moreover, some partial safety factors, such as those on material strengths, may be fixed in advance.

Reliability based structural design codes incorporating partial safety factors in LRFD format have been in use in the North American continent since the 1980s (e.g., AISC 1986; CSA 1992; API 1993; AASHTO 1994; FEMA 2002, etc.) while those involving material, geometric and action type partial safety factors have been around in Europe since the 1990s (e.g., DNV 1992; ISO 1998; JCSS 2001; CEN 2002, etc.).

5. Numerical formulation and results

The aspects of competing performance requirements and the effect of relative failure consequences are brought out in this section through numerical examples involving the reliability based design of prestressed concrete sections corresponding to the dual performance levels of cracking and collapse. The limit states are described first, statistics of basic variables are presented next, and a set of design PSFs optimized for a limited range of structural configurations and corresponding to given target reliabilities are developed. The role of relative failure consequences at the two performance levels in determining which designs are feasible and which limit state ends up governing the design (except at a set of "balance points"), is highlighted. Needless to say, the design equations developed in this section are for illustrative purposes only: actual design equations and associated PSFs would require more detailed analysis and comprehensive data.

5.1. Limit states and basic variables

The limit states of cracking and collapse can be described, respectively, as

$$[9] \qquad g_1 = M'_{\mathrm{serv}} - (M_{\mathrm{DL}} + M_{\mathrm{L0}}) = 0$$

$$[10] \qquad g_2 = M'_{\mathrm{ult}} - (M_{\mathrm{DL}} + M_{\mathrm{Lmax}}) = 0$$

where M'_{serv} and M'_{ult} are the moment capacities (normalized by $f_{ck}bD^2$) corresponding to cracking and collapse, respectively. The cracking limit state is defined as the depth of cracked concrete exceeding the concrete cover, d, i.e., the strain at a depth d from the tensile face exceeding the ultimate strain of concrete in tension. The collapse limit state is defined as the maximum compressive strain in concrete reaching the crushing strain of 0.0035. If required, ordinary reinforcements may yield in either case. The moment due to dead load, M_{DL}, is assumed to be the same in either limit state. The live load moment, M_{L0}, in the cracking limit state is due to working loads while that in the collapse limit state, M_{Lmax}, signifies the lifetime maximum. The statistics and the inter-relation of these two random variables are described subsequently. The statistics of the basic variables are described in Table 1; the range of nominal values (or ratios) are listed in Table 2.

The normalized moment capacity, M'_{cap}, both for limit states of collapse and cracking, is a function of the applied in-plane compression (p'), material properties (f_c, f_y, E, ε_c, ε_t), and geometric quantities (p/f_{ck}, d/D, e/D):

$$[11] \qquad M'_{cap} = \frac{M_{cap}}{f_{ck}bD^2} = M'_{cap}\left(P', f_c, f_y, E, \varepsilon_c, \varepsilon_t, \frac{p}{f_{ck}}, \frac{d}{D}, \frac{e}{D}\right)$$

Of these, four variables are considered as random in this analysis: the normalized prestressing force, P', the compressive strength of concrete, f_c, the yield strength, f_y, and the Young's modulus, E, of the reinforcing steel. As mentioned previously, the prestressing force P is a function of several material and geometric properties such as the area of the prestressing cable, the modulus of elasticity, and yield and ultimate strengths of prestressing steel etc. along with short-term and long-term prestress losses. Prestressing force P can be calculated as a function of these properties taken as random variables, following an approach similar to that used for calculating M_{cap} above. In addition, a more detailed analysis would require consideration of all the loading stages. Since such details would add several more limit states, basic variables, and partial safety factors to the reliability analysis, we simplified the analysis and thus reduced the size of the problem to focus on the performance based design aspect of the paper.

