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# A PROBABILISTIC ESTIMATE OF EXTREME VALUE LOADS ON TRANSVERSE WATER TIGHT BULKHEADS

**Baidurya Bhattacharya**
Department of Civil and Environmental Engineering
University of Delaware, Newark, Delaware, 19716
**Roger Basu**
American Bureau of Shipping,
16855 Northchase Drive, Houston, Texas, 77060
**Sanjay Srinivasan**
Technip Offshore Inc.,
11700 Old Katy Road, Suite 150, Houston, Texas, 77079

## ABSTRACT

Structural integrity of watertight bulkheads (WTBs) is critical for ship survivability in the event of hull damage. Design procedures for WTBs are based on empirical and prescriptive description of loads. However, damage- causing events and damaged ship mechanics exhibit significant variabilities/ uncertainties. Hence, the design and assessment of WTBs should be performed in a probability-based format. This paper outlines the development of a physically-based probabilistic model of transverse WTB loads as an essential input to reliability analysis. For the probability modeling of the loads the hull damage events are assumed to be rare and independent. Therefore damage events are described as a Poisson process and the maximum life-time load on WTBs in damaged condition is derived. The emphasis of this paper is on the probabilistic modeling of loads; hence simple phenomenological expressions of load components are used to underline the cause and extent of randomness in WTB loads. A response surface type approach is suggested for determining ship-specific model parameters. A large Ro-Ro vessel with side shell breach below the waterline is chosen for illustrating the application of the proposed methodology; randomness is considered in ship hydrostatic properties, damage location, length of breach, occurrence and duration of damage events, the environment, curve fitting and modeling errors. Probabilistic estimates of the maximum load on any WTB in the ship are obtained through Monte Carlo simulations, these are compared with available code prescribed design values.

## INTRODUCTION

Although substantial work has been performed on the reliability of primary ship hull structures [1-5], reliability of structures such as bulkheads and decks has so far not received comparable scrutiny. Watertight bulkheads (WTBs) constitute structural boundaries to vital spaces and are crucial for ship survivability in the event of damage involving hull breach. Significant loads on WTBs are likely to occur in damaged condition and failure of these structures could initiate progressive flooding of vital spaces and may ultimately lead to loss of the ship. Damage-causing scenarios include collision, grounding, on-board explosions, weapons effects, extreme wave environments etc.

Current design standards (such as [6] and [7]) for watertight bulkheads are based on historical practices and are empirical and prescriptive in nature and the structural loading or reliability of WTBs has not been addressed in a probabilistic format in any significant way in the literature. Nevertheless, the dynamic response and survivability of damaged ships (particularly Ro-Ro ships) have attracted considerable research in recent years. It is clear that the estimation of response of a damaged ship requires intricate physical models that include nonlinearities due to large-amplitude motions of the ship [8], the effect of water in a flooded compartment on motions [8,9] and the influence of ingress and egress of water through the breach on motions [10], coupling of six degree-of-freedom responses in oblique seas [8] and effect of listing of ships on roll motions [11].

 

Significant variabilities/uncertainties exist in the events that lead to ship hull damage, in the description of such damage and in the response of the ship once such damage occurs. Brown [12] analyzed Lloyds Worldwide Ship data and a Sandia National Laboratory report (that included U.S. Coast Guard data) on ship collisions, and reported that struck ships are frequently moored or anchored. Further, struck ship speed can be modeled by an Exponential random variable with mean 1.7 knots, the collision angle may be modeled as a Normal random variable with mean $90^o$ and standard deviation $28.97^o$ , and the longitudinal strike location may be best modeled as Uniformly distributed although the Sandia report favored a relatively higher probability of midship and forward strike. This may be compared with the findings of TDC [13] who analyzed LRS and IMO data and concluded that the longitudinal location of damage was Uniformly distributed on the struck ship. Analyzing collision scenarios with Monte Carlo simulations, [12] concluded that "probabilistic damage extents are very sensitive to striking ship displacement, striking ship speed and collision angle."

Tagg et al. [14] analyzed 216 damage events from IMO and other databases to obtain statistical estimates of vertical extent of collision damage, and found that the overall damage height above the waterline could be described as a Normal random variable with mean 4m and standard deviation 4.8m. On closer analysis they found that, for a given length of the struck ship, this distribution remained Normal; the mean of the distribution decreased with increasing struck ship length.

Otto at al. [15] studied a 16000 ton "example Ro-Ro passenger ferry" (with length between perpendiculars 173.0m, breadth 26.0m depth 15.7m) operating on a 700nm route between Cadiz and the Canary Islands with 240 voyages per year and 25 hours per trip. They used data from Spanish port statistics to generate traffic data for their analysis and found that the annual collision frequency was 0.0429, with equal likelihood that the example ferry was the striking or the struck vessel. This agrees with the 4.3% per year (based on DNV data) and the 5% per year (based on LRS data) estimates reported in [13]. It may be concluded from the work in [15] that the collision damage length on the example ferry was an Exponential random variable with mean about 3.5m. This also agrees qualitatively with [13] (who analyzed IMO and LRS data) where the distributions of the collision damage longitudinal breach and collision damage transverse penetration exhibit similar exponential characteristics. The grounding frequency of the example ferry in [15] was computed as 0.00578 per year. The authors assumed that damage length, depth and height (in collision or grounding) were statistically independent of each other; this assumption however is not expected to hold in all situations.

Zhu et al. [16] analyzed damage incidents during 1990-99 (inclusive) for Ro-Ro and merchant navy ships with lengths greater than 100m from Lloyds Registry damage database and concluded that grounding rate was approximately 0.02 per year which was "about half the incident rate for ship collision". The difference with the findings of [15] above may be noted in this regard. Zhu et al. [16] further found that grounding damage location was more likely to be the midship and the midship to fore regions of the ship. It may also be concluded from their paper that damage length was Exponentially distributed with mean 0.13 times the ship length, and damage width was Exponential with mean 0.26 times the ship breadth. These Exponential distributions, of course, need to be truncated at 1.0 on the right as a practical matter. The above findings again agree qualitatively with those of TDC [13] (who analyzed IMO and LRS data) where the significant positive skewness in each of the distributions of the grounding damage longitudinal breach and grounding damage vertical penetration is apparent.

It is therefore clear that ship damage must be described in probabilistic terms. Consequently, it is imperative that the assessment and design of WTBs be performed in a reliability framework. This will have the following desirable effects: (i) WTB reliability can be made consistent with that of the primary ship structure; (ii) explicit determination of WTB reliabilities will enable the calibration of new reliability-based design rules; and (iii) a reliability-based approach can reduce weight and cost if current standards for WTB design are overly conservative; alternately, a reliability-based assessment can provide a strong justification for increasing strength and cost [17]. Similar concerns about the design of marine structures in general have been expressed recently by the ISSC Committee on Design Principles and Criteria [18].

