Reliability-Based Load and Resistance Factor Rating Using In-Service Data

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Abstract: Traditional bridge evaluation techniques are based on design-based deterministic equations that use limited site-specific data. They do not necessarily conform to a quantifiable standard of safety and are often quite conservative. The newly emerging load and resistance factor rating (LRFR) method addresses some of these shortcomings and allows bridge rating in a manner consistent with load and resistance factor design (LRFD) but is not based on site-specific information. This paper presents a probability-based methodology for load-rating bridges by using site-specific in-service structural response data in an LRFR format. The use of a site-specific structural response allows the elimination of a substantial portion of modeling uncertainty in live load characterization (involving dynamic impact and girder distribution), which leads to more accurate bridge ratings. Rating at two different limit states, yield and plastic collapse, is proposed for specified service lives and target reliabilities. We consider a conditional Poisson occurrence of identically distributed and statistically independent (i.i.d.) loads, uncertainties in field measurement, modeling uncertainties, and Bayesian updating of the empirical distribution function to obtain an extreme-value distribution of the time-dependent maximum live load. An illustrative example uses in-service peak-strain data from ambient traffic collected on a high-volume bridge. Serial independence of the collected peak strains and of the counting process, as well as the asymptotic behavior of the extreme peak-strain values, are investigated. A set of in-service load and resistance factor rating (ISLRFR) equations optimized for a suite of bridges is developed. Results from the proposed methodology are compared with ratings derived from more traditional methods.

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Introduction

Until fairly recently, most bridges were designed by using allowable stress methods [working stress design (WSD)] in which uncertainties in loads and element resistance were taken into account with a single factor of safety. Safety checks in WSD were performed in the elastic domain with the implicit assumption that the structure thus designed would perform adequately in the inelastic domain near failure. In 1994, as a result of National Cooperative Highway Research Program (NCHRP) Project 12-33, the AASHTO *Load and Resistance Factor Design (LRFD) Highway Bridge Design Specifications* was published (AASHTO 1994). The LRFD specifications are based on structural reliability theory and separately characterize the variability of the loads and resistance. The load and resistance factors (LRFs) were calibrated on the basis of a global population of bridges (Nowak 1995; NCHRP 1999b). The benefits of LRFD over WSD are that safety checks can be performed beyond yield and close-to-collapse conditions and that designs using LRFD have a more uniform level of safety across a range of configurations.

As bridge infrastructures age throughout the world, more and more bridges are being classified as structurally deficient. Unfortunately, because of limited financial resources, bridge owners are not able to immediately repair or, if necessary, replace all the so-called structurally deficient bridges in their inventory. As a result, methods for accurately assessing a bridge's true load-carrying capacity are needed so that the limited funds can be spent wisely.

When bridges are designed, the behavior of the as-built bridge, as well as the nature of the site-specific traffic, can only be estimated. The calibrated load and resistance factors in the AASHTO LRFD specifications are by necessity conservative, and many secondary sources of stiffness and strength are either neglected in design or are too difficult to compute. When load rating a bridge, however, the best model is the bridge itself. By monitoring the bridge, we can gather in-service traffic and performance data and conduct in-service evaluations. Bridge diagnostic and proof load tests are routinely conducted to evaluate in-situ bridge capacity (Fu and Tang 1992; Moses et al. 1994; Chajes et al. 1997; Faber et al. 2000; Chajes et al. 2001), and it should not be surprising that field tests have typically found calculated load-carrying capacities to be underestimates of the safe load-carrying capacities of a bridge (Bakht and Jaeger 1990; Goble et al. 1992; NCHRP 1998; Chajes et al. 2000). *Manual for Bridge Rating*

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through Load Testing was published as an outcome of NCHRP Project 12-28(13)A (NCHRP 1998). This manual provides *deterministic* methods for ascertaining bridge capacities on the basis of field testing and the quantification of site-specific bridge behavior.

Most recently, NCHRP Project 12-46 has led to the development of a *Manual for Condition Evaluation and Load and Resistance Factor Rating of Bridges* (NCHRP 1999a) that is consistent with the LRFD specifications. Like the LRFD specifications, the evaluation procedures developed are probability-based, and the process is called load and resistance factor rating (LRFR). And like LRFD, the LRFR specifications are still based on design parameters and non–site-specific data. Nevertheless, they do open the door for using site-specific information to load-rate bridges. For example, the manual discusses the use of weigh-in-motion data to calibrate site-specific live-load factors (NCHRP 2001).

Methodology for Bridge Rating under Ambient Traffic

When bridges are evaluated, the resulting rating should ideally possess the following characteristics:

1. The rating should be based on a measurable and clearly defined concept of safety and should clearly distinguish safe bridges from unsafe ones under given service conditions and inspection intervals.
2. A higher rating should signify a correspondingly greater margin of safety (and conversely for lower ratings) so that limited bridge management resources may be allocated optimally.
3. The rating method should use site-specific (or region-specific) traffic loading information and should account for uncertainties in strength and future loads.

Following the LRFR lead, and using peak live-load strain data from an instrumented bridge incorporating new sensor technology (described subsequently), a reliability-based rating methodology, hereinafter called *in-service load and resistance factor rating* (ISLRFR) has been developed. This method yields bridge ratings that satisfy the preceding three criteria. As in LRFD and LRFR, the scope is restricted to assessing structural *components*, as opposed to the *system*, and the focus is on flexural behavior, although the methodology can be easily extended to such other limit states as shear, if relevant. Distributions of the maximum live-load effect for various reference periods are projected from the in-service data by using extreme-value theory. The data acquisition procedure requires a minimum of equipment and no load truck, and it causes no traffic restriction. By measuring the actual structural response, the method accounts for both site-specific traffic and as-built bridge response. Since this method uses the actual load-effect data instead of vehicle weights, we can eliminate a substantial portion of the modeling uncertainty that is commonly associated with live-load characterization (e.g., factors related to dynamic impact and girder distribution). The resulting bridge ratings, therefore, are expected to be more accurate than present methods.

Further, because maximum loads over a specified time interval under ambient traffic are used in the present method, the resulting in-service rating factor would provide a rating that is at least as stringent as the so-called inventory rating for unposted bridges. For posted bridges, the rating factor using the proposed method will give a measure of adequacy of the imposed load restrictions. Hence, a bridge that rates above 1.0 with the present method will not require any (new) load restrictions for the duration for which the rating equation is valid, provided that the following are satisfied: (1) traffic observed during in-service measurement reflects the true traffic pattern; (2) vehicles do not become significantly heavier over the years; and (3) the target reliability for the limit state under consideration is acceptable. Application to permit vehicles will require additional procedures.

Finally, it may be relatively time-consuming and expensive to conduct load tests on every bridge in a jurisdiction's bridge inventory. If in-service response from a limited number of sites can be deemed representative of a larger suite of bridges, the rating factors can be optimized for the entire suite of bridges (in much the same principle as in LRFD and LRFR), and the bridge owner may determine the safety of bridges in inventory by using such optimized rating equations.

In-Service Strain Measurement System

The proposed ISLRFR methodology uses a recently developed in-service strain monitoring system (Shenton et al. 2000). The system, which is analogous to a weigh-in-motion system, is used to measure peak live-load bridge strains caused by site-specific traffic over extended periods of time. The prototype system consists of a digital data-acquisition system, a full-bridge strain transducer, battery pack, and an environmental enclosure. The single-channel system was assembled from specially modified instruments, off-the-shelf components, and custom fabricated parts. The primary component of the system is the data-acquisition system, which consists of a specially modified Snap Shock Plus (SSPM4), manufactured by Instrumented Sensor Technologies. The SSPM4 is small and weighs only 204 g (7 oz). It is powered by a single 9-volt battery and has an onboard microprocessor, 16-kilobyte EEPROM memory, 12-bit A-to-D converter, and a serial communication link. Strains are measured with an Intelliducer strain transducer, which is manufactured by Bridge Diagnostics. This sensor requires a regulated 5-volt excitation and is powered by a 9-volt battery pack. The entire system—including the SSPM4, 5-volt regulator, and 9-volt battery pack—fits in a 150 mm × 150 mm × 100 mm environmental enclosure.

As designed, the rapidly deployable stress-in-motion system is perfectly suited for use in routine bridge inspection and field evaluation. The system continuously digitizes an analog signal at 1,200 Hz and waits for a prespecified strain threshold to be exceeded. When this threshold is exceeded, the system evaluates the response and records the time at which the event took place, the peak strain during the event, and the area under the strain-time curve. The system can operate unattended for more than two weeks and can store up to 1,475 data records (events). In this research, only the peak strain during an event and its time stamp are used.