The nominal or design values of the moment capacities, to be used in design equations discussed below, can be obtained by substituting the random quantities in eq. [11] by their design values:

$$[12] \qquad M'_{cap,n} = \frac{M_{cap,n}}{f_{ck}bD^2} = M'_{cap}\left(P'_n, \frac{f_{ck}}{\gamma_c}, \frac{f_{yn}}{\gamma_s}, E_n, \varepsilon_c, \varepsilon_t, \frac{p}{f_{ck}}, \frac{d}{D}, \frac{e}{D}\right)$$

In Indian Standards such as IS 456 (BIS 2000) the compressive stress–strain relationship for concrete is parabolic up to a strain of 0.002, and horizontal from that point on. The design compressive strength of concrete is f_{ck}/γ_c where f_{ck} is the characteristic (i.e., nominal) compressive strength of concrete and γ_c taken to be 1.5 is the material safety factor on concrete strength. The failure strain of concrete in compression is 0.0035. The standard IS 1343(BIS 2003) specifies the minimum grade of concrete as M30 for post-tensioning and M40 for pre-tensioning. The maximum tensile strain in concrete is $\varepsilon_{tmax} = 0.00\ 012$ assuming stress–strain behavior of concrete in tension to be linear (Neville 1995). The design yield stress for reinforcing steel is f_{yn}/γ_s where f_{yn} is the nominal yield strength and γ_s is the material safety factor on yield strength of steel and is taken to be 1.15. The nominal modulus of elasticity of steel, E_n, is 200 000 N/mm^2.

The parabolic stress block for concrete, and the design procedure in the Indian Design Aid SP-16 (BIS 1999) are very similar to the rectangular stress block and the design procedure in ACI 318 (1989). By comparing the results for ultimate moment capacity, curvature etc. obtained using nonlinear stress–strain characteristics of steel and concrete against those calculated using ACI design procedures Naaman (1983) concluded that the ultimate moment capacity obtained by ACI were within 7% of his analysis and on the conservative side of his results indicating that the ACI design procedure and hence the procedure suggested in Indian Standards are sufficiently accurate.

Figure 3 shows an example of the so-called "interaction diagrams" — the normalized moment capacities as functions of the normalized prestressing force, both for collapse and cracking for $p/f_{ck} = 0.2$, $e/D = 0.2$, $d/D = 0.05$, $f_y = 415$, and $f_{ck} = 50$ MPa.

The moment capacities are implicit functions of four basic variables, and their distributions are obtained by numerical simulation, which in turn are used in FORM analyses. Figure 4*a* shows PDFs of M_{serv} and M_{ult} fitted to lognormal distributions — the statistics are as in Table 1 while the nominal values are fixed at: $P'_n = 0.2$, $f_{ck} = 50$ MPa, $f_{yn} = 415$ MPa, and $E_n = 200$ GPa. A chi-squared goodness-of-fit test indicated that the lognormal distribution can be accepted at a very high significance of 0.62 for M_{serv} and 0.72 for M_{ult}. The distribution of the moment capacities thus obtained can be normalized by their respective moment capacities; such normalized statistics of the moment capacities are used subsequently to find the optimal partial safety factors. Figures 5*a*–5*d* show the bias (mean/nominal) and the coefficient of variation (CV = standard deviation/mean) for the two moment capacities for various combinations of P'_n and f_{ck}.

We now specify M_{L0} to be the daily maximum live load and M_{Lmax} to be the lifetime maximum live load in the cracking and collapse limit states, respectively (eqs. [9] and [10]), although, without any loss of generality, they could stand for any non-permanent loads. The dead load moment is the same in both limit states. And although it is not necessary, we assume here that they arise from the same live load process but pertain to vastly different time horizons which makes the two live loads mutually statistically independent. Thus, the cumulative distribution functions (CDFs) of the two loads are related as,

$$[13] \qquad F_{max}(x) = F_0^n(x)$$

The index n is the number of days in the design life of the structure (here taken to be 50 years). We further assume that F_0 is of the Gumbel type (with scale parameter a and mode u), which makes F_{max} to also be of the Gumbel type with the same scale parameter a and the mode increased by an amount $\ln(n)/a$. The ratios of the means and nominals of the two live load moments are $E[M_{L0}]/E[M_{Lmax}] = 0.3035$ and $M_{L0,n}/M_{Lmax,n} = 0.3360$. Figure 4*b* shows the PDFs of the

Table 1. Statistics of basic variables.