This paper crafts a methodology for determination of loads on WTB within a probabilistic framework. A simple physically-based model that describes loads on transverse WTBs in terms of variables describing ship geometry, hydrostatic properties, the seastate and the location, extent and frequency of damage is used. The probabilistic nature of the loads is established by considering randomness in the above variables and in the modeling and curve-fitting errors. This probabilistic load model, in turn, may be used in a full reliability-based design or assessment of WTB structures in damaged condition.

The probabilistic framework for describing WTB loads is discussed in the next section. Following that, equations are developed for loads on transverse water tight bulkheads based on a simple phenomenological model. The emphasis of this study is on randomness in the loads on transverse bulkheads; consequently a simple damage model is adopted. Detailed mechanistic models for loads based on complex and possibly nonlinear response of a damaged ship in seas are beyond the scope of this study.

## APPROACH TO PROBABILISTIC LOAD MODELING

Unlike the prescriptive nature of existing rules, the probabilistic modeling of WTB loads should have a clear analytical and physical basis. Ideally, the load model (i) should be physically-based and rational, (ii) should have explicit functional dependence on damage-related variables, as well as on hydrostatic and environmental descriptors, and (iii) should use only easily available/quantifiable parameters. These would enable the model to be applicable to ships of different sizes and classes, as well as to different types of damage-causing events. These would also allow sensitivity analyses of the loads and WTB reliability in terms of the damage variables, and would accommodate future refinements when new information becomes available.

Based on the requirements listed above, the authors believe that a combination of analytical and response surface-based approaches would be most appropriate. The basic steps are:

1. Identify all relevant variables, and select the important ones to be considered in the model.

2. Identify distinct components of loads acting on the WTB, and develop a rational load combination scheme.
3. Develop simplified description of hull damage in terms of the damage-related variables.
4. Establish, from physical considerations, the functional dependence of each load component on the damage variables. These relations should ideally be valid for all ships of a given class subject to a given type of damage. If necessary, use a response surface type approach to determine unknown parameters of the functions. Also estimate the error statistics in estimating these parameters.
5. Finally, identify or establish the probabilistic description of each of the variables in step 1. Include correlation wherever appropriate. Obtain load statistics using Monte Carlo simulations with an appropriately defined algorithm.

## Loads on Transverse WTBs in Damaged Condition

Loads on WTBs depend on a range of variables including ship geometry, environment and type of damage. These variables can be conveniently grouped under:

1. *Geometry-related variables,* $\underline{G}$*,* describing ship type and loading. These include dimensions, hydrostatic properties, sub-division, construction etc. of the ship, type of cargo and loading pattern etc.
2. *Damage-related variables,* $\underline{D}$*,* describing cause, extent and location of hull damage. These depend on whether the damage is caused by accidents (collision, grounding), internal explosions, weapons effects (mines, torpedo etc.) or extreme natural hazards. These also include the frequency of such damages and the duration of damage events (i.e., the time interval to mission completion, repair or rescue).
3. *Environment related variables,* $\underline{E}$*,* describing wave environment and length of exposure. These include wave height, period, relative wave direction etc. during and following the occurrence of damage.

The generalized load vector on transverse watertight bulkheads at a location $\underline{x}$ and time $\tau$, can be represented as arising from two distinct sources:

$$\underline{F}(\underline{x},\tau) = \underline{F}^L(\underline{x},\tau) + \underline{F}^H(\underline{x},\tau) \qquad (1)$$

Eq (1) shows a vector addition of loads. The superscripts $L$ and $H$ stand for "liquid" and "hull", respectively, as described in the following. In the most general case, $\underline{F}^L$ and $\underline{F}^H$ are stochastic in both space and time. The first term, $\underline{F}^L$, represents the effect of liquids, which may be ingressed water (in damaged condition) or liquid cargo (in intact condition), in direct contact with the bulkhead. $\underline{F}^L$ has static as well as dynamic components and generally act normal to the WTB. The dynamic components may include direct wave action, inertial forces due to motion of the ship and sloshing. The second term, $\underline{F}^H$, represents loads that derive from hull girder response (in intact or damaged conditions) and generally act in the plane of the WTB. These are caused by hull girder bending and torsion, as well as those resulting from dynamic effects such as springing, whipping, and slamming caused by the environment and weapons loads. A method for estimating extreme values of combinations of two or more load components is presented in [19].

From the point of view of assessment of WTBs in damaged condition ($D$), an appropriately defined load-effect, $Q^{L,D}(\underline{x})$ (a scalar, such as maximum pressure head, maximum bending moment, or maximum principal stress etc.), at a given location $\underline{x}$, produced by the loads $\underline{F}^L$ is of concern. $Q^{L,D}(\underline{x})$ depends on $\underline{G}$, $\underline{D}$ and $\underline{E}$ – the damage, geometry and environment related variables, respectively. Owing to the randomness in these variables, it is clear that $Q^{L,D}(\underline{x})$ is random in nature. In order to derive the probabilistic description of transverse WTB loads, an accurate probabilistic description of $\underline{G}$, $\underline{D}$ and $\underline{E}$ and an accurate model of how these variables affect WTB loads, are required. The standard convention of using uppercase letters for random variables and corresponding lowercase letters for their realizations have been used in this paper as much as practicable.

Now, a ship may be subject to several damage events during its lifetime. For example, TDC [13] report that annual probability of ship collision worldwide is 4.3% (based on DNV data) or 5% (based on LRS data). Suppose that $N(t)$ damage events occur during the service life, $t$, at random times $T_1$, $T_2$, ..., $T_N$. The time-dependent reliability of the WTB is then given by:

$$\mathrm{Rel}(t) = P\left[ S(\underline{x},T_i) > Q_{t_i}^{L,D}(\underline{x}), \forall \underline{x} \in \Omega, \text{and } \forall T_i < t; i = 1,\ldots,N(t) \right] \qquad (2)$$

where $S$ is the random strength of the WTB (possibly deteriorating with time); $Q_{t_i}^{L,D}(\underline{x})$ is the load-effect at location $\underline{x}$ due to the damage event at time $T_i$ ; and $\Omega$ is the set of critical locations on the WTB. An example of WTB reliability assessment, based in part on the proposed methodology, under *one* damage event and without considering strength deterioration may be found in [20].