Limit State Equation for Existing Bridges

The resistance (R) of a structural component and the live-load effect (L) on it are both time-dependent parameters. It is common knowledge that bridges lose strength as they age. Furthermore, vehicle loads on a bridge are functions of time. Therefore, the time-dependent limit-state equation for a bridge component can be expressed as $R(\tau)-D-L(\tau)=0$ for $\tau\in[0,t]$; we ignore load combinations involving earthquake, wind, and so on, and concentrate only on traffic loading in this paper. The dead-load effect (D) is generally assumed not to vary with time. The cumulative failure probability, $P_f(t)$, over an interval $[0,t]$, or its complement, the time-dependent reliability function, $\mathrm{Rel}(t)$, is given by

$$P_f(t) = 1 - \mathrm{Rel}(t) = P[R(\tau) - D - L(\tau) \leq 0 \quad \text{for any} \quad \tau \in [0,t]] \tag{1}$$

Evaluating the first passage probability in Eq. (1) is involved but can be vastly simplified if $R(\tau)$ is replaced by a representative resistance, R_e, that is independent of time, τ. The limit-state equation simplifies to

$$R_e - D - L_{\max,t} = 0 \tag{2}$$

and the reliability function becomes $\mathrm{Rel}(t)=1-P[R_e-D-L_{\max,t} \leq 0]$, where $L_{\max,t}$=maximum live load-effect on the bridge during $[0,t]$. In the absence of deterioration, $R_e=R(0)$, the initial resistance. An extremely conservative option, on the other hand, would be to select $R_e=\min[R(\tau),0\leq\tau\leq t]$. In the remainder of this paper, we assume that R is independent of time and that R_e is the random resistance at the time that inspection is performed.

As previously mentioned, significant uncertainties exist in the live-load characterization on a bridge, and an important objective of this paper is to determine a realistic statistical description of $L_{\max,t}$. This result in turn will be used to derive accurate bridge assessment methodologies.

The maximum live-load effect may be caused by a single heavy truck on the bridge or the simultaneous presence of two (or more) trucks on the bridge:

$$L_{\max,t} = \max\{L_{\max,t}^{(1)}, L_{\max,t}^{(2)}, \ldots, L_{\max,t}^{(m)}\} \tag{3}$$

where the superscript (i) indicates the number of trucks present simultaneously on the bridge. For all practical purposes, m is equal to two, but in rare cases it may be higher. In the proposed method, in-service live-load strain data are used directly to derive statistical information on $L_{\max,t}$; hence, it is not necessary to first estimate $L_{\max,t}^{(i)}$, $i=1,2,\ldots$ individually, as would be required if the analysis started with truck weights and location on the bridge deck and went on to finding the structural response.

Since live-load effects are measured directly as strain, considering the preceding strength limit state in the strain domain as well is convenient; hence, the variables R_e, D, and $L_{\max,t}$ are expressed in terms of strain throughout this paper. As long as the structural response is elastic, this formulation is completely equivalent to the more common flexural moment-based approach; a correction is needed for the inelastic domain, as subsequently discussed.

Distribution of Maximum Peak Strains

We let L_i represent the peak strain caused by the ith "loading event"; a loading event is the passing of one vehicle or the simultaneous passing of more than one vehicle over the bridge, as previously discussed. Since the number of events in any bounded time interval is finite, we can describe the loading events over a time interval $[0,t]$ as a marked ordinary point process, $N(t)$, where the marks are the peak-strain responses $L_1, L_2, \ldots, L_{N_t}$, in which N_t denotes the number of events during the interval $[0,t]$. The point process is ordinary (i.e., not more than one event occurs in an infinitesimal interval of time), since the events by definition include the simultaneous presence of more than one vehicle on the bridge and since the recording instrument triggers only beyond a threshold and requires a finite amount of time to reset itself. The marks are random in nature, and the number of events, N_t, is a random variable, as well.

The peak strain *measured* by the in-service monitoring system, L_i', is generally different from (and lower than) L_i, the *true* maximum peak strain because the location of the sensor (i.e., where the maximum peak strain is expected to occur) may not coincide with that of the maximum response (e.g., because of unsymmetrical bending, uncertain support conditions, or longitudinal variations in the structure) for every vehicle event. We let this location-related error be expressed by the random variable $B_{\mathrm{loc}}=L_i/L_i'$ for all i. The maximum live-load strain, $L_{\max,t}$, over the duration t is then a random variable given by

$$L_{\max,t} = B_{\mathrm{loc}} \max\{L_1', L_2', \ldots, L_n'\} \tag{4}$$

We next make the following assumptions:

1. The events occur according to a continuous, nonnegative, and time-dependent rate $\Lambda(t)$. Because of the high trigger value of the data acquisition system, which filters in only the heavier trucks, an observed Λ is usually a fraction of the more traditional average daily truck traffic (ADTT) value. In the most general case, $\Lambda(t)$ is a stochastic process. However, our data did not have a long enough span to determine the temporal randomness in the occurrence rate; hence, we treat Λ as a random variable in this paper. Consequently, the point process $N(t)$ does not have independent increments (traffic pattern and volume do have memory, at least in the short term). However, we assume that, conditional on a fixed value of Λ (i.e., for uniformly flowing traffic with a constant rate), the process $N(t)$ is memoryless, that is, it has independent increments. In other words, $N(t)$ is a conditional or mixed Poisson process.
2. The peak strains (i.e., marks) are identically distributed and statistically independent (i.i.d.) of each other. That is, L_i' and L_j' are independent for $i \neq j$; and all L_i''s are distributed according to $F_{L'}$. The existence of a threshold strain level that triggers the recording device helps ensure the independence of the marks.

The extent to which the data support these assumptions will be investigated in the context of the illustrative example presented subsequently in this paper. A more sophisticated model will be required if there is strong evidence of dependence and nonstationarity in the loading process.

The true cumulative distribution function (c.d.f.) of the live-load strain is unknown; and for any given l, it is estimated as a proportion from the sample:

$$\hat{p}(l) = \frac{1}{n+1}\sum_{k=1}^{n} \mathrm{I}(L_k' \leq l) \tag{5}$$

which in fact is the (biased) average of n independent Bernoulli (i.e., binary) random indicator variables. For any given l, the true value, P, of $\hat{p}$ is unknown (see, e.g., Galambos 1993), and we describe it as a random variable with (prior) probability density function f_P'. On the basis of n observations, $\underline{I}$, of $\underline{\mathrm{I}}$, we can perform a Bayesian updating of the probability law of P and obtain its posterior (updated) density function $f_P''(x)=f_{P|\underline{\mathrm{I}}=\underline{I}}(x)$, given the random sample $\underline{\mathrm{I}}=\underline{I}$, as $f_{P|\underline{\mathrm{I}}=\underline{I}}(p)=f_P''(p)=(1/C)\mathcal{L}(p;\underline{I})f_P'(p)$, where C=normalizing constant and $\mathcal{L}$=likelihood function of observing data, $\mathcal{L}(p;\underline{I})=p^{(n+1)\hat{p}}(1-p)^{n-(n+1)\hat{p}}$

In the absence of any prior information on P, assuming a uniform (0,1) distribution for P is most logical. Hence, the posterior density of P simply becomes the beta density function between the limits 0 and 1:

$$f_P''(x) = \begin{cases} \frac{1}{\mathrm{B}(q,r)} x^{q-1}(1-x)^{r-1}, & 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases} \quad (6)$$

with $q = n\hat{p} + 1$, $r = n(1-\hat{p}) + 1$; where B=beta function; n=number of observations; and estimate $\hat{p}$ is given by Eq. (5). The mean of this distribution is $q/(q+r)$, and its variance is $qr/\{(q+r)^2(q+r+1)\}$.

If the live-load effect data span n_d time units (e.g., days), n_d samples of the random occurrence rate, Λ, may be obtained:

$$\Lambda_i = N^{(i)}/\text{time unit} \qquad i = 1,2,\ldots,n_d \quad (7)$$

where $N^{(i)}$=number of occurrences during ith time unit; Λ_i in turn can be described as the sum

$$\Lambda_i = \frac{1}{\text{time unit}} \sum_{j=1}^{m} N_j^{(i)}, \qquad i = 1,2,\ldots,n_d \quad (8)$$

if each time unit is composed of m subunits (e.g., the 24 hours of a day) and $N_j^{(i)}$ is the total number of occurrences in the jth hour of the ith day. As long as the $N_j^{(i)}$'s $(j=1,\ldots,m)$ are of the same order and not too strongly dependent on one another and as long as m is large enough, their sum, Λ_i $(i=1,\ldots,n_d)$, approaches the normal distribution.