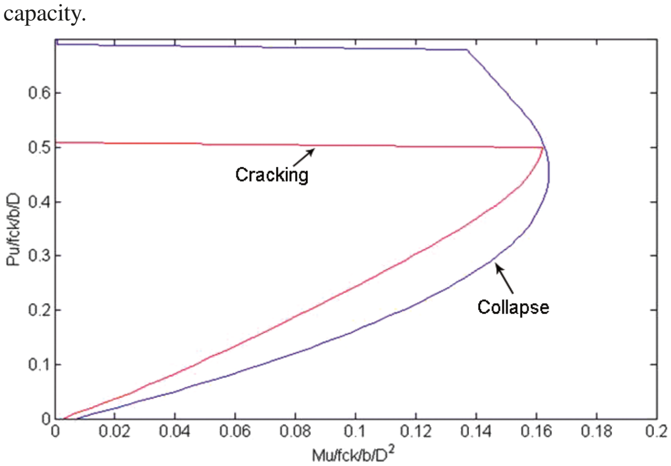
Random variable	Description	Statistics distribution (mean, CV)	Source
P'	Normalized prestressing force	Lognormal ($1.15P_n$, 10%)	Assumed
f_c	Compressive strength of concrete	Normal (f_{ck}+$0.825s_c$, s_c)*	BIS (2000)
f_y	Yield strength of steel	Lognormal ($1.1133f_{yn}$, 0.09)	Barakat et al. (2004); Hamann and Bulleit (1987)
E	Young's modulus	Normal ($1.001103E_n$, 0.01)	Barakat et al. (2004)
$M_{DL}/M_{DL,n}$	Normalized dead load moment	Normal (1, 0.1)	Al-Harthy and Frangopol (1997); Barakat et al. (2004); Hamann and Bulleit (1987)
$M_{L0}/M_{L0,n}$	Normalized daily live load moment	Type 1 (0.9, 0.3)	Al-Harthy and Frangopol (1997); Barakat et al. (2004)
$M_{Lmax}/M_{Lmax,n}$	Normalized lifetime maximum live load	Type 1 (0.996, 0.09)	eq. [13]
$M_{serv}/M_{serv,n}$	Normalized serviceability moment capacity	Lognormal	Obtained through simulations (statistics as in Fig. 5)
$M_{ult}/M_{ult,n}$	Normalized ultimate moment capacity	Lognormal	Obtained through simulations (statistics as in Fig. 5)

Note: CV, coefficient of variation.
*s_c is standard deviation for characteristic strength of concrete as given in IS 1343 (BIS 2003).

Table 2. Deterministic parameters.

Parameter	Description	Values taken
M_{Ln}/M_{Dn}	Nominal lifetime max live to nominal dead load moment ratio	0.25, 0.5, 1.0, 1.5, 2
P	Percent reinforcement	0.2%
f_{ck}	Characteristic compressive strength of concrete	40, 50, 60 MPa
f_{yn}	Nominal yield strength of reinforcing steel	415 MPa
E_n	Nominal Young's modulus of reinforcing steel	200 GPa
$P'_n = P_n/(f_{ck}bD)$	Normalized prestressing force	0.2
e/D	Eccentricity of prestressing force	0.2

Fig. 3. Interaction diagram between prestressing force and moment capacity.



two live loads when the mean daily maximum is standardized to unity. These ratios are consistent with values in the existing literature, e.g., on arbitrary point in time and 50 year maximum live loads, annual extreme, and 50 year extreme wind and snow loads etc. in Ellingwood and Galambos (1982).

