DEVELOPMENT OF PARTIAL SAFETY FACTORS FOR ACCIDENTAL PRESSURE DESIGN CASE OF PRESTRESSED INNER CONTAINMENT SHELLS IN INDIAN NPPS

Baidurya Bhattacharya[1], Aritra Chatterjee[1] , Gunjan Agrawal[2], Apurba Mondal[3]
[1]Dept of Civil Engineering, Indian Institute of Technology Kharagpur, Kharagpur 721302, India
[2]General Analytics Associate, ZS Associates, Pune 411013, India
[3] Deputy Chief Engineer, Nuclear Power Corporation of India Ltd., Anushaktinagar, Mumbai 400085, India
E-mail of corresponding author: baidurya@civil.iitkgp.ernet.in

ABSTRACT

Partial safety factors (PSFs) used in reliability-based design are intended to account for uncertainties in load, material and mathematical modeling while ensuring that the target reliability is satisfied for the relevant class of structural components in the given load combination and limit state. This paper describes the methodology in detail for developing a set of optimal reliability-based PSFs for the design of prestressed concrete inner containment shells in Indian NPPs at collapse limit state under MSLB/LOCA conditions. The mechanical formulation of the flexural limit state is based on the principle behind prestressed concrete design recommended by IS 1343 and SP16. The applied biaxial moments are combined according to Wood's criteria. The optimization of the PSFs is based on reliability indices obtained from importance sampling and a local linear response surface fit; Monte Carlo simulations are performed to determine the capacity statistics and dependence between capacity and applied loads. Numerical examples are provided.

INTRODUCTION

The design of containment shells for Indian Pressurized Heavy Water Reactors (PHWRs) has evolved over the years, originating from a steel cylindrical shell capped with a steel dome (CYRUS Reactor, Trombay), followed by the use of reinforced concrete walls and pre-stressed concrete dome (Rajasthan Atomic Power Station) to the use of Pre-Stressed concrete for the entire shell (Madras Atomic Power Station) and pre-stressed concrete double containment shells (first employed in the Narora and Kakrapar Power Stations). The Kaiga and Rajasthan Atomic Power Plants marked a further improvement in the design philosophy with complete double containment shells having independent domes [1]. The inner containment shells used in recent PHWRs are cylindrical structures of 63 m height, with prestressed concrete spherical domes containing 4 large openings to facilitate the replacement of steam generators. [2]. Until recently, nuclear containment structures in India were designed using the French RCC-G code. The raft of the PWHR at Tarapur was designed using the ASME code and checked against RCC-G [1]. There is yet no formal Indian design standard for containment structures. In 2007, the Atomic Energy Regulatory Board (AERB) of India released the CSE-3 codes [3] which is currently under review.

Significant uncertainties exist in the structural behavior of the IC Shells of PHWRs, arising out of the random nature of material, geometry, prestressing and loadings. As early as 1974, Shinozuka and Shao [4] conducted a probabilistic assessment of prestressed concrete pressure vessels using the first order second moment approximation. Uncertainties in loads and in the material and geometry of the vessels were considered while short term accidental load effects were modeled as Poisson Processes. The overall uncertainty in structural behavior of nuclear containment structures, as in any general structure, is caused by uncertainties in resistance and demand quantities. The uncertainty associated with the resistance of containment shells arises out of uncertainty in the strengths of concrete and steel as well as in shell geometry. While concrete strength has been found to be better controlled in the nuclear power plant industry than in the ordinary building industry, steel strength variability does not display a noticeable reduction. Variability in sectional dimensions is comparatively quite low and has negligible impact on the overall uncertainty in structural resistance [5]. Loads acting on concrete containments intrinsically involve random uncertainties, and therefore need to be treated probabilistically. Different loads have different degrees of randomness and may entail appropriate adjustments in the probabilistic framework, for example, the variability of dead load being substantially lower than that of an accidental pressurization load, the former can be treated as a deterministic quantity for simplification of analysis [6]. Another significant source of uncertainty is the long term behavior of these structures which is highly variable owing to material changes (for example, prestress loss in tendons and creep in concrete) and the occurrence of accidental events [7, 8].

It is most rational to treat uncertainties associated with parameters governing the design and construction of a structure in a probabilistic format, specifically, to model the time-invariant quantities as random variables and the time-dependent ones as stochastic processes. Recognizing the existence of these uncertainties is an admission of the fact that the structure may not always satisfy its performance and safety objectives during its intended design life. The logical extension of this admission is to ensure that the likelihood of unsatisfactory performance be kept acceptably low during the life of the structure.

The subject of structural reliability provides the tools and methodologies to explicitly determine the probability of such failures ("failure" here in the sense of non-compliance or non-performance) by taking into account all relevant uncertainties. These techniques can be used to design new structures with specified (i.e., target) reliabilities, and to maintain existing structures at or above specified reliabilities. Target reliabilities are discussed in the next subsection. Even though such computed probabilities of failure (reliability being 1 minus failure probability) may not have a frequentist or actuarial basis, structural reliability provides a neutral and non-denominational basis to compare different (and often disparate) designs and maintenance strategies on a common basis.

A limit state function (or performance function), $g(\underline{X})$, for a structural component is defined in terms of the basic variables, $\underline{X}$, such that:

$$\begin{aligned} g(\underline{X}) &< 0 \text{ denotes failure} \\ g(\underline{X}) &> 0 \text{ denotes satisfactory performance} \end{aligned} \tag{1}$$

and the surface given by:

$$g(\underline{X}) = 0 \tag{2}$$

is called the limit state equation or limit state surface. The performance function g is typically obtained from the mechanics of the problem at hand. For multiple failure modes or if there are multiple critical sections, Eq. (2) is generalized to an appropriate union of failure events.

The basic variable generally comprise of quantities like material properties, loads or load-effects, environmental parameters, geometric quantities, modeling uncertainties, etc. They are usually modeled as random variables; however, those with negligible `uncertainties may be treated as deterministic. The general expression of failure probability is

$$P_f = P\left(g(\underline{X}) < 0\right) = \int_{g(\underline{x})<0} f_{\underline{X}}(\underline{x})d\underline{x} \tag{3}$$

where $f_X(x)$ is the joint probability density function for $\underline{X}$. The reliability of the structure would then be defined as $Rel = 1 - P_f$.

Like any other design approach, reliability based design is an iterative process: the design is adjusted until adequate safety is achieved and cost and functional requirements are met. The final step of meeting the target reliability can either be direct where the computed structural reliability has to exactly satisfy the target reliability for each relevant limit state or it can be indirect as in partial safety factors (PSF) based design where the structure implicitly satisfies the target reliability within a certain tolerance [9]. The term load and resistance factor design (LRFD) implies the approach followed in the United States where the nominal resistance in the design equation is multiplied by an explicit "resistance factor" but the nominal material properties that go into determining the resistance are not factored. The term PSF based design implies the approach taken in Europe where there is no explicit resistance factor in design, but each material property generally has its own partial safety factor. The latter approach is taken in this work.