Conditioned on fixed values of P, Λ, and B_{loc}, the c.d.f. of the maximum live-load strain during an interval of length t is

$$F_{L\,\max,t}(x|P=p,\Lambda=\lambda,B_{\text{loc}}=b) = \exp\{-\lambda t[1-p(x/b)]\} \quad (9)$$

By removing the conditioning on the three random variables, we obtain

$$F_{L\,\max,t}(x) = \int_b \int_p \int_\lambda F_{L\,\max,t}(x|\lambda,p,b) f_\Lambda(\lambda) f_P''(p) f_{B_{\text{loc}}} d\lambda\, dp\, db \quad (10)$$

With increasing t and under a set of very general conditions, $F_{L\,\max,t}$ approaches one of the three classical extreme-value distributions for largest values. The generalized form of the extreme-value distribution for maxima is

$$H_c(z) = \exp[-(1+cz)^{-1/c}], \qquad 1+cz > 0 \quad (11)$$

The parameter c determines the nature of the distribution: It is of the Gumbel type if $c=0$, the Frechet type if $c>0$, or the Weibull type if $c<0$ [see standard texts such as Galambos (1987) or Castillo (1988) for further details]. The determination of the type and estimation of the parameters of the extreme-value distribution from observed data must be performed with caution (Corotis and Dougherty 2004).

Finally, for steel girder and slab bridges, we emphasize that it may not always be possible to ascertain from the observed strain data whether they are a result of a bridge acting compositely or noncompositely. Compositely designed bridges are assumed to act compositely unless load tests show otherwise. Many noncompositely designed bridges, however, may act compositely under service loads, but the composite action will likely be lost as the load approaches the failure load (Bakht and Jaeger 1990). Even though some guidelines are available regarding the threshold shear stress at which this transition occurs, for expample, in NCHRP (1998), they do not appear to have been based on substantial testing programs; and in any case, no consensus seems to exist in the professional community on the threshold shear stress. Furthermore, it is reasonable to assume that the structural response characteristics of a bridge that has been in service for several years will likely continue to be in that state until the next scheduled inspection. Thus, although in a more sophisticated analysis, the projected distribution of $L_{\max,t}$ for noncomposite bridges could be transformed appropriately to account for the loss of composite behavior at high loads, we will not do so in this paper.

Development of Rating Equation for Suite of Bridges

In terms of LRFR methodology (NCHRP 1998), the rating factor (RF) for an existing bridge is

$$\text{RF} = \frac{\phi R_n - \gamma_D D_n}{\gamma_L L_n} \quad (12)$$

where R_n=nominal resistance; D_n and L_n=nominal (or characteristic) values of dead and live load effects, respectively; ϕ=resistance factor, and λ_D and λ_L=load factors for rating. Elastic buckling is generally not encountered in bridge flexural members; hence, for the first-yield limit state, the nominal resistance, R_n, is equal to the nominal yield strength, Y_n. For the plastic-collapse limit state, the nominal resistance is $R_n = f_p Y_n$, where f_p is an amplification factor accounting for postyield reserve strength.

If a bridge is not instrumented, its nominal live load L_n needs to be estimated indirectly. Because L_n is a load-effect, it already includes dynamic impact effects.

Target Reliability Index and Rating Interval

The reliability of the bridge (or a structural component of the bridge), $\text{Rel}(t) = 1 - P_f(t)$, is a nonincreasing function of time; and for a given t, it is often expressed in terms of the reliability index, β, which is related to the reliability as $\beta = \Phi^{-1}(\text{Rel})$, where Φ=normal distribution function. The reliability index, β, is a popular measure of reliability; and it usually ranges from 2 to 5 for most structural components. To be meaningful, a value of β should be accompanied by the relevant time period, failure mode, load combination, and types of uncertainty considered in the analysis.

The target or minimum acceptable reliability, β_T, for a given failure mode is intended to ensure that the bridge (or the component, as relevant) has an adequate level of safety up to the end of a reference period, which for new bridges is typically 75 years (design life), and for existing bridges is typically 2 to 5 years. The fact that the bridge is already in existence or that in-service data is used should not be the sole reason for reducing the target reliability when choosing a target reliability for bridge rating. What in-service bridge monitoring achieves is a better understanding of (and possibly a decrease in) the variability of live load effects. Finding that the probability distribution of the measured live load is significantly to the left of that used in new design is also not unusual. Such favorable changes in the distribution of live load can lead to a higher reliability or a more benign rating for an existing bridge. Furthermore, the bridge owner has flexibility in setting the time interval for which the rating—and consequently the target reliability—will remain valid.

The target reliability, β_T, used implicitly in LRFD of new bridge components in flexure is 3.5 (Nowak 1995; NCHRP 1999a,b). This value is based on calibration with a representative sample of existing bridges. When target reliability is *calibrated* to existing service-proven design standards, the calibration itself becomes dependent the method of reliability analysis, the assumptions regarding random variables (Ditlevsen 1993), the mechanistic model, and so on. In other words, the calibrated target reliability is not absolute—it changes if any of the model assumptions are altered (NCHRP 2001), for example, if live-load distribution is changed from lognormal to Gumbel (Li 2004).

For evaluating existing bridges, a value of $\beta_T=2.5$ has been suggested (NCHRP 1998; Ghosn 2000) mainly because of economic considerations. It has been argued that the marginal cost of increasing bridge reliability *before* construction (i.e., at the design stage) is small compared with that for an *existing* bridge (through repair or rehabilitation). And since the total expected cost over the *remaining* life of the bridge has to be minimized in this situation, the revised optimal target reliability would clearly be lower than that of a new design in this approach. Although cost-based optimization of target reliability of structural systems is a rational method (Frangopol et al. 1997; Wen 2001a), it was not adopted in this paper because of two considerations. First, as stated in NCHRP (2001), some of the key cost data involved in this approach are difficult to obtain and were not available for this project; second, the cost-based optimal reliability formulation always has a lower bound on the reliability that often governs it and is set by sociopolitical considerations—such as the value of human life, the perceived risks of engineering activities, and the acceptable fatal accident rate—that do not have a well-defined cost metric. For example, regardless of cost considerations, ISO (1998) sets a clear upper limit of annual probability of structural failure as $10^{-6}/p_D$ where p_D=probability that a person present in the structure at the time of collapse will be killed.

We can obtain the cost-independent lower acceptable limit for target reliability for bridge rating if we adopt a risk-based approach and make the reasonable assumption that bridges in the nation's inventory have overall performed satisfactorily and that the public almost universally does not consider the use of bridges to be a particularly risky activity. In other words, the risk of bridge failure is certainly within society's tolerable limit, and people would like to keep it that way. The *risk* of an undesirable event, F, is defined as the product $p_F \times C_F$ where p_F=probability that the event F will occur and C_F=consequence if it does. Failure consequence includes fatalities, economic loss, punitive damages, loss of life quality, and so on (Ditlevsen 2003). In a risk-based approach to design, the risk of failure is kept within a tolerable limit, R_0, which depends on society's attitude toward cost versus benefit from an activity and on additional factors, including the nature of warning before failure and the degree of structural redundancy, the voluntary versus involuntary nature of risk and so on (Whitman 1984; Hauptmanns and Werner 1991; Moan 1997; Stewart and Melchers 1997; van Breugel 1998; Bhattacharya et al. 2001). Since we have assumed that the risk of bridge failure is within the society's tolerable limit and since failure consequences are essentially the same whether a bridge is old or new, we must conclude that the target reliability for computing rating factors has to equal the computed design-basis reliability of bridges. We therefore retain the target value $\beta_T=3.5$ against ultimate failure of bridge components; doing so makes the current approach significantly more stringent than LRFR, since the acceptable failure probability is approximately an order of magnitude *lower* than that in the LRFR manual.

Finally, rating a bridge with reference to only one failure mode and hence only one set of target reliability and reference periods may not be adequate for all purposes (Aktas et al. 2001). Setting safety targets at two or more levels is consistent with the performance-based trend that has evolved in structural engineering over the last three decades (Ellingwood 2000). For example, Wen (2001b) proposed a bilevel reliability requirement for structures against natural hazards corresponding to "incipient damage" and "incipient collapse" limit states. Collins et al. (1996) proposed dual-level reliability-based seismic design criteria: a serviceability-type limit state to ensure (nearly) elastic response during small to moderate earthquakes and an ultimate limit state to control the nonlinear inelastic behavior caused by severe earthquakes. Nowak et al. (1997) recommended a (lifetime) target component reliability index of 3.5 in the ultimate limit states for bridge structures; for serviceability limit states, they recommended a target component (i.e., girder) reliability index of 1.0 in tension and 3.0 in compression. Ghosn and Moses (1998) listed four limit states for a highway bridge: first member failure (m), system ultimate (u), system functionality (f), and damaged condition (d); and they propose the following relations among the four target reliability indices: $\beta_u-\beta_m \geqslant 0.85$, $\beta_f-\beta_m \geqslant 0.25$, and $\beta_d-\beta_m \geqslant -2.7$.

In view of the preceding discussion, we propose the following two-level rating of bridge components using in-service live load data. The assumption is that the consequence of exceeding the yield limit state is roughly an order of magnitude less than that of collapse.

(1) $\beta_T=2.5$ for the *first-yield* limit state in flexure under the action of maximum live-load effect over a duration not exceeding two years
(2) $\beta_T=3.5$ for the *plastic-collapse* (i.e., *ultimate*) limit state in flexure under the action of maximum live-load effect over the reference period.