5.2. Optimized partial safety factors

The design equations in serviceability and ultimate limit states, respectively, can be written in the same format

$$[14]\quad M'_{serv,n} \geq \gamma_{Ds} M_{DL,n} + \gamma_{Ls} M_{L0,n}$$

$$[15]\quad M'_{ult,n} \geq \gamma_{Du} M_{DL,n} + \gamma_{Lu} M_{Lmax,n}$$

where γ_D is dead load factor, γ_L is live load factor, with the subscript s indicating serviceability and u indicating ultimate limit states, $M'_{serv,n}$ is nominal serviceability moment capacity, $M'_{ult,n}$ is nominal ultimate moment capacity, $M_{DL,n}$ is nominal dead load moment, $M_{L0,n}$ is nominal daily live load moment, and $M_{Lmax,n}$ is nominal daily live load moment.

Normalizing the limit state equations with the respective design equations allows us to use normalized statistics of the strength and load variables,

$$[16]\quad g_{sn} = \frac{M_{serv}}{M_{serv,n}} - \frac{M_{DL}/M_{DL,n} + (M_{L0}/M_{L0,n})(M_{L0,n}/M_{DL,n})}{\gamma_{Ds} + \gamma_{Ls}(M_{L0,n}/M_{DL,n})} = 0$$

$$[17]\quad g_{un} = \frac{M_{ult}}{M_{ult,n}} - \frac{M_{DL}/M_{DL,n} + (M_{Lmax}/M_{Lmax,n})(M_{Lmax,n}/M_{DL,n})}{\gamma_{Du} + \gamma_{Lu}(M_{Lmax,n}/M_{DL,n})} = 0$$

and to focus on the role of the PSFs and the nominal load ratios in determining the reliability index for each limit state:

$$[18]\quad \beta_s = \Phi^{-1}(g_{sn} > 0) = \beta(\gamma_{Ds}, \gamma_{Ls}, M_{L0,n}/M_{Dn})$$

Fig. 4. Probability density functions for (*a*) normalized moment capacities in collapse (Mult) and cracking (Mserv) and (*b*) daily maximum and lifetime maximum live load moments ($\mu_{M_{L0}} = 1.0$).