Closed-form solutions to Eq. (3) are generally unavailable. Two different approaches are widely in use: (i) analytic methods based on constrained optimization and normal probability approximations, and (ii) simulation based algorithms with or without variation reduction techniques and both can provide accurate and efficient solutions to the structural reliability problem. The first kind, grouped under First Order Reliability Methods (or

FORM), holds an advantage over the simulation based methods in that the design point(s) and the sensitivity of each basic variable can be explicitly determined. However, FORM can prove to be costly or even infeasible if the size of the reliability problem goes up (in terms of basic variables and/or number of limit states) or if the limit state is not analytic in the basic variables, and is not used in this work. Monte Carlo simulations with Importance Sampling have been used to compute failure probabilities.

Target Reliability

It has become increasingly common to express safety requirements, as well as some functionality requirements, in reliability based formats. A reliability based approach to design, by accounting for randomness in the different design variables and uncertainties in the mathematical models, provides tools for ensuring that the performance requirements are violated as rarely as considered acceptable.

The cause, reference period, and consequences of violation of different performance requirements may vary, and if a reliability approach is taken, the target reliability in each performance requirement must take such difference into account [9-12]. For example, If the structure gives appropriate warning before collapse, the failure consequences reduce and that in turn can reduce the target reliability for that mode [11, 13]. Functionality target reliabilities may be developed exclusively from economic considerations. The safety target reliability levels required of a structure, on the other hand, cannot be left solely to the discretion of the owner, or be derived solely from a minimum total expected cost consideration, since structural collapse causing a large loss of human life and/or property may not be acceptable either to the society or the regulators. Design codes, therefore often place a lower limit on the reliability of safety related limit states [9, 14]. For optimizing a structure with multiple performance requirements, Wen [(15)] suggested minimizing the weighted sum of the squared difference of the target and actual reliabilities.

ISO 2394 [10], and later JCSS [(11)], proposed three levels of requirements with appropriate degrees of reliability: (i) serviceability (adequate performance under all expected actions), (ii) ultimate (ability to withstand extreme and/or frequently repeated actions during construction and anticipated use), (iii) structural integrity (i.e., progressive collapse in ISO 2394 and robustness in JCSS). Target reliability values were suggested based on the consequences of failure for ultimate limit states and relative cost of safety measure for serviceability limit states. The Canadian Standards Association [16] defines two safety classes and one serviceability class (and corresponding annual target reliabilities) for the verification of the safety of offshore structures (i) Safety class 1- great risk to life or high potential for environmental pollution or damage, 2) Safety class 2-small risk to life or low potential for environmental pollution or damage, and 3) Serviceability Impaired function and none of the other two safety classes being violated. Det Norske Veritas [13] specifies three types of structural failures for offshore structures and target reliabilities for each corresponding to the seriousness of the consequences of failure. The American Bureau of Shipping [17] identified four levels of failure consequences for various combinations of limit states and component class for the concept Mobile Offshore Base and assigned target reliabilities for each. Ghosn & Moses[(18)] suggest three levels of performance to ensure adequate redundancy of bridge structures corresponding to functionality, ultimate and damaged condition limit states, while Nowak et al.[(19)] recommend two different reliability levels for bridge structures corresponding to ultimate and serviceability limit states. Nuclear power plant containment structures are designed for earthquakes at two different levels of intensity and correspondingly to two different criteria for failure [3, 20, 21]. Damage, if any, caused by the Operating Basis Earthquake (OBE) must not lead to loss of functionality of the nuclear power plant; whereas the Safe Shutdown Earthquake (SSE) that has a higher intensity and longer recurrence interval than OBE, is allowed to cause the power plant to shutdown but must not cause any radioactive leakage to the environment or loss of structural integrity.

The fact that the consequences of failure of a critical component in a nuclear reactor are potentially catastrophic has driven the interest in building so called “inherently safe reactors” which are publicly perceived to have a zero probability of failure. The application of a probabilistic risk assessment framework to such structures then has the dual purpose of determining the probability of meeting the stipulated conditions under which “inherently safe” performance is guaranteed, and the probability of departure from acceptable performance under these conditions. Values of maximum acceptable probability levels set by regulatory authorities of different nations for accidents that cause severe damage to the reactor core have ranged from $1\text{x}10^{-6}$ per year to $1\text{x}10^{-4}$ per year [22].

Given the inability to predict the occurrence or magnitude of earthquakes, the uncertainties involved from source to site, and the potential for massive damage, it is not surprising that performance based design has been most enthusiastically espoused in the seismic engineering community, as evident in SEAOC [(23)], ATC-40 [(24] and FEMA 273/274 [25]. Performance levels for seismic design are commonly defined in terms of increasing severities, e.g., (i) Immediate Occupancy (IO), the state of damage at which the building is safe to occupy without any significant repairs, (ii) Structural Damage (SD), an intermediate level of damage in which significant structural and non-structural damage has occurred without loss of global stability, and (iii) Collapse Prevention (CP), representing extensive structural damage that causes global instability [26]. A comparison of the performance of structures designed to one ultimate design earthquake vs. those designed to dual level performance levels indicated that the latter produces relatively stronger structures [15]. A similar finding was echoed by Ghobarah [27] who opined that the reason for the revision of the then design standards to more reliable performance based methods was that after severe earthquakes (such as Northridge and Kobe), while structures designed to the existing codes performed well with respect to safety, the extent of damage and the economic costs were unexpectedly high.

Reliability of Prestressed Concrete Sections

The tensile strength of concrete is negligible compared to its compressive strength. In ordinary reinforced concrete, the reinforcing steel is used to carry the tensile stresses, and the concrete near the tensile face may crack. Prestressing is intended to artificially induce compressive stresses in the concrete to counteract the tensile stresses caused by external loads, such that the loaded section remains mostly if not entirely in compression [28].

Prestressed concrete (for shells, slabs, girders etc.) is often adopted when in addition to satisfying strength requirements, the member is also required to be slender (e.g., from aesthetic or weight considerations) and/or to limit cracking (e.g., to satisfy leak-tightness). Prestressed concrete members are relatively lightweight as they are built from high strength steel and high strength concrete, more resistant to shear, and can recover from effects of overloading. However, prestressed concrete structures are more expensive, have a smaller margin for error, and the design process of prestressed members is more complicated. Although the loss of prestress with time is built into the design, unintended loss of prestress arising from corrosion of the tendons, slippage etc. can have catastrophic consequences.

Prestressed concrete sections may fail in several possible ways (such as a combination of flexure, shear and torsion, bursting of end blocks, bearing, anchorage or connection failures, excessive deflections etc.). This work however, only looks at ultimate flexural limit state defined by collapse of concrete due to crushing.