It is up to the bridge owner to define the adequacy of the bridge in terms of successful rating in either one or both of the preceding criteria.

Optimum Load and Resistance Rating Factors

As in LRFD, where a design equation is optimized for a suite of bridges, the rating equation should preferably be valid for at least a sizable fraction of a given bridge inventory. The limit-state equation, Eq. (2), can be normalized by the rating equation, Eq. (2), to yield

$$\frac{X_1}{\phi} - \frac{X_2 + B_{\text{site}}\left(\dfrac{L_n}{D_n}\right)X_{3,t}}{\gamma_D + \left(\dfrac{L_n}{D_n}\right)(\text{RF})\gamma_L} = 0 \tag{13}$$

where the dimensionless random variables are normalized strength, X_1, discussed next; normalized dead load $X_2=D/D_n$; normalized live load $X_{3,t}=L_{\max,t}/L_{n,\text{true}}$; and a new modeling uncertainty term, B_{site}, discussed subsequently.

For the first-yield limit state, the resistance in Eq. (2) is simply $R_e=Y$, where Y is the random yield strength; hence, X_1 is the normalized yield strength, Y/Y_n. For the plastic-collapse limit state, however, $R_e=F_pY$, where F_p is the random plastic-strength factor, making the normalized resistance equal to $X_1=(F_p/f_p)(Y/Y_n)$. The statistics of X_1 are not necessarily the same for the two limit states.

As previously stated, only a few representative bridges in an inventory are expected to be instrumented to obtain live-load

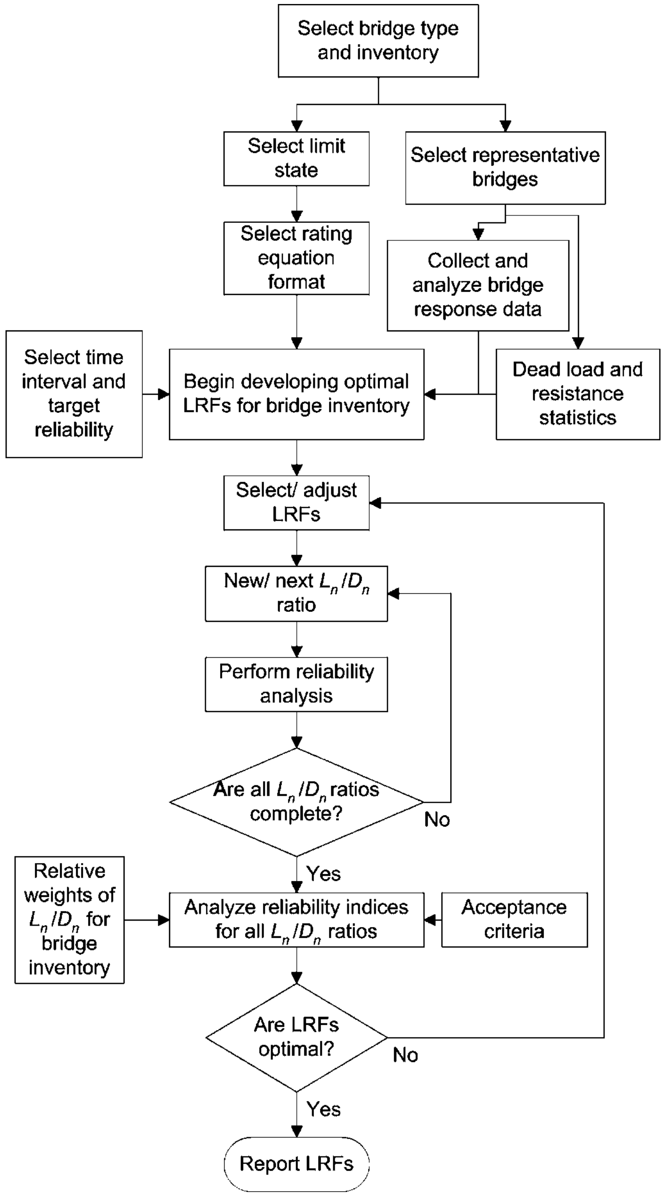

Fig. 1. Flowchart for determining optimal load and resistance factors

effect statistics; therefore, in-service data may not be available for all bridges. For uninstrumented bridges, the nominal live load has to be estimated indirectly [e.g., by using software version 5.08 of *Bridge Rating and Analysis of Structural Systems (BRASS)* Wyoming Department of Transportation, Cheyenne, Wyo., 1992] instead of basing it on some site-specific statistic $L_{n,\text{true}}$ such as the m-year mean maximum or m-year return-period load effect. The associated random error is denoted by $B_{\text{site}}=L_{n,\text{true}}/L_n$ in Eq. (13). Finally, we note that although the *nominal* dead load D_n is always estimated indirectly, we assume that the error is negligible.

The desired objective is that the rating equation be applicable to a suite of bridges of a given type, regardless of span length, support type, or number of spans; and that it holds as long as the statistics of the normalized random variables X_1, X_2, X_3, B_{loc}, and B_{site} remain the same for the bridges being considered. Then the only term in Eq. (13) that may vary from bridge to bridge is the characteristic load ratio, L_n/D_n. Therefore, we need to derive a set of LRFs that are in some sense optimized and that relate to all possible values of L_n/D_n.

Fig. 1 shows the scheme for obtaining optimized LRFs for rating. The acceptance criteria used in the optimization procedure are

$$\min \sum_{i=1}^{k} (\beta_i - \beta_T)^2 w_i,$$

where

$$\beta_i = f(\text{RF}, \phi, \gamma_D, \gamma_L; (L_n/D_n)_i), \qquad \sum_{i=1}^{k} w_i = 1 \tag{14}$$

subject to RF=1; $\phi \leqslant 1$, $\gamma_D \geqslant 1$ $\gamma_L \geqslant 1$, where k=total number of nominal load ratios (L_n/D_n) in suite of bridges; and w_i= relative frequency (weight) of the ith nominal load ratio. The constraints on the LRFs are intended to conform with accepted engineering practices. The constraint on RF ensures that bridges that rate 1.0 just satisfy the target reliability (on the average), so that rating factors above 1.0 indicate reliabilities above β_T, and vice versa. If rating only the instrumented bridge is desired, a unique set of LRFs may be obtained (bypassing the optimization) that exactly satisfy the target reliability. In this case, a rating factor of 1 implies a reliability exactly equal to the target value.

Numerical Example of Bridge Rating Using In-Service Data

We next demonstrate the proposed rating procedure is with a brief example involving highway bridges. For this purpose, all highway bridges in the state of Delaware were assumed to constitute the bridge inventory for which the optimal rating equation would be developed. The bridge selected for instrumentation and data acquisition was Bridge 1-791 which is a three-span continuous, slab-on-steel girder structure carrying two lanes of Interstate 95 over Darley Road in Delaware. In-service strain data were recorded at midspan of the critical girder of the approach span (beneath the right travel lane) during an approximately 11-day period in August 1998 [Fig. 2(a)]. A trigger level was set at 85 $\mu\varepsilon$ so that only the larger truck events would be recorded. A histogram of the data is shown in Fig. 2(b), which represents 533 loading events; the distribution of the corresponding 532 interarrival times is shown in Fig. 2(c). The data were analyzed to evaluate the probability distribution of maximum load effects for various intervals: 1 year (annual inspection), 2 year (normal inspection cycle) and 10 year (potential repair cycle), as described in the following.

We emphasize that developing optimized LRFs for a suite of bridges requires careful selection of representative bridges for deriving live-load statistics—it is for the purpose of illustration alone that we are relying on only one bridge. There is also no guarantee that a single set of rating equations will actually be found valid for the entire bridge inventory of the state of Delaware.