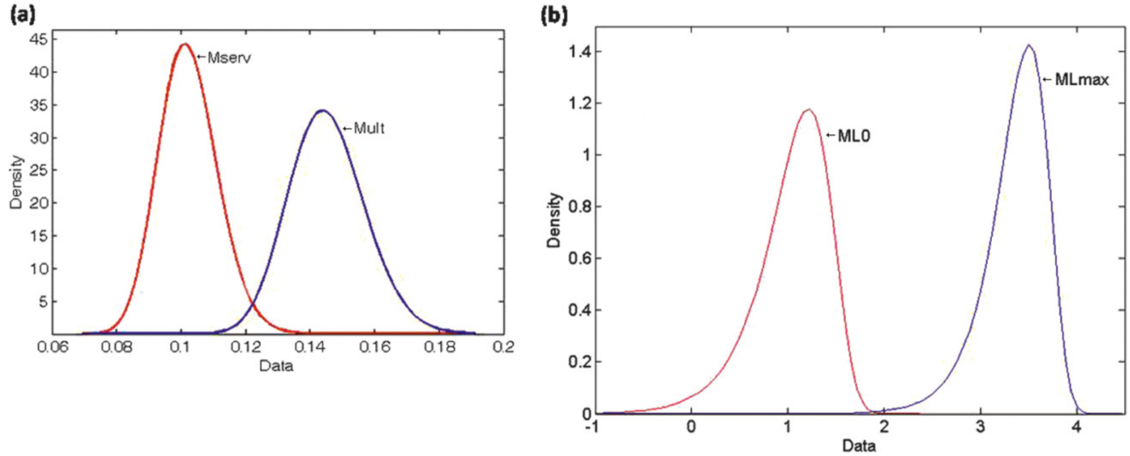
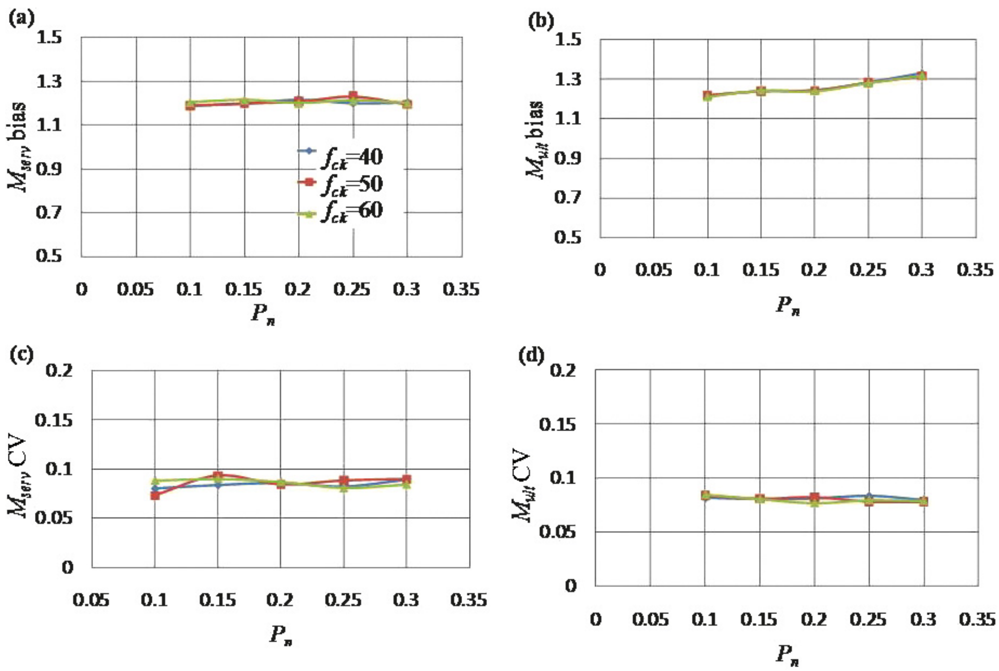


Fig. 5. (*a*), (*b*) bias and (*c*), (*d*) coefficient of variation, respectively, of serviceability and ultimate moment capacities for various combinations of P_n and f_{ck}. The fixed parameters are: $e/D = 0.2$, $p = 0.2$, and $d/D = 0.05$.



$$\beta_u = \Phi^{-1}(g_{un} > 0) = \beta(\gamma_{Du}, \gamma_{Lu}, M_{Lmax,n}/M_{Dn}) \tag{19}$$

If there are n_r different nominal load ratios, r_i, with weights w_i, the optimal PSFs are the solution of the following problem:

$$\min\left[\sum_{i=1}^{n_r} w_i\left(\beta_x(\gamma_D, \gamma_L, r_i) - \beta_{Tx}\right)^2\right] \text{ where } \sum_{i=1}^{n_r} w_i = 1, x = s \text{ or } u \text{ subject to } \gamma_D > 1, \gamma_L > 1 \tag{20}$$

The constraints on the load factors are intended to be consistent with accepted engineering practice. Five nominal lifetime maximum live to nominal dead load moment ratios, M_{Ln}/M_{Dn}, have been taken: 0.25, 0.5, 1.0, 1.5, and 2.0; the corresponding weights are 0.1, 0.45, 0.3, 0.1, and 0.05 (Ellingwood et al. 1980). Strictly speaking these weights are for reinforced concrete beams, but are assumed to be applicable in the present case.