Several reliability based studies on partially prestressed concrete sections have been conducted in the past. Al-Harthy and Frangopol [29] studied prestressed beams designed to the 1989 ACI 318, considering 3 different limit states (ultimate flexure, cracking in flexure and permissible stresses), random dead and live loads, material and geometric properties, prestressing forces and modeling uncertainty. Their studies concluded that the reliability indices implied by the 1989 ACI 318 design standard are non-uniform over various ranges of loads, span lengths and limit states. Hamann and Bulleit [30] examined the reliability of under reinforced high-strength concrete prestressed beams designed in accordance with the 1983 ACI-318 standard, considering only the ultimate flexural limit state of beams subjected to dead and snow loads. While Al-Harthy and Frangopol included all the material and geometric random variables in a FORM analysis, Hamann and Bulleit first estimated the moment capacity through Monte Carlo simulations, fitted the data to standard distributions, and then performed a first order second moment reliability analysis on the linear limit state.

Reliability for Class-1 structures, particularly concrete containment structures for nuclear power plants, is a much researched subject primarily due to the dire failure consequences of the containment structure in terms of environmental impact, radiation effect on human health and other economic costs. Hwang et al. [6] described a Load and Resistance Factor Design (LRFD)-based approach to determine the critical load combinations for design of concrete containment structures. The limit state, corresponding to ultimate strength of concrete, was defined in the 2-D space of membrane stress and bending moment in the shell, leading to an octagonal limit state surface. Pandey [8] and Varpasuo [31] also worked on the on the reliability of concrete containments, their limit states forming sides of the octagonal limit state considered by Hwang et al [6].

Mechanics of Pre-Stressed Concrete Sections

In this work, we look at collapse limit state of partially prestressed sections in flexure which corresponds to crushing of concrete in compression (reinforcements may yield). Bidirectional flexure on shell elements corresponding to nuclear power plant inner containment structures with voids have been modeled using Wood's criteria ([32]) summarized in Appendix A. The material properties of concrete and steel and the mechanistic formulation of both the limit states are discussed next.

In Indian Standards such as IS 456 ([33]) the compressive stress-strain relationship for concrete is taken to be parabolic up to a strain of 0.002, and horizontal from that point on. The nominal compressive strength of concrete is taken to be $f_{cn} = f_{ck} /1.5$ where f_{ck} is the characteristic compressive strength. The design compressive strength of concrete is f_{cn} /γ_c, where γ_c is the material safety factor on concrete strength. The value of γ_c is usually taken to be

1.5 for normal design condition and as 1.15 for abnormal design condition. The failure strain of concrete in bending compression is 0.0035. IS 1343([34]) specifies the minimum grade of concrete as M30 for post-tensioning and M40 for pre-tensioning.

The stress-strain behavior of concrete in tension is linear [35] and the tensile strength is taken to be $f_{c\gamma} = 0.7\sqrt{f_{ck}}$ and the modulus of elasticity of concrete in tension is assumed to be same as the secant modulus of concrete in compression which is $E_c = 5000\sqrt{f_{ck}}$ The maximum tensile strain in concrete is then,

$$\varepsilon_{t\max} = \frac{f_{cr}}{E_c} = 0.00012 \tag{4}$$

The design yield stress for reinforcing steel is f_{yn}/γ_s where f_{yn} is the nominal yield strength and γ_s is the material safety factor on yield strength of steel and is taken to be 1.15 for normal design conditions and 1.0 for abnormal conditions. The nominal modulus of elasticity of steel, E_n, is 200000 N/mm^2.

The moment capacity of a partially prestressed concrete section, given the amount of prestressing force and the geometric and material properties can be obtained in the form of an interaction diagram using strain compatibility equations and force balance. Interaction diagrams are plots of normalized compressive force, $P' = P / (f_{ck}bD)$ and normalized moment capacity, $M' = M / (f_{ck}bD^2)$ where b and D are the width and the depth of the section, respectively.

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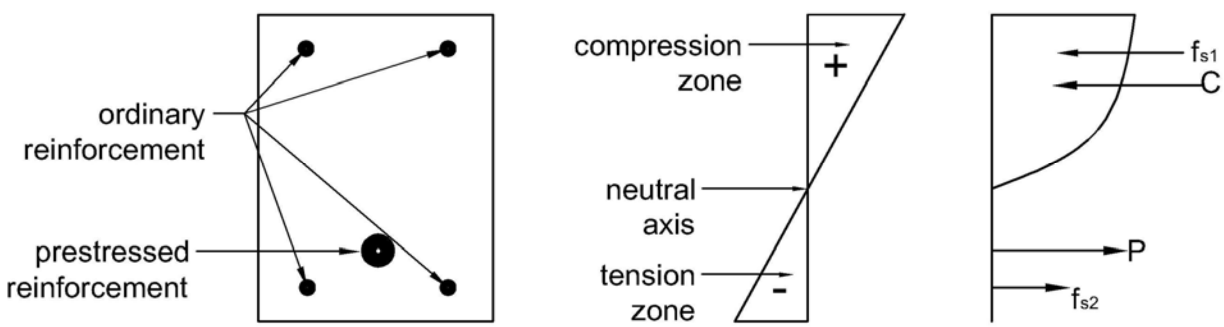

Fig.1: Force balance and moment computations for partially prestressed section

Figure 1 shows the strain and stress diagrams for an example section similar to the ones used in this work - with one set of prestressing tendons and two layers of ordinary reinforcement. In the figure, C= compressive force in concrete, f_{s1} = force in top reinforcement, f_{s2} = force in bottom reinforcement and P = prestressing force.

For given amount of prestress the position of the neutral axis is determined iteratively by balancing the tensile and compressive forces on the section. The moment capacity can then be found by taking the moment of the forces about any convenient point. In determining the collapse moment capacity, two cases are possible (Figure 2): the neutral axis (NA) outside and the neutral axis inside the section. In the former, the entire section is in compression and in the latter, concrete has cracked and is assumed not to carry any load in the tensile zone.