The maximum live-load model developed in the section on the distribution of maximum peak strains was based on some key assumptions, namely, the loading events occur according to a conditional Poisson process and the peak strains associated

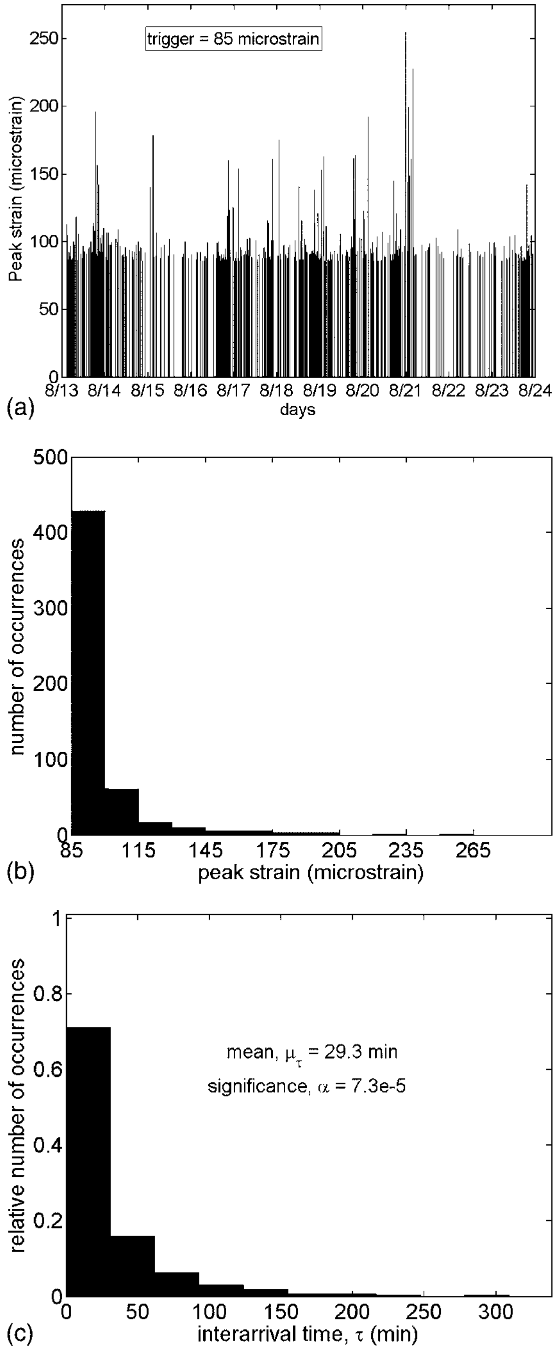

Fig. 2. Bridge 1-791: (a) timeline of loading events spanning two weeks in August 1998; (b) peak strain distribution; and (c) interarrival time distribution

with each event constitute an i.i.d. sample. It is desirable, at the outset, to investigate the extent to which the data support these assumptions.

Tests of Independence of Loading Process

Simple nonparametric tests for serial dependence, namely, the turning points test, the difference-sign test, and the rank correlation test are performed on the observed sample of peak strains $L_1, L_2, \ldots, L_{N_t}$. These tests evaluate the null hypothesis of how truly random a given sample is and report the level of significance to which the hypothesis can be accepted. Each of these

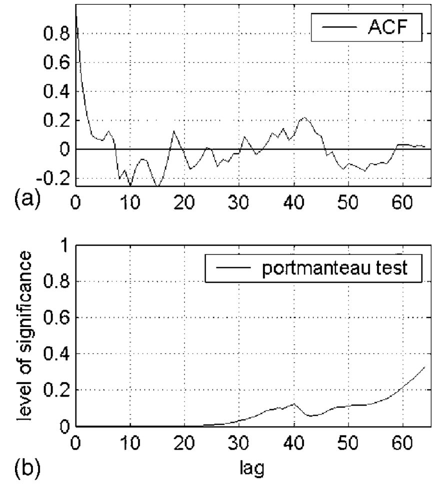

Fig. 3. (a) Autocorrelation function of four hour count series at different lags and (b) portmanteau test for partial sum up to various lags, showing the level of significance at which the identically distributed and statistically independent hypothesis can be accepted

tests is based on the property that the respective test statistic approaches the normal distribution with parameters that depend only on n, the size of the sample. In each of the three tests, if consecutive values in the sample are identical, they should be merged and considered as only one sample point. These tests are described in such classic texts as Kendall and Stuart (1968) and Brockwell and Davis (1991). Table 1 clearly shows assumption (2) in the section on distribution of maximum peak strains that the marks are i.i.d. and cannot be rejected at very high significance levels using the rank correlation test (75%) and difference-sign test (65%) and at a moderate level of significance (~15%) using the turning point test.

Since the peak-strain time series $(L_1, L_2, \ldots, L_{N_t})$ was recorded at random instances of time, the autocorrelation function of the peak-strain process cannot be determined from the data. However, the increments, ΔN_i, in successive disjoint intervals, $[(i-1)\Delta t,\ i\Delta t]$, of the associated counting process, $N(t)$, can be studied as a time series sampled at a constant frequency:

$$\Delta N_i = N[(i\Delta t)] - N[(i-1)\Delta t] \tag{15}$$

According to the assumption (1) in the section on distribution of maximum peak strains, given one sample function of $N(t)$, i.e., for a fixed value of the random rate Λ the increments, ΔN_i are independent and identically distributed. This assumption is tested for four different values of interval length Δt: 3 hours, 4 hours, 6 hours, and 24 hours; the results of the three nonparametric tests listed previously are shown in Table 1. The results consistently suggest a lack of dependence of the increments for each of the time intervals.

As a further confirmation, the autocorrelation function (ACF) of the four-hour count series is shown in Fig. 3(a). The portmanteau test measures the asymptotic normality of the sample ACF of a given time series. Let $\rho(k)$ denote the sample autocorrelation function at lag k of a zero-mean i.i.d. sequence $y_1, y_2, \ldots, y_n$ with finite variance. Then for large n, the autocorrelation sequence $\{\rho(k)\}$ itself is approximately i.i.d. with normal $(0, 1/n)$ marginal distribution (Brockwell and Davis 1991). Then the sum, Q, given by

Table 1. Nonparametric Tests for Independence in Observed Peak Strain Data and Associated Counting Process

	Turning point test $\mu=2(n-2)/3$, $\sigma^2=(16n-29)/90$				Difference sign test $\mu=(n-1)/2$, $\sigma^2=(n+1)/12$				Rank correlation test $\mu=n(n-1)/4$ $\sigma^2=n(n-1)(2n+5)/8$			
Sample	X	μ	σ	α (%)	X	μ	σ	α (%)	X	μ	σ	α (%)
Peak strain $n=533$ (511)	353	339.3	9.5	15.1	252	255	6.5	64.6	63,307	65,152	5,784	75.0
3-hour count $n=87$ (79)	46	51.3	3.7	15.0	36	39.0	2.6	24.5	1,234	1,540.5	354.3	38.7
4-hour count $n=65$ (63)	42	40.7	3.3	68.6	29	31	2.3	38.6	831	976.5	252.9	56.5
6-hour count $n=44$ (40)	22	25.3	2.6	20.1	17	19.5	1.8	17.6	308	390	128.7	52.4
24-hour count $n=11$ (11)	5	6	1.3	43.4	4	5	1	31.7	20	27.5	19.3	69.7

Note: $n=$ original sample size; $X=$observed statistic; $\mu=$true mean; $\sigma=$true standard deviation; and $\alpha=$level of significance at which the null hypothesis (sample truly random) is accepted. The number in parentheses is the reduced value after consecutive identical observations are merged into one data point.

$$Q(h) = n\sum_{k=1}^{h} \rho^2(k) \tag{16}$$

approaches the chi-square distribution with h degrees of freedom. The i.i.d. assumption in the portmanteau test can be rejected at significance α if $Q>\chi^2_{1-\alpha}(h)$, where $\chi^2_{1-\alpha}(h)$ is the chi-square deviate at exceedance probability α with h degrees of freedom. Fig. 3(b) shows that the test statistic indeed approaches the chi-square distribution as the number of terms in the partial sum increases, thus supporting the i.i.d. hypothesis for the four hour counts.

The interarrival times, τ_i, for the 533 events occurring over approximately 11 days are analyzed next. The distribution is plotted in Fig. 2(c), which suggests an exponentially decaying density function of the random variable, τ. However, a chi-square test of exponential goodness of fit (10 equiprobable intervals) yielded a rather low level of significance of 0.007% with 8 degrees of freedom. This result suggests a degree of dependence in the point process $N(t)$ that presumably would decrease with increasing threshold, u; however, we do not probe this aspect further in this paper.

We can now conclude that the data are largely consistent with the main assumptions made in developing the live-load probabilistic model. The following section is devoted to establishing the parameters of the proposed live-load model, given in Eq. (10) and incorporating the uncertainties listed in the section on the distribution of maximum peak strains.