The life-time maximum load-effect at a location $\underline{x}$ is:

$$Q_{\max,t}^{L,D}(\underline{x}) = \max_{i=1,2,\ldots,N(t)} \left\{ Q_{T_i}^{L,D}\left(\underline{x};\underline{G}_i,\underline{D}_i,\underline{E}_i\right) \right\} \qquad (3)$$

where it is emphasized that the load-effect at the $i^{\text{th}}$ damage event depends on the values of the geometry-related, the damage-related and the environment-related variable at the time $T_i$ of the damage. We assume that (i) hull damage events are sufficiently rare and the duration of damage event is negligible compared to the life of the ship, (ii) after each damage event the hull can be repaired to an intact condition provided the ship is not lost, and (iii) the occurrence (or non-occurrence) of damage in past voyages does not affect the likelihood of future damages. Hull damage events then occur according to a homogeneous Poisson process with constant rate $\lambda$, so that the probability distribution function of the maximum load-effect evaluated at any $q$,

 

$$F_{Q^{L,D}_{\max,t}}(q;\underline{x}) = \frac{1}{c}\sum_{n=1}^{\infty}\frac{1}{n!}e^{-\lambda t}(\lambda t)^n \prod_{i=1}^{n} F_{Q^{L,D}_{i}}(q;x,\underline{D}_i,\underline{G}_i,\underline{E}_i) \tag{4}$$

The constant $c$ is simply $P[N(t) \geq 1]$; it is included to ensure proper normalization since the case of damage-induced loads due to no damage event is degenerate. $F_{Q^{L,D}_i}(q;\underline{x},\underline{D}_i,\underline{G}_i,\underline{E}_i)$ is the probability distribution of the load effect due to the $i^{\text{th}}$ damage event. Although the random variables $\underline{G},\underline{D}$ and $\underline{E}$ are assumed to be statistically independent in different damage events, significant dependence may exist among them in the *same* damage event, owing, for example, to the possibilities that in a more extreme environment or that due to an unfavorable cargo loading pattern the extent of damage may be higher.

We now turn to estimating the properties of $Q^{L,D}(\underline{x})$ for any $i$; the subscript $i$ is omitted when there is no scope of confusion. Consistent with existing design practices, we describe $Q^{L,D}(\underline{x})$ and $Q^{L,D}_{\max,t}(\underline{x})$ in terms of equivalent pressure head of water in the remainder of this paper.

## Loads due to Ingressed Seawater

For a damaged and flooded compartment, $Q^{L,D}(\underline{x})$ is the result of seawater ingress, and it depends on several components including the original draft ($T$), the parallel sinkage ($\Delta T$) and additional water heads due to heel ($\Delta h_{heel}$), trim ($\Delta h_{trim}$), roll ($\Delta h_{roll}$), pitch ($\Delta h_{pitch}$) etc. of the damaged ship, as well as any additional water head ($\Delta h_{wave}$) due to direct wave pile up.

The original (intact) draft depends only on ship type and loading ($\underline{G}$) and not on location ($\underline{x}$), damage ($\underline{D}$) and environmental ($\underline{E}$) variables. The parallel sinkage, however, depends on the ship type and loading as well as on the damage variables. Heads due to heel and trim at a point depend on $\underline{G}$, $\underline{D}$ and also on location $\underline{x}$ of the point in consideration. The heads due to roll and pitch arise from the external environment and are time-dependent. Heads due to roll and pitch motions at a point, $\underline{x}$, depend on $\underline{G}$, $\underline{D}$, $\underline{E}$ as well as on the location of the point. Finally, the additional head due to direct wave pile up (if any) is conservatively given by the largest wave amplitude during the damage event, and depends on the sea state and the duration of the damage event, $\Delta V_i$ :

$$\Delta h_{wave} = \varsigma_{a,\max} = \max\{\varsigma_a(t;E_i), t \in (T_i, T_i + \Delta V_i)\} \tag{5}$$

where $\zeta_a$ is wave amplitude. In the following sections, the proposed methodology for obtaining probabilistic estimates of maximum WTB loads will be illustrated using a simplified large Ro-Ro vessel. We reiterate that the focus is more on the probabilistic framework than on the mechanistic intricacies such as non-linear dynamic effects. Simple expression of loads in terms of damage-related variables are adopted wherever appropriate. Special care is taken to include only readily available geometric, environmental and hydrostatic properties in the analytical expressions, so that (a) they are acceptable and easy to use, and (b) they can be general enough to have applicability over several classes of ships.

# A NUMERICAL EXAMPLE OF SHIP DAMAGE AND RESULTANT LOADS

The example Ro-Ro vessel mentioned above has a double bottom at elevation 5.5 ft, and has six compartments created by five transverse watertight bulkheads at –420 ft, –240 ft, –85 ft, 120 ft and 265 ft (centerline = 0 ft, aft positive). The ship does not have any *watertight* longitudinal bulkhead. The nominal volume permeability of these compartments can be taken as 0.9. The hydrostatic response of the intact and the damaged ship is computed using the proprietary software GHS [21]. Relevant hydrostatic properties of the vessel in intact condition are given in Table 1.

**Table 1: Nominal hydrostatic properties of the Ro-Ro ship**

| Geometric property | Nominal value |
|---|---|
| Length, $L$ | 900 ft (274 m) |
| Breadth, $B$ (at midsection) | 100 ft (30.5 m) |
| Height, $H$ | 90 ft (27.4 m) |
| Draft, $T$ (at midsection) | 33 ft (10.1 m) |
| Displacement, $\Delta$ | 60000 LT ($5.9\times10^5$ kN) |
| Waterplane area, $A_w$ | 80000 ft$^2$ ( 7436 m$^2$) |
| Longitudinal center of flotation, $L_{CF}$ (from amidships) | 44.3 ft aft (13.4 m aft) |
| Moment to trim per unit length, $M_{TL}$ | 132000 LT-ft/ft ($1.31\times10^6$ kNm/m) |
| Transverse metacentric height, $H_{GM}$ | 5.5 ft (1.7 m) |

## Description of Damage

The following simplifying assumptions are made about hull damage:

1. The damage occurs in the form of a breach in the side-shell (consistent with [22] who report that 58% of structural damage pertains to the side shell and another 19% to the framing). The breach occurs below the waterline, which is a conservative assumption.
2. Asymmetric flooding is assumed to be absent. This is consistent, for example, with the U.S. Navy's design philosophy [6] that longitudinal watertight bulkheads are to be avoided, otherwise counterflooding measures are installed. This assumption removes $\Delta h_{heel}$ from the model.
3. Once the ship is damaged, it is assumed to stop or lose main propulsion if it was already not stationary. In a regular sea, this will cause the ship to attain a beam sea configuration. This is consistent with the observation in [12] that roughly half the collision accidents occur when the struck ship is stationary. Consequently, $\Delta h_{pitch}$ may be assumed as zero. This is also consistent with load cases 5 and 6 among the

loading patterns listed in the ABS rules for bulk carriers [7].

4. The flooding occurs almost instantaneously, i.e., rate-dependent phenomena (such as those discussed in [23]) are neglected. The time-dependence of $\Delta h_{roll}$ and $\Delta h_{wave}$ may be eliminated by replacing them with their respective maximum amplitudes (cf Eq (5)). This is a conservative assumption.