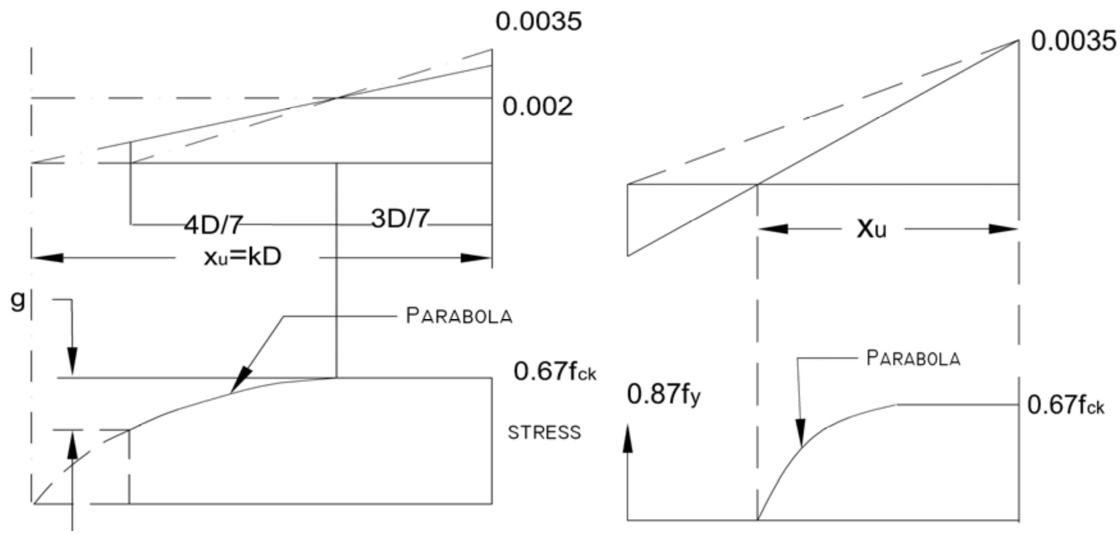

Fig.2: Strain and stress distributions on section for neutral axis outside (left) and inside (right) the section for limit state of collapse

Figure 3 shows an example prestressed concrete element corresponding to the shell structure of nuclear power plant inner containment structures. Two layers of ordinary reinforcement top and bottom can be seen and JI and JO correspond to prestressing cables in the North-South and East-West directions respectively. In the co-ordinate system adopted, these two are considered as the x and y directions.

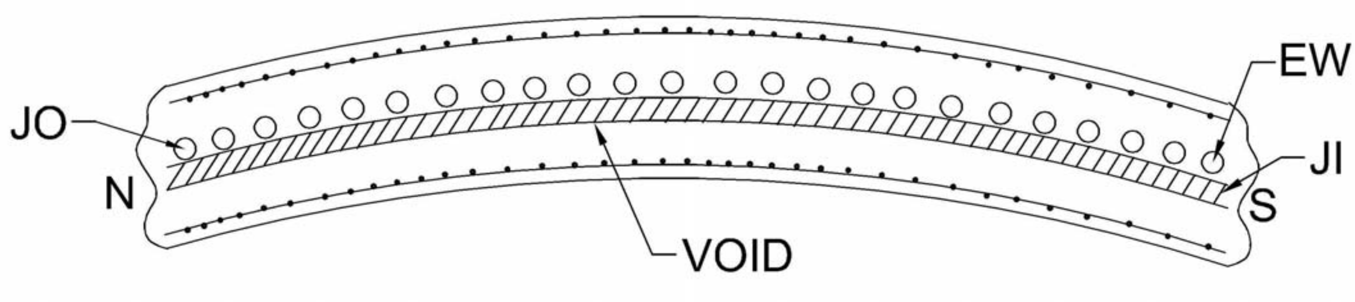

Fig.3: Prestressed concrete shell

When calculating the flexural strength of an element such as this, the space taken by the prestressing cable JI has to be considered as a void in concrete, i.e. while calculating the contribution of concrete to the strength the area considered is the total area minus the area of the void. For the typical prestressed containment shell, the void depth is approximately 0.15 times the total depth of the element.

The forces and moments acting on the section (for example Dead Load, Ordinary Live Load, Construction Live Load, Pre Stressing Load and Accidental Pressure Load) have two normal components (xx and yy) and one shearing (xy) component. Additionally, areas occupied by pre stressing cables are considered as voids in concrete. The section is thus under bi-directional flexural loading.

The components of the externally applied loads (N_{xx} and N_{yy}) on the section act in the same direction as the pre-stressing cables. As a result, the external forces cause the section to be in compression, thus acting like a external pre stressing forces. These forces therefore affect the moment capacity of the section. On the other end, applied moments on the section are caused due to the same set of forces. Thus both the capacity of the system and the loading on the system are affected by a common source and therefore it is possible that the applied moment and the moment capacity will show some degree of correlation.

The design check is carried out in the principal plane with respect to stresses. Applied moments are converted to this plane according to the basic rules of tensorial transformation. The moment capacities of the section in each of the 2 principal directions are computed from interaction diagrams with transformed dimensions, reinforcement areas and voids. The applied moments in directions X and Y are obtained using Wood's Criteria [32], summarized in Appendix A) which outlines a procedure to obtain applied moments in x and y direction at the bottom and the top of the section (M^*_{xtop}, $M^*_{xbottom}$, M^*_{ytop}, $M^*_{ybottom}$), eliminating the torsional moment component M_{xy}.

Since the structural analysis used to compute stress resultants is completely linear in nature, the different stress resultant components ($N_{xx}, N_{yy}, N_{xy}, M_{xx}, M_{xy}, M_{xy}$) in a given load case are statistically fully dependent on one another. Additionally, stress resultants due to different load cases are completely independent. The correlation matrix that is developed for analysis is based on these two assumptions.

RELIABILITY ANALYSIS AND CALIBRATION OF PSFs

Limit States and Basic Variables

Since this work concerns the reliability of PC shells in biaxial flexure, the limit states in x and y directions can be written respectively as:

$$g_x = M_{\text{cap},x} - M_{\text{app},y} = 0 \tag{5}$$

$$g_y = M_{\text{cap},y} - M_{\text{app},y} = 0 \tag{6}$$

so that failure of the section is given by:

$$\{\text{Failure}\} = g_x < 0 \cup g_y < 0 \tag{7}$$

and the failure probability can be written as:

$$P_f = \int_{\underline{x} \in \{\text{Failure}\}} f_{\underline{X}}(\underline{x})\, d\underline{x} = \int_{\text{all } \underline{x}} \mathbb{I}\left[\{\text{Failure}\}\right] f_{\underline{X}}(\underline{x})\, d\underline{x} \tag{8}$$

The indicator function, $\mathbb{I}$, on the right hand side of Eq. [4.4] evaluates the expression within brackets so that:

$$\mathbb{I}[\bullet] = \begin{cases} 1, \text{ if } [\bullet] \text{ is true} \\ 0, \text{ if } [\bullet] \text{ is false} \end{cases} \tag{9}$$

and is a convenient way to convert the domain of integration from the failure region to the entire range of $\underline{x}$ which is useful in simulation based estimates as described subsequently.

$M_{\text{cap},x}$ and $M_{\text{cap,y}}$ are the moment capacities in x and y directions respectively. Likewise, $M_{\text{app},x}$ and $M_{\text{app,y}}$ are the applied moments. As stated above, the moment capacity corresponds to collapse of the section. From this point forward, unless otherwise mentioned, all moments in this work are normalized by $f_{ck}bD^2$ and all forces by $f_{ck}bD$.