Statistics of Maximum Live-Load Effect

Point estimates of the c.d.f., $\hat{p}$, of the peak strains caused by the loading events [Fig. 2(b)] are listed in Table 2. On the basis of these observations, a Bayesian updating of the c.d.f. is performed [Eq. (6)]; the mean and the coefficient of variation (c.o.v.) of the updated distribution are listed in Table 2 at various values of l. Because the data spanned 11 days, 11 point estimates of the ran-

Table 2. Estimates of Daily Maximum Peak Strain

			Cumulative distribution function of load effect			
				Updated beta distribution parameters for P [Eq. (6)]		
Interval	Right endpoint (l)	Counts (k)	Point estimate $\hat{p}(l)=\Sigma k/(n+1)$ [Eq. (5)]	Mean	Coefficient of variance	Predicted maximum load effect $F_{L\max,1\ day}(l)$ [Eq. (10)]
<85	85	0	0	—	—	—
85–100	100	428	0.8015	0.8019	2.15%	0.0252
100–115	115	61	0.9157	0.9159	1.31%	0.0776
115–130	130	17	0.9476	0.9477	1.02%	0.1550
130–145	145	9	0.9644	0.9645	0.83%	0.2461
145–160	160	5	0.9738	0.9738	0.71%	0.3354
160–175	175	5	0.9831	0.9832	0.56%	0.4782
175–190	190	3	0.9888	0.9888	0.46%	0.6021
190–205	205	3	0.9944	0.9944	0.32%	0.7674
205–255	255	2	0.9981	0.9981	0.19%	0.9146

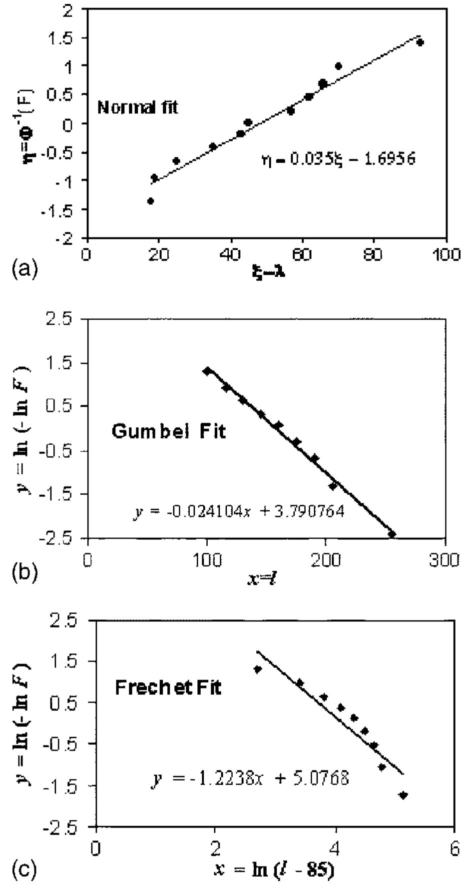

Fig. 4. (a) Normal fit for random occurrence rate; (b) Gumbel fit of daily maximum live-load strain; and (c) Frechet fit for daily maximum live-load strain

dom occurrence rate, Λ, were available [Eq. (7)]. As previously discussed, Λ approaches the normal distribution, which is clearly demonstrated in the normal probability fit shown in Fig. 4(a). The probability paper fit yielded the following parameters for Λ: mean=48.5 events per day and c.o.v.=59.0%. Because of a lack of suitable data, location-dependent randomness was ignored in Eqs. (9) and (10), that is, B_{loc} was taken to be deterministic and equal to 1. The unconditional c.d.f. of $L_{\max,1d}$ at each value of l [Eq. (10)] is listed in the last column of Table 2; a total of 10,000 Monte Carlo simulations were used in estimating Eq. (10) in each case.

A previous assertion indicated that as the time interval increases, the probability distribution of the maximum load approaches one of the classical extreme-value distributions [Eq. (11)]. Of the three classical extreme-value distributions for largest values, the Gumbel (i.e., Type I maximum) and the Frechet (i.e., Type II maximum) distributions were tried for $L_{\max,1d}$ (Fig. 4). The third, the Weibull distribution for maxima, was not tried here because it is limited on the right, although this property of the Weibull distribution can be attractive in situations where geometric, posting, or other constraints put a well-defined upper limit on the vehicular load that can be placed on the bridge. The Gumbel fit was clearly better in the present case and was adopted for $L_{\max,1d}$ in this paper:

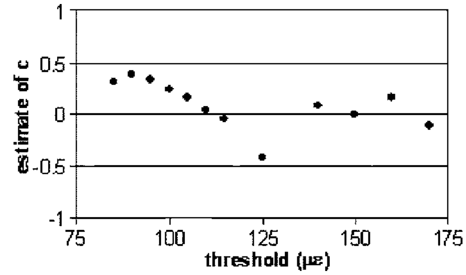

Fig. 5. Peaks-over-threshold analysis of peak-strain data for various thresholds

$$F_{L_{\max,1d}}(x) = \exp[-\exp(-\alpha_{1d}(x-u_{1d}))] \qquad (17)$$

where α and u are the shape and scale parameters, respectively. The mean and c.o.v. of this distribution were $\mu_{1d}=181.2$ microstrain and $V_{1d}=32.8\%$, respectively.

As a further verification of the Gumbel model for the maxima, we employ a peaks-over-threshold (POT) analysis of the loading data. POT is an elegant tool for predicting the asymptotic distribution of the largest values from an i.i.d. sample $\{X\}$ distributed according to F_X [other civil engineering applications of the method can be found in Simiu and Heckert (1996) and Naess (1998)]. If F_X belongs to the domain of attraction of one of the classical extreme-value distributions for largest values, then the exceedances, $Y=X-u$, over a sufficiently high threshold, u, of the sequence $\{X\}$ approach the generalized Pareto distribution, $G(y)=P[Y\leq y|Y>0]=1-[1+(cy/a)]^{-1/c}$, $a>0$, $1+(cy/a)>0$, where the parameter c has the same significance as in Eq. (11). We use the de Haan (1994) estimator for c:

$$\hat{c} = M_n^{(1)} + 1 - \frac{1}{2\{1-[M_n^{(1)}]^2/[M_n^{(2)}]\}} \qquad (18)$$

where

$$M_n^{(r)} = (1/k)\sum_{j=0}^{k-1}[\ln(X_{n-i,n}) - \ln(X_{n-k,n})]^r, \quad r=1,2$$

k=number of data above u; and the highest, second-highest, kth highest, and $(k+1)$th-highest variates are denoted by $X_{n,n}$, $X_{n-1,n}$, $X_{n-(k-1),n}$ and $X_{n-k,n}\equiv u$, respectively. A POT analysis of the peak-strain data [Fig. 2(a)] shows the exponent c in Eq. (11) to converge toward zero with a minimum value of −0.43 and a maximum of 0.37, as the threshold increased from 85 $\mu\varepsilon$ to 170 $\mu\varepsilon$ (Fig. 5), thus pointing once again to the Gumbel property.

Since we assumed the loads to be i.i.d., it is consistent to assert that the daily maxima are independent and identically distributed, as well. Consequently, the maximum strain for any other interval $t=r$ days (in integral multiples of days) is also Gumbel-distributed. The shape parameter, α, remains unchanged for this new distribution; and the location parameter, u, moves to the right:

$$u_t = u_{1d} + \frac{1}{\alpha_{1d}}\ln(r) \qquad (19)$$

The Gumbel distribution is quite often adopted for extreme loads in civil engineering applications, for example, for bridge live loads (Imai and Frangopol 2001); for structural live loads (Ellingwood 1996); for maximum value of a variable action on a

Table 3. Maximum Live-Load Statistics for Different Time Intervals

Time t	Maximum live load effect, $L_{\max,t}$				Normalized $X_{3,t}=L_{\max,t}/L_{n,\text{true}}$	
	Location parameter u	Shape parameter α	Mean	Coefficient of variance (%)	Mean	Coefficient of variance (%)
1 year	402.0	0.0241	426.0	12.5%	1.02	12.5%
2 years	430.8	0.0241	454.7	11.7%	1.09	11.7%
10 years	497.6	0.0241	521.5	10.2%	1.25	10.2%

Note: $L_{n,\text{true}}$=two-year return period value.

structure within a chosen reference time (ISO 1998; JCSS 2001); and for annual maximum wind speed, wave height, and current in the case of marine structures (DNV 1992).

The mean and c.o.v. of the maximum live-load effect for various time intervals, t, are listed in Table 3 (selection of the nominal value is discussed in the next subsection). With increasing t, the maximum live-load effect distribution shifts to the right and becomes narrower. The latter property has sometimes been cited against the use of extreme-value distributions in modeling maximum loads, although incorrectly. If samples are taken repeatedly from a stationary truck population and if a record of only the heaviest truck up to the current instant is retained, there is *less certainty* at the beginning that a very heavy truck has entered the sample when the sample is, say, a few weeks old. But as the sample size becomes larger (say, several years worth), it becomes increasingly *more certain* that some very heavy truck has indeed appeared in the sample, thus erasing all record of less heavy trucks. The statistics in Table 3 may then be compared with those used in the LRFR manual. The maximum live load in LRFR is assumed to be lognormal-distributed with a bias of 1.0 and a c.o.v. of 18% (regardless of reference period). The nominal value of the maximum live load in LRFR is that caused by the AASHTO 3S2 vehicle. The proposed methodology thus predicts the maximum live load in this example with significantly less uncertainty than that assumed in calibrating the LRFR manual (c.o.v. of around 10∼13% instead of 18%). Part of this reduction can be ascribed to (1) circumvention of modeling uncertainty in structural analysis, e.g., those related to girder distribution and impact factors, since load effect is measured and used directly in the proposed method; and (2) use of site-specific (or region-specific) in-service data, as was also suggested by NCHRP (2001). Nevertheless, we emphasize again that this illustrative example used data from only one bridge gathered over only 11 days; and it is possible that when data from several sites and seasons are gathered for use in a practical application, the composite distribution for a given t may be wider than that shown in Table 3.