The target reliability index in ultimate limit state, β_{Tu}, for structural components is commonly taken between 3 and 4 (depending on failure mode, consequence, level of warning, mitigation costs etc. as reviewed in Bhattacharya et al. 2001), and for the purpose of this example, we adopt the value of 3.5. Once β_{Tu} is selected, the target in the serviceability limit state, β_{Ts}, should not be chosen arbitrarily, but needs to be consistent with the marginal costs incurred for improving reliability and the relative failure consequences at the two limit states. Following JCSS (2001), we adopt a value of 2.0 for β_{Ts}, which also puts the relative failure consequence,

$$[21]\qquad C_r = \frac{\Phi(-\beta_{Ts})}{\Phi(-\beta_{Tu})}$$

at approximately 100. Table 3 lists the optimal PSFs for three different values of f_{ck} corresponding to these two target reliabilities and the statistics, deterministic parameters, and load ratios discussed above. The last row of Table 3 lists the recommended PSFs for all concrete grades.

5.3. Governing performance requirement

Following the load and resistance factor rating methodology for evaluating bridges in service, we adopt the reliability based rating factor approach to measure the excess capacity of a component designed according to a partial safety factor based format (NCHRP 2001; Bhattacharya et al. 2005, 2008). For a load combination involving dead load and a non-permanent load, the rating factor (RF) is the ratio of the design capacity (in excess of the design dead load) and the design value of the non-permanent load:

$$[22]\qquad \mathrm{RF} = \frac{C_n - \gamma_D D_n}{\gamma_L L_n}$$

A rating factor of 1 implies the component just satisfies the target reliability it was designed for. The higher the rating factor, the greater is the reserve capacity and hence greater is the actual reliability compared to the target value. The relationship between target reliability and rating factor has been investigated in detail in Bhattacharya et al. (2005).

We now compare the rating factors of a given component in ultimate and serviceability limit states designed to eqs. [14] and [15]. We start with a particular value of the nominal dead load, $M_{DL,n}$, then obtain $M_{Lmax,n}$ corresponding to one of the ratios described above, and calculate the design $M'_{ult,n}$ from eq. [15]. This way, $\mathrm{RF_u}$, the rating factor in ultimate limit state, is identically equal to 1.

Once the nominal ultimate moment capacity, $M'_{ult,n}$, of a section is obtained, the serviceability design capacity, $M'_{serv,n}$, for the same section can be read from the interaction diagram (Fig. 3). The nominal dead load, $M_{DL,n}$, is the same in both design equations. As stated above, the nominal live load in serviceability is $M_{L0,n} = 0.3360\, M_{Lmax,n}$. The rating factor in serviceability limit state is then

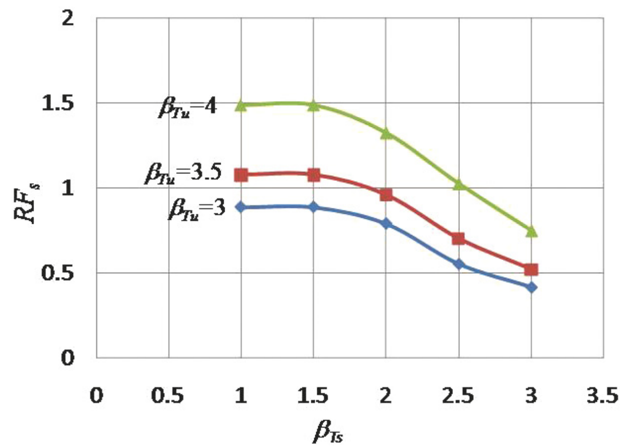
$$[23]\qquad \mathrm{RF_s} = \frac{M'_{serv,n} - \gamma_{Ds} M_{DL,n}}{\gamma_{Ls} M_{L0,n}}$$

We calculate the serviceability rating factor $\mathrm{RF_s}$ for each value of $M_{Lmax,n}/M_{DL,n}$ listed above and using the corresponding weights from the table, the average $\mathrm{RF_s}$ is obtained. The average $\mathrm{RF_s}$ is plotted in Fig. 6 for various combinations of the target reliabilities β_{Tu} and β_{Ts}. It is important to remember that each combination of β_{Tu} and β_{Ts} corresponds to a particular consequence ratio. We should also emphasize that each combination of β_{Tu} and β_{Ts} gives rise to a new set of optimal PSFs.