Before going into the details of the individual terms above, it is important to recall that the moment capacities and applied moments are mutually statistically dependent since the capacities are functions of the axial loads which in turn are linearly related to the applied moments in each load case. In addition, the capacities in the x and y directions are strongly correlated as they are functions of the same material properties and some of the same axial loads.

As explained in Appendix A, $M_{\text{app},x}$ and $M_{\text{app,y}}$ are defined respectively as $\max(\text{abs}(M^*_{xtop}),\text{abs}(M^*_{xbottom}))$ and $\max(\text{abs}(M^*_{ytop}),\text{abs}(M^*_{ybottom}))$ and are functions of the applied moments M_{xx} ,M_{yy} and M_{xy} caused by all load cases relevant to the load combination at hand. For example, we can have the load combination Dead (D) + Prestressing (P_s) + Ordinary Live (L_o) + Temperature (T) + Accidental Pressure (P_a) giving us:

$$\left.\begin{aligned} M_{xx} &= M_{xx,D} + M_{xx,Ps} + M_{xx,Lo} + M_{xx,T} + M_{xx,Pa} \\ M_{yy} &= M_{yy,D} + M_{yy,Ps} + M_{yy,Lo} + M_{yy,T} + M_{yy,Pa} \\ M_{xy} &= M_{xy,D} + M_{xy,Ps} + M_{xy,Lo} + M_{xy,T} + M_{xy,Pa} \end{aligned}\right\} \tag{10}$$

The normalized moment capacity, M_{cap}, is a function of the applied in-plane compression, material properties ($f_c, f_y, E, \varepsilon_c, \varepsilon_t$) and geometric quantities ($p / f_{ck}, d / D, e / D, t_{void} / D$):

$$M_{cap} = M_{\text{cap}}\left(P, f_c, f_y, E, \varepsilon_c, \varepsilon_t, \frac{p}{f_{ck}}, \frac{d}{D}, \frac{e}{D}, \frac{t_{void}}{D} \right) \tag{11}$$

Of these, the random terms are: the applied in-plane compressive force, P , the compressive strength of concrete, f_c , the yield strength, f_y , and the Young's modulus, E, of the reinforcing steel. The compressive force P in turn is the algebraic sum of forces from all load cases in the load combination considered.

The nominal or design values of the moment capacities, to be used in design equations discussed below, can be obtained by substituting the random quantities in Eq. [4.9] by their design values:

$$M_{cap,n} == M_{\text{cap}}\left(P_n, \frac{f_{cd}}{\gamma_c}, \frac{f_{yn}}{\gamma_s}, E_n, \varepsilon_c, \varepsilon_t, \frac{p}{f_{ck}}, \frac{d}{D}, \frac{e}{D}, \frac{t_{void}}{D} \right) \tag{12}$$

As stated above, in accidental pressure load case, the material safety factor on concrete compressive strength, γ_c is commonly taken to be 1.15, while that on yield strength of reinforcing steel, γ_s is commonly 1.0.

The moment capacities $M_{cap,x}$ and $M_{cap,y}$ thus defined are implicit functions of four basic variables; their distributions and correlations with applied moments are obtained by numerical simulation, which in turn are used in the reliability analyses.

Monte Carlo Simulations and Importance Sampling

Except in very special situations, closed form solution to the structural reliability problem (Eq. (8)) does not exist and numerical approximations are needed. The true probability of failure, P_f ,

$$P_f = \int_{\text{all } \underline{x}} \mathbb{I}\left[\{\text{Failure}\}\right] f_{\underline{X}}\left(\underline{x}\right) d\underline{x} = \int_{\text{all } \underline{u}} \mathbb{I}\left[\{\text{Failure}\}\right] f_{\underline{U}}\left(\underline{u}\right) d\underline{u} \tag{13}$$

can be estimated using basic (or "brute-force" or "crude") Monte Carlo simulations (MCS) in practice as:

$$\hat{P}_f = \frac{1}{N}\sum_{i=1}^{N} \mathbb{I}\left[g_x\left(T\left(\underline{U}_i\right)\right) < 0 \cup g_y\left(T\left(\underline{U}_i\right)\right) < 0 \right] \tag{14}$$

where a zero-mean normal vector $\underline{U}$ with the same correlation matrix ρ as the basic variables is generated first and then transformed element by element according to the full distribution transformation:

$$T(\underline{u}) = \underline{x} \;\Rightarrow F_{X_i}(x_i) = \Phi(u_i) \tag{15}$$

The use of the same ρ for $\underline{U}$ as for $\underline{X}$ results in error, but the error is generally small [36]. N is the total number of times the random vector $\underline{U}$ is generated, and $\underline{U}_i$ is the i^{th} realization of the vector. It is well-known that the basic Monte-Carlo simulation-based estimate of P_f has a relatively slow and inefficient rate of convergence. The coefficient of variation (COV) of the estimate is :

$$\text{c.o.v.}(\hat{P}_f) = \sqrt{(1-P_f)/(NP_f)} \approx \sqrt{1/(NP_f)} \tag{16}$$

which is proportional to $1/\sqrt{N}$ and points to an inefficient relation between sample size and accuracy (and stability) of the estimate.

Such limitations of the basic Monte Carlo simulation technique have led to several "variance reducing" refinements. Notable among them is Latin hypercube sampling (e.g., [37]), importance sampling (e.g.[38]) along with its variants (e.g., [39], [40]) which, if performed carefully, can significantly reduce the required sampling size. Nevertheless, importance sampling and other variance reducing techniques should be performed with care, as their results may be quite sensitive to the type and the point of maximum likelihood of the sampling distribution, and an improper choice can produce erroneous results.. In this work, we have adopted Importance Sampling to estimate the failure probability in Eq. (13).

The mathematical formulation of importance sampling is simply obtained by modifying the basic expression of failure probability (Eq. (8)) as:

$$P_f = \int_{\underline{x}\in\{\text{Failure}\}} f_{\underline{X}}(\underline{x})\,d\underline{x} = \int_{\underline{x}\in\{\text{Failure}\}} \frac{f_{\underline{X}}(\underline{x})}{f_{\underline{H}}(\underline{x})} f_{\underline{H}}(\underline{x})\,d\underline{x} \tag{17}$$

where $f_{\underline{H}}$ is any PDF not equal to zero in the region of interest. A judicious choice of $f_{\underline{H}}$ can ensure low variance of the estimated failure probability. By a simple change of the variable of integration, the failure probability estimate is as before the computation of the expectation of the indicator function but now modified with a correction factor ($f_{\underline{U}} / f_{\underline{H}}$):

$$\hat{P}_f = \frac{1}{N}\sum_{i=1}^{N} \mathbb{I}\left[g_x\left(T\left(\underline{h}_i\right)\right) < 0 \cup g_y\left(T\left(\underline{h}_i\right)\right) < 0 \right] \; \frac{f_{\underline{U}}\left(\underline{h}_i\right)}{f_{\underline{H}}\left(\underline{h}_i\right)} \tag{18}$$

It is important to note that this expectation as computed with respect to the sampling density $f_{\underline{H}}$ and the estimate of failure probability is obtained by simulating vectors of $\underline{H}$. The choice of $f_{\underline{H}}$ is extremely important, and depending on the limit state function, an improper choice may lead to errors in the estimate of P_f.