The above assumptions may be relaxed in a more rigorous analysis.
The contributors to flooding load therefore reduce to:

$$Q^{L,D} = f\begin{pmatrix} T(\underline{G}), \Delta T(\underline{G},\underline{D}), \Delta h_{trim}(\underline{G},\underline{D},\underline{x}), \\ \Phi^*(\underline{G},\underline{E},\underline{x}), \varsigma_{a,\max}(\underline{E}) \end{pmatrix} \quad (6)$$

where, $\Phi^*$ = maximum roll amplitude and $\zeta_{a,\max}$ = maximum wave amplitude during a damage event.
Based on the above simplifications, ship damage can be completely quantified by: (i) $X_D$, the longitudinal location of the center of damage, (ii) $L_D$, the length of damage, and (iii) $L_F$, the flooded length. It should be noted that the three variables above are not all independent. For example, $L_F$ depends on damage location as well as size, and also on the geometric variables, $\underline{G}$, such as ship subdivision.
The functional dependence between each the load components $T$, $\Delta T$ and $\Delta h_{\mathrm{trim}}$ and the variables $\underline{G}$, $\underline{D}$, and $\underline{E}$ is determined in a response surface type analysis. Damaged ship response is analyzed for all possible values of $x_D$ and $l_D$, details of these computations can be found in [24].
Parallel sinkage as functions of damage location and damage length for the example Ro-Ro vessel are shown, respectively, in Figure 1(a) and (b). It is clear that sinkage is not strongly dependent on either variable indicating that sinkage depends primarily on the volume of ingressed seawater and not specifically on damage location. It is much easier to estimate the flooded length of a damaged ship instead of the flooded volume. The flooded length, $l_F$, is therefore chosen as the damage-related variable here at a cost to accuracy.
Trim angle as functions of damage location and damage length are shown, respectively, in Figure 2(a) and (b). Like sinkage, trim is found to be almost independent of damage length. Unlike sinkage, however, it is clear that trim is strongly dependent on damage location. For completeness, the relation between trim angle and the flooded length is shown in Figure 2(c). Even though a degree of dependence between $|\theta|$ and $l_F$ is evident, this is ignored in the scope of this work.

## Sources of Randomness

The sources of randomness in WTB loads due to ship damage can be grouped into the following categories:

1. Randomness in $\underline{G}$, $\underline{D}$ and $\underline{E}$: Randomness in $\underline{G}$ occurs due to variabilities in ship loading. Randomness in $\underline{D}$ is due to uncertainties/variabilities in cause, type, location and source of damage, and in the duration of the damage event. Randomness in $\underline{E}$ results from wave height, period, relative direction etc being random processes.
2. Error in ship response modeling, $\underline{B}_{error}$: The computed ship response deviates from the actual due to several reasons such as model idealization, numerical errors etc.
3. Curve-fitting error, $\underline{\varepsilon}_{fit}$: A least-square equation usually simplifies the relation between the dependent and the independent variables, and thus introduces error in the prediction.

The model predicted load given by Eq (6) are functions of the random quantities $\underline{G}, \underline{D}$ and $\underline{E}$. In addition, sinkage, trim and roll amplitudes are functions also of the least square coefficients and random curve-fitting errors. Finally, the model predicted loads are related to the actual loads through the use of modeling error variables:

$$Q = Q_{model} \cdot B_{error} \quad (7)$$

where $B_{error}$ is the relevant modeling error variable. Therefore, the head of water at a transverse distance $y$ (from the longitudinal centerline) on a bulkhead located at $x_b$ from amidships is:

$$H(x_b, y) = H_x(x_b) + y \sin\Phi^* \quad (8)$$

where $H_x(x_b)$ is the component that varies longitudinally given by:

$$H_x(x_b) = T + \left\{\Delta T e^{\varepsilon_{fit_1}}\right\} B_1 + x_b \sin((\Theta + \varepsilon_{fit_2}) B_2) + \varsigma_{a,\max} \quad (9)$$

The parallel sinkage, $\Delta T$, in Eq (9) can be shown to equal [24]:

$$\Delta T = \alpha_{11} \frac{PLBT}{A_w} \left(\frac{L_F}{L}\right)^{\alpha_{12}} \quad (10)$$

where, $P$ = volume permeability of damaged compartment, $\alpha_{11}$ and $\alpha_{12}$ are non-dimensional constants obtained from non-linear regression analysis.

The trim angle, Θ, in Eq (9) can likewise be given as [24]:

$$\Theta = -\alpha_{21} \frac{P\rho gBT}{M_{TL}} L_{CF} + \alpha_{22} \frac{P\rho gLBT}{M_{TL}} \left(\frac{X_D}{L}\right) \quad (11)$$

where $M_{TL}$ = moment required to trim the ship per unit vertical distance from its original intact position ($M_{TL}$ has dimensions of force) $\rho$ = density of seawater, $g$ = acceleration due to gravity, $\overline{x}$ = longitudinal distance between the c.g. of the ingressed water, $G$, and the ship's original center of flotation, $F$. A least square analysis of the data gives the following results: $\alpha_{11} = 2.0$, $\alpha_{12} = 1.6$, $\alpha_{21} = 2.3$ and $\alpha_{22} = 10.35$ for $n_1 = 100$, $n_2 = 100$. These are expected to be valid for large Ro-Ro vessels in general, but likely to be different for other classes of ships
.