Determination of Nominal Live Load

The "true" nominal live load used in rating equations (for all reference periods and limit states) can be set arbitrarily as long as it is internally consistent with the statistical analysis of maximum live-load effect. Therefore, as suggested in the section on optimum load and resistance rating factors, we adopt the predicted two-year return period load effect, $L^*_{2\text{ year}}$, as the true nominal live-load effect, $L_{n,\text{true}}$. By definition, $L^*_{2\text{ year}}$ is exceeded on average once every two years (the usual inspection interval) and is equal to the median annual maximum. The same statistics for the normalized live-load effect, $X_{3,t}$, may be used for bridges in the inventory that have a similar traffic pattern. In the current example, this quantity is $L^*_{2\text{ year}}=417.2\ \mu\varepsilon$ for Bridge 1-791. The statistics of $X_{3,t}$ for various time periods are listed in Table 3.

For an unmonitored bridge, as mentioned in a previous section, the true nominal live load can only be *estimated*; this estimate is denoted by L_n, and the site-dependent live-load modeling uncertainty is $B_{\text{site}}=L_{n,\text{true}}/L_n$ [Eq. (13)]. We choose to define L_n for unmonitored bridges as the effect of the HL93 configuration currently used in the design of new bridges. We note that the numerical value of L_n (410 $\mu\varepsilon$ for Bridge 1-791 using *BRASS* with a 10% impact factor, 1.34 distribution factor, and assuming noncomposite behavior as designed) is of the same order as the $L^*_{2\text{ year}}$. Because of a lack of further data, we neglect any uncertainty and bias in B_{site} and simply consider it deterministic and equal to 1 in this example.

It is important to establish the robustness of the proposed method for predicting maximum live-load statistics to changes in the in-service monitoring system. If, for example, the threshold strain (i.e., trigger) were set at 100 $\mu\varepsilon$ only 105 events would have been recorded over the 11 days in August 1998 instead of the 533 recorded, and the random arrival rate would instead have a mean of 9.3/day and a c.o.v. of 123%. The analysis, nevertheless, would yield a new nominal live-load effect of 427.8 $\mu\varepsilon$ and would yield the same mean and c.o.v. of $X_{3,1\text{ year}}$ as 1.02 and 12.5%, respectively. Table 4 lists the effect of raising the thresh-

Table 4. Effect of Data Acquisition Trigger on Live-Load Statistics

Trigger ($\mu\varepsilon$)	Total number of occurrences	Rate of occurrence (Λ)			Normalized annual maximum live load ($X_{3,1\text{ year}}$)	
		Mean (per day)	Coefficient of variance (%)	Mean	Nominal live load ($\mu\varepsilon$)	Coefficient of variance (%)
85	533	48.5	58.9	417.2	1.02	12.5%
100	105	9.3	123	427.8	1.02	12.5%
115	44	3.0	200	433.5	1.02	12.5%
130	27	2.2	127	420.3	1.02	12.3%

Table 5. Statistics of Strength Parameters

Limit state	R_e Eq. (2)	R_n Eq. (12)	Yield strain Y	Plastic factor F_p	Normalized resistance X_1 Eq. (13)	Remarks
First yield	Y	Y_n	LN (1.05 Y_n, 11.7%)	NA	$X_1=Y/Y_n$, $X_1\sim LN(1.05,11.7\%)$	$Y=S/E$, where $S\sim LN(1.05S_n,10\%)$, $E\sim LN(1.00E_n,6\%)$
Plastic collapse	F_pY	f_pY_n	Same as above	LN (1.03 f_p, 7.1%)	$X_1=(Y/Y_n)(F_p/f_p)$ $X_1\sim LN(1.09,13.7\%)$	Assumed $F_p/f_p=PF$, $P\sim LN(1.03,5\%)$, $F\sim LN(1.00,5\%)$,

Note: $LN(\mu,V)$ implies lognormal random variable with mean$=\mu$ and c.o.v.$=V$; subscript n implies nominal value; NA=not applicable; S=yield stress; E=elastic modulus; P=modeling (professional) error; and F=fabrication error. Statistics of S, E, P, and F are from Ellingwood et al. (1980). No aging effect is considered.

old from 85 $\mu\varepsilon$ to 130 $\mu\varepsilon$. Reassuringly, the effect of this filtering was found to be minimal on the distribution of the predicted maximum load effect even after 95% of the events had been filtered out in the end. For the remainder of this example, however, we continue with the original trigger of 85 $\mu\varepsilon$ and the resultant predictions (Table 3).

Optimal Rating Equation

According to the Delaware Department of Transportation (DelDoT) records, the state has 333 single-span bridges and 317 multispan bridges. To avoid including culverts in the study, these numbers exclude aqueducts and bridges that are shorter than 5 m. Most of the bridges fall in the 6 to 18 m (20 to 60 ft) range (Hastings 2001). The ratio L_n/D_n, that is, the ratio of the estimated nominal live-load effects to nominal dead-load effects, calculated with *BRASS* (with L_n corresponding to HL93), varied from 1.0 to 4.0 for the entire highway bridge inventory for Delaware. The relative frequency, w_i, of the L_n/D_n ratios 1.0, 1.5, 2.0, 2.5, 3.0, and 4.0 are, respectively, 4%, 9%, 13%, 18%, 23%, and 33%.

This study did not involve any experimental analysis of dead load or resistance; the statistics of these quantities are adopted from those published and widely used by the professional community. Table 5 lists the resistance statistics for both first-yield and plastic-collapse limit states, along with the sources and assumptions. No aging effect is considered. The dead-load statistics, taken from NCHRP (1999b), were as follows: normalized dead load, X_2, is normally distributed with mean 1.04 and c.o.v. 9%.

The optimal rating equation LRFs for different time intervals and the two limit states are shown in Table 6. In each case, the limit-state probability [Eq. (13)] was computed with the help of the first-order reliability method (FORM), in which the Rackwitz-Fiessler algorithm was used for mapping the basic variables to the uncorrelated standard normal space (Rackwitz and Fiessler 1978). The total weighted squared deviation [Eq. (14)] and the lowest and highest reliability index in each of the five optimal cases are also listed in Table 6, which indicates that each of the five sets of factors ensure near-uniform reliability across the spectrum.

In addition to rating a bridge, ascertaining an explicit relation between the rating factor and a metric of safety would be desirable. Fig. 6 shows the quantitative relation between rating factor and reliability index for various limit states and load ratios, thereby providing a rational interpretation of excess load carrying capacity (or a lack thereof) in terms of an explicit measure of safety.

Comparison with Traditional Bridge Rating

Table 7 lists rating factors that use the proposed ISLRFR method for Bridge 1-791 and that correspond to three different criteria: yield limit state for a 2-year reference period and plastic-collapse limit state for 2-year and 10-year reference periods (relevant LRFs are taken from Table 6). The dead-load effect on 1-791, computed by *BRASS*, is 96 $\mu\varepsilon$. The bridge is constructed of A36-grade steel, and its nominal yield resistance is taken to be 1,241 $\mu\varepsilon$. Pending more sophisticated analysis, the plastic strength factor, f_p, (Table 5) is simply taken as the ratio of the girder's ultimate to yield moment capacities assuming a bilinear moment-curvature relationship, which in this example is $f_p=1.16$. It is clear that Bridge 1-791 rates very satisfactorily in all three limit states under ambient site-specific traffic. The governing limit state in this case is 10-year ultimate (RF=1.44). However, it is up to the bridge owner to decide on a suitable acceptance criterion that is based on the three rating factors.

Table 7 also lists rating factors under the action of different design trucks using the proposed ISLRFR rating equations. These are then compared with the rating factors given by two other methods [the LRFR manual (NCHRP 1999a) and *BRASS* using load factor design] for the same design trucks. Neither of these

Table 6. Proposed Rating Equation LRFs under Ambient Traffic for Highway Girder Bridges in Delaware

						Optimization [Eq. (14)]		
Limit state	β_T	Time (year)	ϕ	γ_D	γ_L	Objective	Minimum β_i	Maximum β_i
First yield	2.5	1	0.90	1.15	1.35	0.00039	2.49	2.54
		2	0.90	1.10	1.45	0.0011	2.46	2.54
Plastic collapse	3.5	1	0.85	1.20	1.60	0.00034	3.49	3.54
		2	0.85	1.15	1.70	0.00061	3.46	3.53
		10	0.85	1.20	1.85	0.00011	3.49	3.52

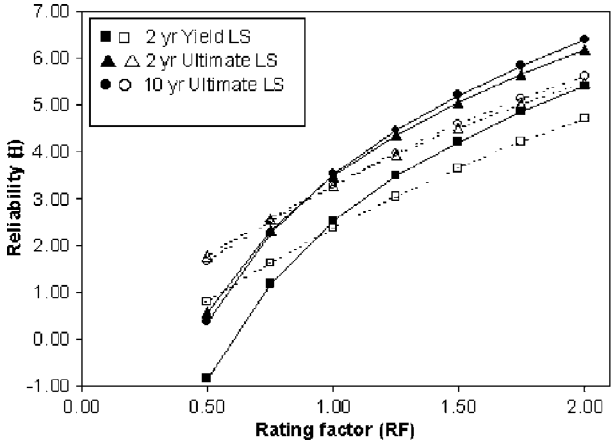

Fig. 6. Relation between rating factor using in-service load and resistance factor rating and reliability index (filled *symbol* and solid line for L_n/D_n=4.0; open symbol and dashed line for L_n/D_n=0.5)

methods have the ability to rate a bridge by using in-service data or for various projected time intervals. For any given design truck, the spread among the three proposed ratings factors is much less than that found in *BRASS* ratings.