In this work, $\underline{H}$ has been taken as a jointly Normal random vector with the same correlation matrix ρ as $\underline{U}$, but with a mean vector that is closer to the failure region. This mean vector is chosen carefully by comparing the IS results with basic MCS results for the range of problems encountered. The variance of the estimate in Eq. (18) is:

$$\text{var}\,(\hat{P}_f) = \frac{1}{N^2}\sum \text{var}\left(\mathbb{I}_i \, \frac{f_{\underline{U}}(\underline{h}_i)}{f_{\underline{H}}(\underline{h}_i)} \right) \tag{19}$$

which can be estimated during the sampling as:

$$\hat{S}^2(\hat{P}_f) = \frac{\sum \mathbb{I}^2 f_{U}^{\,2} / f_H^2}{N^3} - \frac{1}{N^2}\left(\frac{\sum \mathbb{I}\, f_U / f_H}{N} \right)^2 \tag{20}$$

giving the coefficient of variation (COV) of the failure probability estimated through importance sampling as:

$$\hat{V}(\hat{P}_f) = \frac{\hat{S}(\hat{P}_f)}{\hat{P}_f} \tag{21}$$

One of our stopping criteria for the Importance Sampling simulation in this work involves an upper limit on the COV of the estimated failure probability.

Partial Safety Factors and their Optimization

Reliability based partial safety factor (PSF) design is intended to ensure a nearly uniform level of reliability across a given category of structural components for a given class of limit state under a particular load combination [41]. We approach the topic of optimizing PSFs by noting that any arbitrary point, $\underline{X}^a$, on the limit state surface by definition satisfies:

$$g(\underline{X}^a) = 0 \tag{22}$$

Conversely, a "design point" $\underline{X}^d$ on the limit state surface can be carefully chosen so that it "locates" the limit state in the space of basic variables such that a pre-defined target reliability is ensured for the design. The ensuing design equation:

$$g(\underline{X}^d) = 0 \tag{23}$$

is essentially a relationship among the parameters of the basic variables and gives a minimum requirement type of tool in the hand of the design engineer to ensure target reliability for the design in an indirect manner. Since nominal or characteristic values of basic variables are typically used in design, Eq. (22) may be rewritten as:

$$g\left(\frac{X_1^n}{\gamma_1},\ldots,\frac{X_k^n}{\gamma_k},\gamma_{k+1}X_{k+1}^n,\ldots,\gamma_m X_m^n\right) \geq 0, \text{ where } \frac{X_k^n}{\gamma_k} = X_k^d \tag{24}$$

where the superscript n indicates the nominal value of the variable. We have partitioned the vector of basic variables into k resistance type and $m - k$ action type quantities. The partial safety factors, γ_i , are typically greater than one: for resistance type variables they divide the nominal values while for action type variables they multiply the nominal values to obtain the design point:

$$\begin{aligned} &\text{resistance PSFs}: \gamma_i = \frac{X_{i,n}}{X_i^d}, i = 1,\ldots,k \\ &\text{action PSFs}: \gamma_i = \frac{X_i^d}{X_{i,n}}, i = k+1,\ldots,m \end{aligned} \tag{25}$$

If the design equation (23) can be separated into a strength term and a combination of load-effect terms, the following safety checking scheme may be adopted for design:

$$R_n\left(\frac{S_i^n}{\gamma_i^s}, i = 1,\ldots,k\right) \geq l\left(\sum_{i=1}^{m-k}\gamma_i^q Q_i^n\right) \tag{26}$$

where R_n = the nominal resistance and a function of factored strength parameters, l = load-effect function, S_i^n =nominal value of i^{th} strength/material parameter, γ_i^s = i^{th} strength/material factor, Q_i^n = the nominal value of the i^{th} load and γ_i^q = i^{th} load factor. Note that there is no separate resistance factor multiplying the nominal resistance (as in LRFD) since material partial safety factors have already been incorporated in computing the strength.

The nominal values generally are fixed by professional practice and thus are inflexible. Some of the m partial safety factors (often those associated with material properties) can also be fixed in advance. The remaining

PSFs can be chosen by the code developer so as to locate the design point, and hence locate the limit state as alluded to above, and hence achieve a desired reliability for the structure. Such an exercise can be conveniently performed if the strength and load effect terms can be separated as above in which case the limit state equation can be normalized by the design equation:

$$\frac{M_{\text{cap}}}{M_{\text{cap}}^{n}} - \frac{M_{\text{app}}}{M_{\text{app}}^{n}} = 0 \tag{27}$$

The reliability problem now becomes:

$$\begin{aligned} &\text{Find } \gamma_1^s,...,\gamma_k^s,\gamma_1^q,...,\gamma_{m-k}^q \text{ such that} \\ &P\left[\frac{M_{\text{cap}}}{M_{\text{cap}}^{n}(\gamma_1^s,...,\gamma_k^s)} - \frac{M_{\text{app}}}{M_{\text{app}}^{n}(\gamma_1^q,...,\gamma_{m-k}^q)} \le 0\right] = \Phi(-\beta_T) \end{aligned} \tag{28}$$

where β_T is the target reliability index. Of course, this is an under-defined problem and even though some of the PSFs may be fixed in advance as stated above, it has an infinite number of solutions. Additional considerations are needed to improve the problem definition. Such considerations naturally arise when PSFs are needed to be "optimized" for a class of structures and are discussed next.