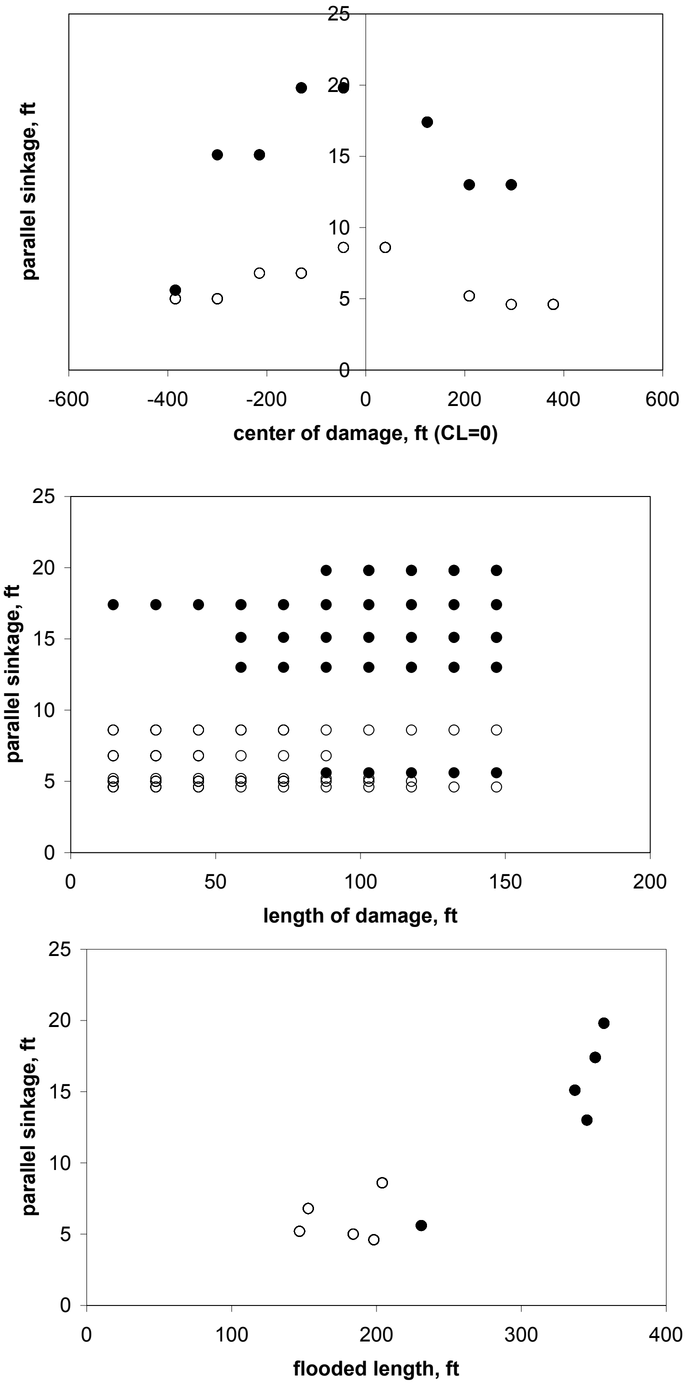


**Figure 1 Sinkage as a function of the damage parameters (∘ = one-compartment flooding, • = two-compartment flooding)**

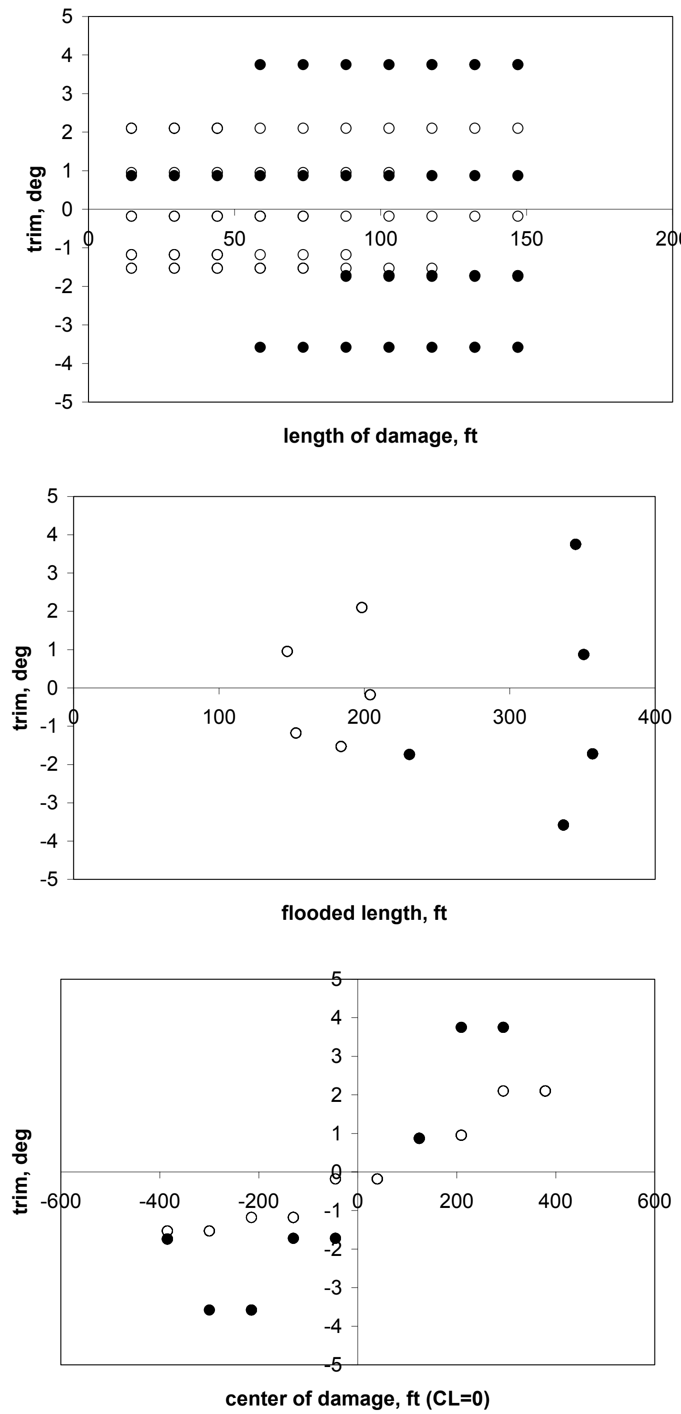


**Figure 2: Trim as a function of the damage parameters (∘ = one-compartment flooding, • = two-compartment flooding)**

The final term, $\Phi^*$, in Eq (9) is the maximum roll amplitude given by [24]:

$$\Phi^* = \frac{\exp\left(-\dfrac{1}{2}\dfrac{\omega^2}{g}(T+\Delta T)\right)}{\sqrt{\left(1-\dfrac{\omega^2}{\Omega_D^2}\right)^2+\nu_\phi^2\dfrac{\omega^2}{\Omega_D^2}}}\frac{\omega^2}{g}\varsigma_{a,\max}B_3 \tag{12}$$

where, $\omega$ = wave frequency and $\Omega_D$ = roll natural frequency of the damaged ship, $\omega^2/g$ = wave number, and $\nu_\phi$ = damping factor. $\zeta_{a,\max}$ is the largest wave amplitude occurring during the damage event, given by Eq (5). Note that the roll response in damaged condition is likely to be significantly different from that in intact condition, and may be attributed to the fact that quantities like $\nu_\phi$, $\Omega_D$ and draft are generally affected by ship damage. Note also that, by definition, the maximum roll amplitude, $\Phi^*$, is a non-negative quantity.

As noted previously, the uppercase letters $P$, $D_0$, $A_w$, $L_F$, $M_{TL}$, $X_D$, $\Delta T$ and $\Omega_D$ are used in place of their lowercase counterparts in Eqs (9) and (12) as they are now treated as random variables. It is important to note that $x_b$, the location of the affected bulkhead, is a function of the location and the size of damage as well as the ship subdivision and geometry. The curve-fitting errors, $\varepsilon_{fit_1}$ and $\varepsilon_{fit_2}$, pertain to the formulas developed for sinkage and trim, respectively. In the present model, modeling errors, represented by random variables, $B_1$, $B_2$, $B_3$ respectively, are assumed to be present only in sinkage, trim and roll angle computations.