At this point, the rationale behind applying the proposed equations to idealized design trucks such as HS20 must be clearly elucidated. The proposed rating equations are founded on the fact that load and strength variables are inherently uncertain. It is therefore difficult to assign, much less interpret, uncertainties in load effects of *idealized* trucks. One possible interpretation is to consider the truck load as deterministic (i.e., with zero c.o.v.); however doing so would require a new set of LRFs and would invariably lead to an artificially high reliability-based rating factor. Another alternative is to put a deterministic HS20 in one lane and a random lane load in the next, but the definition of the adjacent lane load would be arbitrary. A third alternative, to treat the design truck load as random with the nominal value as its annual maximum median and its c.o.v. equal to that in the observed traffic appears most rational in this context: it allows using the same rating equation and allows comparison with more traditional methods and thus seems to be the only consistent explanation.

In addition to 1-791, the highway bridge that yielded the data used in the above analysis, another structure—Bridge 1-704—also on Interstate 95, was rated with the proposed ISLRFR method. Bridge 1-704 is located over Christina Creek on I-95 South in Newark, Delaware, and is a slab-on-steel girder bridge consisting of 3 simply supported spans (each with 12 girders). It is responsible for carrying a large amount of commuter traffic between Maryland and Delaware. Bridge 1-704 is treated as noncomposite in this example, since it was designed noncompositely, although load tests in 1995 showed that it acted compositely. Its dead-load effect, computed by *BRASS*, is 189 $\mu\varepsilon$. The bridge is constructed of A32 grade steel, and its nominal yield resistance is taken to be 1,103 $\mu\varepsilon$. As previously, the plastic strength factor, f_p, is simply taken as the ratio of the girder's ultimate to yield moment capacities assuming a bilinear moment-curvature relationship, which in this example is 1.14.

Site-specific data were not originally available for Bridge 1-704; and as previously indicated, the nominal live-load effect was estimated as the action of HL93 (512.4 $\mu\varepsilon$ with an impact factor of 10% and distribution factor of 1.51). The rating factors using the proposed ISLRFR method for Bridge 1-704 under the action of HL93 with LRFs taken from Table 6 were found to be 1.06 (for the 2-year yield limit state), 0.98 (for the 2-year ultimate limit state), and 0.89 (for the 10-year ultimate limit state). Thus, the bridge appeared to rate satisfactorily only in the 2-year yield limit state, and not in the ultimate limit states under HL93 load. For comparison, the rating factors using the LRFR manual (NCHRP 1999a) were 1.14 (inventory) and 1.48 (operating) under HL93, whereas while rating factors using *BRASS* were 1.13 (inventory) and 1.89 (operating) under HS20.

Hence, we decided to obtain the site-specific in-service live-load effect for Bridge 1-704; and following the method outlined in this paper, the nominal in-service live-load effect (two-year return period value) was found to be 214.5 $\mu\varepsilon$ (Li 2004). The ISLRFR rating factors under in-service site-specific traffic corresponding to all three limit states were then obtained: 2.52 (2-year yield), 2.33 (2-year yield ultimate), and 2.12 (10-year yield ultimate). Bridge 1-704 clearly has more than

Table 7. Rating Factors for Bridge 1-791 Using Three methods under Different Loading Conditions

Method	Limit state	Rating factors under different loading conditions: In-service site-specific (417.2 $\mu\varepsilon$)[a]	HL93 (409.8 $\mu\varepsilon$)[b]	HS20 (322.7 $\mu\varepsilon$)[b]	Type 3 (246.5 $\mu\varepsilon$)[b]	Type 3S2 (217.7 $\mu\varepsilon$)[b]	Type 3-3 (196.8 $\mu\varepsilon$)[b]
Proposed ISLRFR	2-year yield limit state β_T=2.5	1.67	1.70	2.16	2.83	3.20	3.54
	2-year ultimate limit state β_T=3.5	1.57	1.60	2.03	2.66	3.01	3.33
	10-year ultimate limit state β_T=3.5	1.44	1.46	1.86	2.43	2.75	3.04
LRFR manual[c]	5-year ultimate β_T=2.5	NA	1.84, 2.39[d]	2.27	2.98	3.37	3.73
BRASS	Operating	NA	NA	2.84	3.84	4.22	4.67
	Inventory	NA	NA	1.70	2.30	2.53	2.79

Note: ISLRFR=in-service load and resistance factor rating; LRFR=load and resistance factor rating; NA=not applicable; and *BRASS*=bridge rating and analysis of structural systems.

[a]Nominal live-load effect equal to 2-year return period value based on in-service site-specific measurement.

[b]Nominal live-load effect computed using *BRASS*, with an impact factor of 10% and a distribution factor of 1.34.

[c]NCHRP 1999a.

[d]1.84 is the inventory rating, and 2.39 is the operating rating.

adequate safety under ambient traffic. The use of site-specific data thus prevented an unnecessary posting of this bridge.

Conclusions and Future Work

The recent LRFR method uses a probabilistic approach to ensure that existing bridges can be rated and compared against a common target reliability level. Although neither the LRFD nor the LRFR procedures are based on site-specific information (although the LRFR manual does discuss some applications of site-specific data), they do set the stage for using field data to more accurately rate bridges.

This paper has presented a methodology that allows the use of in-service peak-strain data to evaluate the safety of existing bridges in a fully probabilistic manner. A considerable part of the effort has involved statistical characterization of the live-load effect the basis of on extreme-value theory. Load effects (strain response) attributable to ambient traffic were monitored, and only those above a high threshold (trigger) were considered as "loading events" and were recorded. Loading events were assumed to occur according to a conditional Poisson point process, that is, one with a random rate. The strain response associated with these loading events was assumed to consist of an i.i.d. sample. Simple nonparametric tests were performed to test the validity of these assumptions in the collected data. A limited degree of dependence was observed in the data; it was not probed further in this paper, but it may be the subject of future work. The asymptotic behavior of extremes from the sample was investigated. A Bayesian updating was performed on the sample distribution function. The maximum live-load-effect distribution was projected for reference periods ranging from 1 year to 10 years; the Gumbel distribution best described the data. The distribution of extreme loads was shown to be rather insensitive to the trigger value of the data-acquisition system.

The proposed methodology is consistent with both the LRFD and LRFR procedures; and because it is based on actual bridge response, it eliminates a substantial part of live-load modeling uncertainties, such as those related to dynamic impact and girder distribution factors. It can lead to more accurate condition assessments. In this particular example, the c.o.v. of the live-load effect varied around 10 to 13% (depending on the reference time) instead of the 18% used in LRFR calibration. The methodology can be used to rate the specific bridge that has been instrumented; it can also be used to rate a group of bridges having similar traffic characteristics by instrumenting a small but representative subset of the group. Unlike more traditional methods, the proposed method can be applied to various projected time intervals and allows safety checks at two different limit states—yield and ultimate. Rating equations optimized for a group of bridges for both yield and ultimate limit states, with target reliability indices of 2.5 and 3.5 and reference periods of 2 years and up to 10 years, respectively, were developed. The proposed ISLRFR is more stringent than existing methods: the acceptable failure probability in the ultimate limit state used herein is about an order of magnitude lower than that in the LRFR manual. The example presented shows how ratings obtained by using the proposed procedure relate favorably to ratings derived from more traditional methods; although this result cannot be claimed as general without exhaustive comparisons.

Two sources of modeling uncertainty, B_{loc} and B_{site} [Eqs. (4) and (13), respectively] were identified in this paper but were ignored in the numerical examples for lack of supporting data. The role of aging was acknowledged but was not considered in this paper. The postyield amplification factor was taken in the numerical examples to be the ratio of ultimate to yield moment capacities; nevertheless, a more sophisticated analysis may be required. The departure from composite behavior in noncompositely designed bridges at sufficiently high stresses was acknowledged but was not probed further. The proposed methodology was illustrated by using in-service data from only one bridge for a period of only 11 days. This limited amount of data is clearly inadequate for practical implementation of the methodology: Bridge traffic may vary significantly from bridge to bridge and may have seasonal variations and show a general upward trend over the years. Hence, implementation of the proposed method will require estimates of the minimum number of bridges and the minimum durations for collecting in-service data that can be deemed sufficient. These aspects should be addressed in future work.

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