It is common to expect that the design equation be valid for *r* representative structural components (or groups), and let w_i be the weight (i.e., relative importance or relative frequency) assigned to the i^{th} such component (or group). These *r* representative components may differ from each other on account of different locations, geometric dimensions, nominal loads, material grades etc. For a given set of PSFs, let the reliability of the i^{th} group be β_i. Choosing a new set of PSFs gives us a new design, a new design point, and consequently, a different reliability index. If there has to be one design equation, i.e., one set of PSFs, for all the *r* representative components, the deviations of all β_i's from β_T must in some sense be minimized. The design equation (Eq. (24) or Eq. (26)), when using the optimal PSFs obtained this way, can ensure a nearly uniform reliability for the range of components. Several constraints may be introduced to the optimization problem to satisfy engineering and policy considerations (as summarized in [42]). Moreover, some partial safety factors, such as those on material strengths, may be fixed in advance as stated above. The PSF optimization exercise adopted in this paper has the following form:

$$\begin{aligned} &\min\left[\sum_{i=1}^{r} w_i \left(\beta_i\left(\gamma_1^q,...,\gamma_{m-k}^q\right) - \beta_T\right)^2\right] \text{ where } \sum_{i=1}^{r} w_i = 1 \\ &\text{subject to: } \min(\beta_i) > \beta_T, \quad i = 1,...,r \\ &\gamma_i^q \ge \gamma_i^{\min}, \quad i = 1,...,m-k \\ &\gamma_i^q \le \gamma_i^{\max}, \quad i = 1,...,m-k \\ &\gamma_i^s = m_i, i = 1,...,k \end{aligned} \tag{29}$$

NUMERICAL EXAMPLE

An example problem based on the prestressed IC shells of typical 220 MWe Indian PHWRs has been set up to demonstrate the methodology developed in this paper. A combination involving 5 load cases namely Dead Load (D), Pre-Stressing Load (P_s), Ordinary Live Load (L_0), Accidental Temperature Load (T) and Accidental Pressure Load (P_a) has been considered. For each load case sets of six stress resultants (N_{xx}, N_{yy}, N_{xy}, M_{xx}, M_{yy}, M_{xy}) have been obtained from linear elastic finite element analyses. The FE model consisted of about 2500 elements.

Four structural groups of the IC Shell have been selected for finding optimal PSFs (Group 1: dome general area between two SG openings, Group 2: SG opening, Group 3: dome general area between SG opening and ring beam, Group 4: IC wall). The section depths (*D*) are respectively 500, 1200, 500 and 610 mm. For each group the critical element has been identified as the one having the lowest capacity demand ratio – by considering all nominal stress resultants for the given load combination. The objective of the example is to obtain a set of partial safety factors for the 5 applied loads that satisfy a set of optimality criteria.

The statistical parameters and nominal values used in the problem are listed in Tables 1 – 4. The computed correlation coefficients for Group 1 obtained between the moment capacities and applied moments are listed in Table 5. Noticeable here is the high positive correlation between moment capacity and prestressing force and the high negative correlation between the moment capacity and the accidental pressurization force. The moment capacities in x and y directions are almost fully mutually dependent. These are consistent with our intuitive expectations from the mechanics of the problem. The bias and COV of the moment capacities obtained for each group are provided in Table 6. The optimization parameters (cf. Eq. (29)) are listed in Table 7. While estimating the objective function in Eq. (29), a linear response was fitted around the given decision variable vector in order to smooth the sampling related fluctuations in the estimated reliability indices. Table 8 lists the optimal results for this example problem.

Table 1: Statistics of Basic Variables

Random Variable	**Description**	**Statistics Distribution(mean, COV)**	**Source**
$M'_{LC,xx}$ $M'_{LC,xy}$ $M'_{LC,yy}$	normalized applied moments	for each load case (LC) in given load combination, combined according to Wood's criteria	FEM Analysis of IC Shell Model
$P'_{LC,xx}$ $P'_{LC,xy}$ $P'_{LC,yy}$	normalized force	for each load case (LC) in given load combination	FEM Analysis of IC Shell Model
$M'_{cap,xx}$ $M'_{cap,yy}$	normalized moment capacity	obtained through Interaction diagram (fn of $P'_{LC,xx}$, $P'_{LC,xy}$ and $P'_{LC,yy}$)	Structural Analysis of Prestressed concrete section
f_c	compressive strength of concrete	Normal, (max(f_{ck}+0.825s_c, f_{ck}+4), s_c)*	[33]
f_y	Yield strength of steel	Lognormal(1.1133f_{yn},0.09)	[43], [30]
E	Young's modulus	Normal(1.001103E_n,0.01)	[43]
* s_c=standard deviation for characteristic strength (in MPa) of concrete as given in IS 1343([34])			

Table 2: Deterministic Parameters

Parameter	**Description**	**Values taken**
p	Percent reinforcement	0.2%
f_{ck}	Characteristic compressive strength of concrete	45 MPa
$f_{cn} = f_{ck}/1.5$	Nominal compressive strength of concrete	30 MPa
f_{yn}	Nominal yield strength of reinforcing steel	415 MPa
E_n	Nominal Young's modulus of reinforcing steel	200 GPa
e/*D*	Eccentricity of prestressing force	0
d/*D*	cover depth	0.05
void range	no concrete due to PS cables	0.5*D* to 0.6 *D*

Table 3: Distribution types of loads

Load type	**Distribution type**	**C.O.V**	**Bias**
Dead	Normal	0.1	1.0
Pre-Stress	Lognormal	0.15	1.2
Live (Ordinary)	Lognormal	0.15	1.0
Temperature	Gumbel	0.15	0.9
Accidental Pressure	Gumbel	0.15	0.8

Table 4: Nominal load effects for the critical element in each group

Load case	Load effect					
	Nxx (ton/m)	Nyy(ton/m)	Nxy (ton/m)	Mxx (ton-m/m)	Myy (ton-m/m)	Mxy (ton-m/m)
	Group 1					
D	-9.61E+00	-1.22E+01	9.64E+00	-2.14E-01	-2.27E-01	3.91E-02
Ps	-4.44E+02	-4.77E+02	1.25E+02	1.53E+00	1.90E+00	-1.39E+00
Lo	-9.61E-01	-1.03E+00	2.82E-01	-3.37E-02	-3.63E-02	1.03E-02
T	3.12E+00	2.75E+00	1.34E+00	3.38E+00	3.42E+00	-1.27E-01
Pa	2.08E+02	2.26E+02	-7.09E+01	5.96E+00	6.29E+00	-1.30E+00
	Group 2					
D	-5.41E+01	-1.54E+01	2.80E+00	-8.11E+00	-1.31E+00	2.74E+00
Ps	-1.02E+03	-6.20E+02	-5.63E+01	3.55E+01	-5.00E+00	6.20E+00
Lo	-2.58E+00	-6.15E-01	3.18E-01	1.20E-02	1.27E-02	6.93E-02
T	1.56E+00	-1.28E-01	-4.86E-01	1.17E+01	2.78E+00	-1.90E+00
Pa	5.80E+02	1.50E+02	-7.01E+01	1.64E+01	-3.10E+00	-2.56E+01
	Group 3					
D	-2.65E+01	2.12E+00	6.12E+00	6.72E-01	-1.24E-01	-4.42E-02
Ps	-5.23E+02	-2.43E+02	2.42E+01	1.85E+01	8.95E+00	-2.07E+00
Lo	-1.04E+00	-3.24E-01	6.12E-02	-2.36E-02	-1.36E-02	-4.47E-04
T	-1.09E-01	5.88E+00	1.86E+00	4.56E+00	3.66E+00	-8.39E-02
Pa	2.46E+02	6.23E+01	-1.91E+01	-1.47E-01	1.98E+00	5.85E-01
	Group 4					
D	3.73E+00	-4.10E+01	2.24E-01	-1.34E-01	-6.23E-01	-1.18E-02
Ps	-6.70E+02	-5.32E+02	-1.89E+01	1.17E+00	-1.39E+01	-1.89E-01
Lo	2.55E-01	-8.37E-01	-2.62E-03	-4.52E-03	-2.28E-02	-4.45E-06
T	7.88E+00	5.93E-02	1.48E-02	5.01E+00	5.61E+00	-1.06E-02
Pa	2.85E+02	1.80E+02	1.77E+00	2.91E+00	1.23E+01	-8.57E-02