## Monte Carlo Simulations for Maximum WTB Loads in Damaged Condition

Recall that the loads on transverse WTBs in damage condition as given in Eqs (8) through (12) above pertain to *one* damage event. The objective in probability-based design and assessment, on the other hand, is to estimate the *maximum* load during the ship's design/ remaining life (Eq (3)). Hence, this section describes a Monte Carlo simulation based approach for obtaining the probabilistic estimate of the life-time maximum load on transverse WTBs in damaged condition. A total of $N_{MCT}$ time histories are generated for the ship. For each time history, the occurrence of damage events are simulated as Poisson arrivals (i.e., the inter-arrival times $\Delta t_i$ are i.i.d. Exponentials); this is continued until the design/ remaining life of the ship, $t$, is exhausted. For each damage event, the geometric, damage and environmental random variables are simulated, and the resultant WTB loads are computed. For each time history, the maximum WTB loads ($h_x$ and $\phi^*$) are recorded. The location of the affected bulkhead is not considered so that the envelope to the maximum loads can be obtained. Those time histories where no damage event occurs are discarded since the objective is to determine the statistics of the maximum loads (cf. Eq (4)); hence the statistical results in the following should be qualified as conditional on at least one damage event occurring during the ship's lifetime [24].

The statistical properties for the random variables considered in the present analysis including their sources are listed in Table 2. Reasonable values have been assumed for those properties which were not available to the authors and are indicated as such in the Table. All random variables are mutually independent unless explicitly mentioned in the Table. Among the geometric variables in the model, only $T$, $A_w$, $P$, $\Omega_D$, $M_{TM}$ and $\nu_\phi$ are considered as random variables in this analysis. A moderately high correlation has been assumed between undamaged draft and undamaged waterplane area. A large uncertainty has been assumed in $\Omega_D$, the natural roll frequency in damaged condition that is consistent with the findings of the references listed at the beginning of this paper. The mean of $\Omega_D$ of has been taken to be about half of $\omega_0$ which is the nominal value in undamaged condition given by $\omega_0 = \sqrt{[g(h_{GM})/r_\phi]}$ (where $h_{GM}$ = transverse metacentric height, $g$ = acceleration due to gravity, $r_\phi$ = radius of gyration about the longitudinal axis). The independent damage variables, $X_D$ and $L_D$ are considered as random variables in this analysis. As stated previously, the right limit on $L_D$ is chosen so that 3-compartment flooding is prevented. The dependent damage variable, $L_F$, is therefore also a random variable. A large uncertainly has been assumed in the duration of the damage event, $\Delta V$.

The environmental variables in the model are $\omega$ and $\varsigma_{a,\max}$; both are determined from the prevailing wave spectrum during the damage event. The ship's composite route scatter diagram is given in terms of the long term joint distribution of the significant wave height, $H_s$, and peak spectral period, $T_p$; the distribution is based on the model in [25] as shown in Table 2. For purposes of illustration, the Bretschneider spectrum that uses only these two parameters is adopted [27]:

$$S(f) = 0.257\frac{h_s^2}{t_s^4 f^5}\exp\left[-1.03\left(\frac{1}{t_s f}\right)^4\right], t_s \approx t_p/1.1 \tag{13}$$

where $h_s$ is in m, $t_s$ is in sec and $f$ is in hertz. Statistics of the short term wave period, $T_0$, and wave amplitude, $\zeta_a$, corresponding to specific realizations of $H_s$ and $T_p$ are then found from $m_0$ and $m_2$, the area and the second moment of the spectrum, respectively. In particular, the average period is $\overline{T}_0 = \sqrt{m_0/m_2}$; and the variance of $\zeta_a$, assuming it is Rayleigh distributed, is $\sigma_{\zeta_a}^2 = m_0(2-\pi/2)$. The distribution of maximum wave amplitude during a damage event is then determined in this analysis as (cf. Eq (5)):

$$F_{\varsigma_{a,\max}}(h) = \left[1-\exp\left\{-\frac{h^2}{2m_0}\right\}\right]^{n_w}, n_w = \Delta v/\overline{T}_0 \tag{14}$$

It is assumed that the random wave heights that occur during a damage event are mutually independent, and that they are identically distributed provided the sea-state stays unchanged during the damage event. Finally, the wave frequency, $\omega$, in Eq (12) is simply given by the average period: $\omega = 2\pi/\overline{T}_0$.

**Table 2: Statistical properties of geometric environmental and damage variables used in Monte Carlo simulations**

| Variable | Distribution | Parameters* | Correlation* | Remarks |
|---|---|---|---|---|
| Draft, $T$ | Normal | $\mu = 31$ ft (9.45 m) , $V = 5\%$ | $\rho$ between $D_0$ and $A_w$ is 0.6 | (a), (b), (c) |
| Waterplane area, $A_w$ | Normal | $\mu = 80000$ ft$^2$ (7436 m$^2$), $V = 5\%$ | | |
| Permeability, $P$ | Normal | $\mu = 0.7$, $V = 20\%$ | | (a), (b) |
| Damaged natural frequency, $\Omega_D$ | Lognormal | $\mu = 0.13$ rad/s, $V = 50\%$ | | (a), (b), (c) |
| Moment to trim per unit length, $M_{TL}$ | Lognormal | $\mu = 10500$ LTft/in ($1.26\times10^6$ kNm/m), $V = 5\%$ | | (a), (b) |
| Damping factor, $\nu_\phi$ | Lognormal | $\mu = 0.15$, $V = 20\%$ | | (b) |
| Longitudinal center of damage, $X_D$ | Uniform | $\Delta = -\,l/2$ to $l/2$ | | (d) |
| Length of damage, $L_D$ | Truncated exponential | $\mu = 0.08\,l$, $\Delta = 0$ to $0.16\,l$ | | (e) |
| Damage occurrence rate, $\lambda$ | Deterministic | 0.10 per year | | (f) |
| Ship lifetime, $t$ | Deterministic | 20 years | | |
| Duration of damage event, $\Delta V$ | Lognormal | $\mu = 1$ day, $V = 50\%$ | | (b) |
| Significant wave height, $H_s$ | Lognormal, if $H_s < 6.6$ ft (2 m) | $\mu = 5.6$ ft (1.70 m), $V = 67\%$ | Expressed through conditional statistics of $T_p$ | (g) |
| | Weibull, if $H_s >$ 6.6 ft (2 m) | $\mu = 5.1$ ft (1.55 m), $V = 66\%$ (i.e.. $\alpha = 1.55$) | | |
| Peak period, $T_p$ | Lognormal | $\mu' = 1.59 + 0.42\ln(h_s + 2)$, $\sigma'^2 = 0.005 + 0.085\exp(-0.13\,h_s^{1.34})$ ; $h_s$ in $m$, $t_p$ in s | | |
| Wave height, $\zeta_a$ | Rayleigh | $\sigma^2$ given by Bretschneider spectrum | | |
| Sinkage curve-fitting error, $\varepsilon_{fit_1}$ | Normal | $\mu = 0$, $\sigma = 0.28$ | | (h) |
| Sinkage curve-fitting error, $\varepsilon_{fit_2}$ | Normal | $\mu = 0$, $\sigma = 1.03$ | | (h) |
| Sinkage modeling error, $B_1$ | Lognormal | $\mu = 1.0$, $V = 5\%$ | | (i) |
| Trim modeling error, $B_2$ | Lognormal | $\mu = 1.0$, $V = 5\%$ | | (i) |
| Roll modeling error, $B_3$ | Lognormal | $\mu = 0.95$, $V = 3\%$ | | (i) |