Table 3. Optimal PSFs on dead and live loads.

f_{ck}	Ultimate limit state (β_{Tu} = 3.5)	Serviceability limit state (β_{Ts} = 2)
40	1.14, 1.19	1.03, 1.16
50	1.13, 1.18	1.00, 1.22
60	1.15, 1.18	1.00, 1.10
Recommended PSFs for all concrete grade	1.15, 1.20	1.00, 1.15

Fig. 6. Average $\mathrm{RF_s}$ for various combinations of β_{Ts} and β_{Tu} ($\mathrm{RF_u}$ = 1 in all cases).



Let us first look at the curve corresponding to β_{Tu} = 3.5 in Fig. 6. On this curve, for β_{Ts} = 2, the rating factor in serviceability is approximately 1.0. Thus, a component that just satisfies the ultimate limit state target reliability of 3.5, also just satisfies the serviceability requirement if β_{Ts} is 2.0. In other words, given the statistics of the random variables, and the constraints in eq. [20], both limit states are active if the target reliabilities are 3.5 and 2.0 — the "balance point" in dual performance-level design. If we move left from this point along the curve of β_{Tu} = 3.5, the average $\mathrm{RF_s}$ exceeds 1.0 and the ultimate limit state starts to govern the design. Failure will therefore occur at the ultimate limit state and the component should be designed for it. Moving right along the curve of β_{Tu} = 3.5, on the other hand, indicates that the section cannot satisfy serviceability requirements at all if β_{Ts} is 2.0 or higher.

In general, all combinations of β_{Tu} and β_{Ts} (i.e., different failure consequence ratios) will render one of the limit states inactive except at most at a discrete set of points. When the design is such that the ultimate limit state governs and the design serviceability RF is substantially higher than 1.0, routine maintenance policy, intended to ensure serviceability may be made less stringent. Looking at the curve corresponding to β_{Tu} = 4.0, it is clear that the ultimate limit state governs for β_{Ts} < 2.5. Both limit states are active at β_{Tu} = 4.0, β_{Ts} = 2.5, i.e., when the consequence ratio is about 200. It is impossible to satisfy serviceability requirements if the ultimate target reliability is 4.0 but the consequence ratio is less than 200. The plot corresponding to $\beta_{Tu} = 3.0$ indicates that the serviceability limit state always governs and that the serviceability requirements may never be satisfied as long as the PSFs are constrained as in eq. [20].

An ideal pair of target reliability indexes would have to satisfy three criteria, namely, minimum value specified by codal provisions, an acceptable consequence ratio (eq. [21]), and RF greater than but close to 1 for both the limit states.

6. Conclusion

Performance based design is not a new paradigm — every structure has to satisfy a set of (sometimes conflicting) performance requirements. It has become common to express all safety requirements and some serviceability requirements in terms of reliability. We looked at reliability based design of flexural sections at cracking and collapse performance levels and partial safety factors were developed for each limit state. The target reliability at ultimate was chosen based on accepted safety criteria while that at serviceability was deduced from a relative consequence ratio. However, not all combinations of target reliabilities can produce a feasible design if the load statistics cannot be controlled. Except at a discrete set of "balance points," a design is dominated by one of the two requirements as highlighted by the variation of average serviceability rating factor. The combination of a high relative consequence ratio and stringent design requirements at the ultimate level may have implications on maintenance strategies (for serviceability at least and hence on life-cycle cost optimization) for the structure.

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