Table 5: Typical correlation matrix (group 1)

	Dead	Prestress	Live	Temperature	Pressure	M_{capxx}	M_{capyy}
Dead	1.0000	-0.0028	0.0005	0.0054	-0.0006	0.099	0.0117
Prestress	-0.0028	1.0000	-0.0028	-0.0093	0.0029	0.9156	0.9072
Live	0.0005	-0.0028	1.0000	0.0049	-0.0053	0.0036	0.0023
Temperature	0.0054	-0.0093	0.0049	1.0000	0.0060	-0.0176	-0.0156
Pressure	-0.0006	0.0029	-0.0053	0.0060	1.0000	-0.2935	-0.2972
M_{capxx}	0.0099	0.9156	0.0036	-0.0176	-0.2935	1.0000	0.9956
M_{capyy}	0.0117	0.9072	0.0023	-0.0156	-0.2972	0.9956	1.0000

Table 6: Bias and COVs of moment capacities

Group	M_{capxx}		M_{capyy}	
	Bias	COV	Bias	COV
1	1.82	0.159	1.41	0.157
2	1.65	0.133	1.42	0.171
3	1.40	0.137	1.59	0.149
4	1.58	0.133	1.54	0.145

Table 7: Optimization parameters

Parameter	Value
Target reliability, β_T	3.5
Tolerance on target reliability, $\Delta\beta$	1.0
Weights on four Groups, w_i	0.25, 0.25, 0.25, 0.25
Material PSF on concrete strength, γ_c	1.3
Material PSF on steel strength, γ_s	1.0
Lower bounds on load PSFs	1.0, 1.0, 1.0, 1.0, 1.3
Upper bounds on load PSFs	1.2, 1.2, 1.3, 1.4, 1.8

Table 8: Optimal results

Parameter	Optimal values
Beta values at optimum (Groups 1 – 4 respectively)	2.41, 3.88, 4.51, 4.25
Objective value (weighted squared error)	0.82
Optimal PSFs (D, P_s, L_o, T, P_a)	1.19, 1.09, 1.24, 1.35, 1.51

CONCLUSIONS

A set of optimal partial safety factors (for collapse limit state) ensuring a nearly uniform level of reliability across 4 groups of structural elements in a typical IC Shell of an Indian NPP have been obtained. The complete methodology for the same was developed from first principles. Correlations between demand and capacity terms owing to the structural mechanics underlying the problem were taken into account and the methodology developed accordingly. Analysis of the structural behavior of prestressed concrete section was formulated using recommendations provided in IS 1343 and SP 16. Monte Carlo simulations using (1) Importance Sampling and (2) a linear response surface fit for variance reduction was used to compute probabilities of failure. The load factors obtained in this example problem are in agreement with design practices from around the world, except the temperature load factor is typically lower than found here since thermal loads are categorized as secondary loads caused by geometric constraints and local yielding and micro-cracking ultimately result in redistribution of forces.

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APPENDIX A: WOOD'S CRITERIA FOR MOMENT COMBINATION

The design check is carried out in the principal plane with respect to stresses, which is inclined at an angle θ given by:

$$\tan 2\theta = \frac{2N_{xx}}{N_x - N_y}$$

On this plane shearing stresses are absent and the perpendicular (principal) stresses are given by:

$$N_1 = \frac{N_x + N_y}{2} + \sqrt{(\frac{N_x - N_y}{2})^2 + N_{xy}{}^2}$$

$$N_2 = \frac{N_x + N_y}{2} - \sqrt{(\frac{N_x - N_y}{2})^2 + N_{xy}{}^2}$$

The applied moments are converted to this plane according to standard tensor transformation procedures that lead to the following expressions:

$$M_{XX} = \frac{M_{xx} + M_{yy}}{2} + \frac{(M_{xx} - M_{yy})\cos 2\theta}{2} - M_{xy}\sin 2\theta$$

$$M_{YY} = \frac{M_{xx} + M_{yy}}{2} - \frac{(M_{xx} - M_{yy})\cos 2\theta}{2} - M_{xy}\sin 2\theta$$

$$M_{XY} = \frac{(M_{xx} - M_{yy})\sin 2\theta}{2} + M_{xy}\cos 2\theta$$

The moment capacity in direction1 and 2 (or X and Y) are then computed from interaction diagrams with transformed dimensions, reinforcement areas and voids. The applied moments in directions X and Y are obtained using Wood's Criteria. This outlines a procedure to obtain applied moments in X and Y direction at the bottom and the top of the section according to the following procedure:

Bottom Reinforcement

The bottom reinforcement can be calculated for following set of moments in x- and y- directions

$$M_x^* = M_{XX} + |M_{XY}|$$

$$M_y^* = M_{YY} + |M_{XY}|$$

If both M_x^* & M_y^* calculated as per the above equation are found to be negative, then both are assigned a zero value and not utilized for design. If M_x^* is negative, then

$$M_y^* = M_{YY} + \left|\frac{M_{XY}^2}{M_{XX}}\right| \text{and}\, M_x^* = 0$$

If M_y^* is negative, then

$$M_x^* = M_{XX} + \left|\frac{M_{XY}^2}{M_{YY}}\right| \text{and}\, M_y^* = 0$$

<u>*Top Reinforcement*</u>
The top reinforcement can be calculated for following set of moments in x- and y- directions

$$M_x^* = M_{XX} - |M_{XY}|$$

$$M_y^* = M_{YY} - |M_{XY}|$$

If both M_x^* & M_y^* calculated as per the above equation are found to be negative, then both are assigned a zero value and not utilized for design. If M_x^* is negative, then

$$M_y^* = M_{YY} - \left|\frac{M_{XY}^2}{M_{XX}}\right| \text{and}\, M_x^* = 0$$

If M_y^* is negative, then

$$M_x^* = M_{XX} - \left|\frac{M_{XY}^2}{M_{YY}}\right| \text{and}\, M_y^* = 0$$

The limit state in x direction (i.e, principal plane 1) and y directions are respectively:

$$M_{cap\_x} - \max(\text{abs}(M_{xtop}^*), \text{abs}(M_{xbottom}^*))$$
$$M_{cap\_y} - \max(\text{abs}(M_{ytop}^*), \text{abs}(M_{ybottom}^*))$$