* $\mu$=mean, $V$=coefficient of variation (c.o.v.), $\sigma$=standard deviation (s.d.), $\rho$ = correlation coefficient, $\Delta$ = range, $\alpha$ = shape parameter, $\mu'$ =mean of ln( ), $\sigma'^2$ =variance of ln( )

(a) Statistical properties of the hydrostatic variables are based on the trim and stability booklet of the Ro-Ro ship. Note that mean values are not necessarily equal to the respective nominal values. However, these statistics are only for illustration purposes and should not be used without verification.

(b) Distribution type (i.e., Normal or Lognormal etc.) has been assumed.

(c) Correlation coefficient has been assumed.

(d) Based on [12], [13] and [22].

(e) Based on [15], [16] and [13].

(f) Assumed to be the sum of frequencies of collision, grounding, fire, slamming and other damages and that these lead to partial flooding. Collision and grounding frequencies are taken to be 5% per year and 2% per year respectively, based on [15], [16], [13]. The remaining frequencies have been assumed.

(g) The wave statistics are based on [25]. Uppercase letters denote random variables, and corresponding lowercase letters denote their realizations.

(h) Estimated from the data generated for this paper. Normal distribution is commonly assumed for curve fitting error.

(i) Based on errors associated with other similar models listed in [26]. Lognormal distribution is commonly assumed for modeling errors.

The curve fitting errors, $\varepsilon_{fit_1}$ and $\varepsilon_{fit_2}$, are zero mean random variables, and usually taken to be Normally distributed. The three modeling error random variables, $B_1$, $B_2$ and $B_3$, are each Lognormal as modeling uncertainties commonly are, and their statistics are adopted based on uncertainties in similar models used elsewhere in the literature (Table 2).

Table 3 lists the first four moments of $H_x$ and Φ* obtained from 1000, 10000, 100000 and 1 million Monte Carlo simulations, each starting with the same seed; the numerical convergence is found to be rapid for the first three moments. A moderate correlation is observed between the two random variables. Approximately 86.4 % of the simulated time histories, denoted by $N_D$ in the Table, were found to yield at least one damage event; this is in fact an estimate of $c$ in Eq (4), and agrees with the theoretical result of $c = 1 - \exp(-\lambda t)$.

Figure 3 shows the histograms of $H_x$ and Φ* drawn from the 1000 simulation case used in Table 3. The distribution of $H_x$ is seen to be slightly skewed to the right (corroborated by coefficient $g_3$ in TT3) with a kurtosis slightly higher than the Gaussian shape. The exponential nature of the distribution of maximum roll amplitude, on the other hand, is clearly visible.

An accurate determination of the probability distribution that best fits the data is crucial to a full characterization of the loads and to the subsequent structural reliability analysis. In order to select the best probability distribution for the simulated data, Chi-squared goodness of fit tests were performed for the two load components. The candidate distributions were Normal, Lognormal, Weibull, Exponential, Gamma and Gumbel. The same set of data as in Figure 3 were used for the purpose (883 realizations each); 10 equiprobable intervals were chosen in each case and statistics were adopted from the 1 million simulation case. The Gamma distribution was clearly the best for $H_x$, yielding a total Chi-squared statistic of 10.95 and a level of significance of 27.9% with 9 degrees of freedom. For Φ*, the Exponential distribution was clearly the best, yielding a total Chi-squared statistic of 14.25 and a level of significance of 11.4% with 9 degrees of freedom. A confirmation of the above goodness of fit is shown in Figure 4 where the Gamma distribution function for $H_x$ and the Exponential distribution for Φ*are plotted with respective statistics taken from Table 3 (the 1 million simulation case).

Finally, the above probabilistic estimates of maximum flooding water level are compared with design values given by naval [6] and commercial [7] rules. As stated in assumption 2 in Section 3.1, heel due to unsymmetric flooding is neglected. Hydrostatic analysis of the example Ro-Ro vessel yielded the highest flooding water level at any transverse bulkhead due to trim and sinkage as 75 feet (2 compartment flooding). Per [6] procedure the design roll amplitude of a damaged ship with displacement 60,000 Lt is 6 degrees; adding four feet due to direct wave action, this gives the highest water level on the transverse WTB near the side shell as 84 ft. ABS rules [7] for vessels intended to carry ore or bulk cargoes (492 feet or more in length) are used to determine maximum water level at an intact transverse bulkhead. As stated in assumption 3 listed in Section 3.1, pitch motion due to waves is assumed zero. Hydrostatic analysis of the Ro-Ro ship gives a maximum trim of 3.75 degrees (two compartment flooding). The maximum water level at the forward most transverse bulkhead in a ballast tank near the side shell due to static water head, trim, roll and wave induced pressure (inertia and added pressure head) is determined as 81 ft. The prescribed roll angles in both design documents is significantly to the right of the distribution of Φ* derived above. Although the two design heads are remarkably close to each other and it is interesting to note that they are located in the right tail of the probability distribution of $H_x$, a systematic study over a large population of ships is required before a generalization of this observation can be attempted.

## CONCLUSION AND FUTURE WORK

A rational probability-based method for modeling flooding load on transverse watertight bulkheads was presented in this paper. This model is general and can be expanded to different classes of ships and different damage scenarios. Complete phenomenological expressions were derived for simplified cases; the methodology was illustrated using a generic Ro-Ro ship. Unlike existing prescriptive rules for WTB design against accidental flooding, the proposed method establishes explicit dependence of flooding loads on ship damage parameters, environmental variables and ship hydrostatic properties. Consequently, probabilistic estimates of flooding loads can be estimated from first principles by considering randomness in the above parameters, and can be incorporated in a full reliability analysis of WTB structures. A detailed numerical example demonstrated the proposed methodology: correlation among hydrostatic properties, a random rate of occurrence and duration of damage events, randomness in location and size of damage, a long-term joint probability distribution of environmental parameters, dependence between damage duration and maximum wave loads, and random modeling errors were considered. Statistics of lifetime maximum loads on transverse WTBs were obtained through Monte Carlo simulations: correlation among load components was not ignored and their probability distributions were estimated. These were then compared with code predicted nominal values.

Further work is required in the areas of:

1. More sophisticated hydrostatic modeling including more subdivisions and side-protection, unsymmetrical flooding and heel response is required. In-plane loads on transverse WTBs from the hull girder in damaged condition needs to be considered. Dynamic effects including pitch, sway, surge etc. motions and rate effects need to be considered as well.
2. More detailed damage description needs to be incorporated. Statistics of geometric properties in damaged state, such as roll damping factor and roll natural frequency, need to be derived.
3. Sensitivity analyses of various sources of randomness need to be studied systematically particularly in the context of reliability of transverse WTB structures.

**Table 3: Statistics of lifetime maximum loads $H_x$ and $\Phi*$**

| $N_{mct}$ | $N_D$ | Height of max flooding water, $H_x$ | | | | Max roll amplitude $\Phi^*$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $\mu$ (ft) | $\sigma$ (ft) | $g_3$ | $g_4$ | $\mu$ (deg) | $\sigma$ (deg) | $g_3$ | $g_4$ | $\rho$ |
| $10^3$ | 883 | 54.3 | 8.79 | 0.369 | 3.18 | 0.179 | 0.185 | 2.35 | 11.9 | 0.214 |
| $10^4$ | 8679 | 54.1 | 9.04 | 0.329 | 3.20 | 0.191 | 0.210 | 2.82 | 16.3 | 0.251 |
| $10^5$ | 86543 | 54.1 | 9.12 | 0.311 | 3.17 | 0.193 | 0.215 | 3.06 | 20.4 | 0.248 |
| $10^6$ | 864520 | 54.1 | 9.11 | 0.309 | 3.18 | 0.193 | 0.214 | 3.13 | 24.6 | 0.250 |

$\mu$=mean, $\sigma$=standard deviation, $\rho$ = correlation coefficient, $g_3$=coefficient of skewness =E[$Y$-$\mu$]$^3/\sigma^3$, $g_4$ = coefficient of kurtosis =E[$Y$-$\mu$]$^4/\sigma^4$; $\rho$ is between $H_x$ and $\Phi^*$

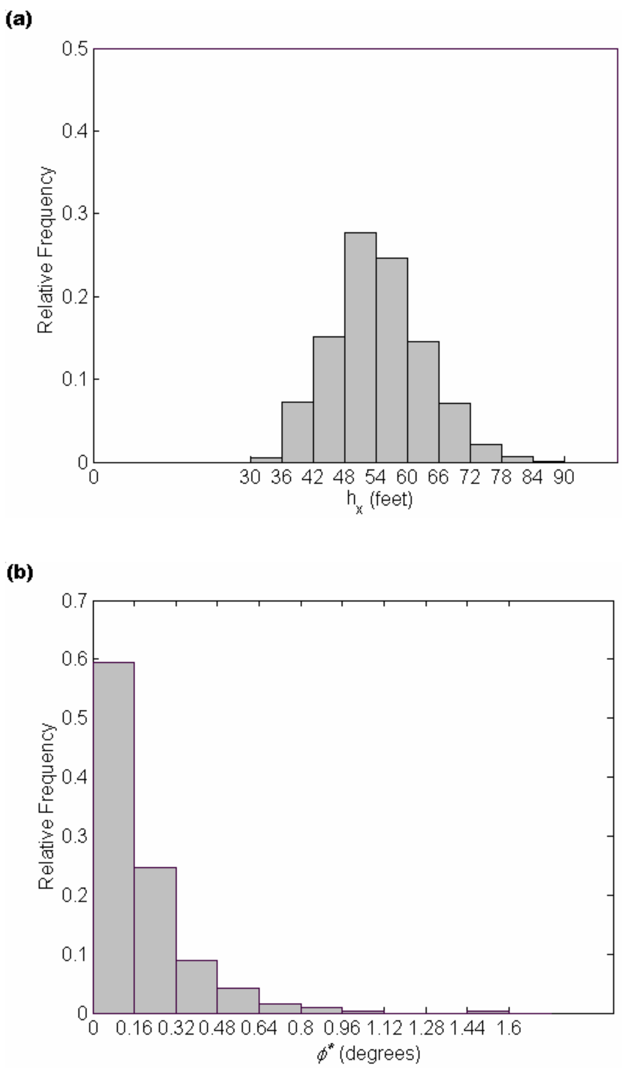


**Figure 3: Relative frequency histogram for maximum lifetime loads on WTBs (883 realizations each)**

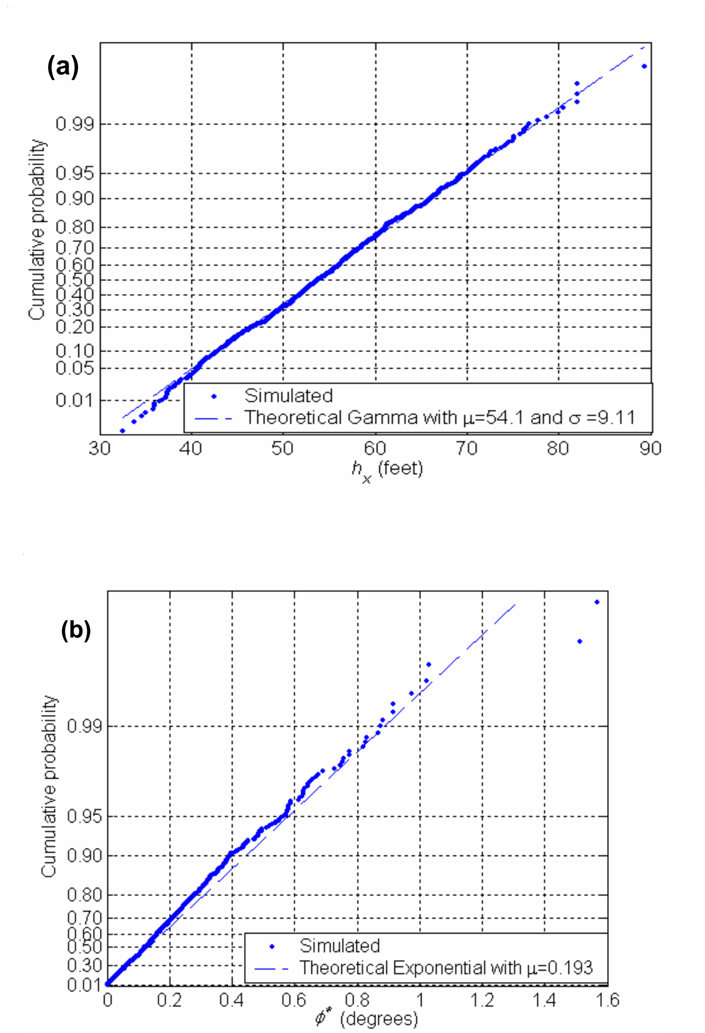


**Figure 4: Comparison of simulated data with best fit theoretical distributions**

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