CHAPTER 7: RISK AND RELIABILITY IN BRIDGES

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Abstract

The chapter opens with a discussion on all sources of uncertainty that affect the bridge structure first at the design stage, and then during its lifetime after it has been constructed and put into service. The probabilistic modeling of these uncertainties are taken up next. Following this, the main subject of the chapter, reliability of bridge structures is approached. Concepts of limit states are introduced and various limit states relevant to bridge structures are listed. Limit state exceedance probabilities, i.e., the probabilities of failure, are defined. Various computational techniques of for computing reliabilities are discussed. Once bridge reliability is computed, the question of whether it is safe enough is brought in. The notion of risk and what constitutes acceptable risk are introduced. Acceptable failure probabilities, and thus target reliabilities to be achieved in bridge design, are then discussed. The next section centers around translating the above concepts of uncertainty, limit states, probabilities of failure and acceptable risk into a practical tool in the form of design equations. The development of reliability based partial safety factor design equations are discussed for given load combinations and load and strength statistics. Once the bridge is put into service, its load characteristics may change and the structure may be subjected to various forms of deterioration. The time dependent behaviour of bridge reliability due to aging and how maintenance can ensure that reliability does not fall below acceptable limits are highlighted. The utility of partial safety factor type rating for in-service bridges is discussed.

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1 Overview

In probability-based design, we explicitly account for uncertainties/vaiabilities in (i) the inputs (loads etc.), (ii) the properties (including strength), and (iii) the model of the system. We then design or assess the system so that it satisfies its safety and performance objectives with acceptable probabilities for its intended function, under expected service condition and projected service life. When a design code is used instead of an explicit probability based approach, such uncertainties are often accounted for indirectly in the form of partial safety factors, load combination schemes and other codal provisions.

Consider the entire life of a bridge from its conception. It starts with functional requirements. Say, the bridge should be part of the National Highway network at some location, over a major river, be able to carry 4 lanes of unrestricted traffic, plus shoulders on both sides. The waterway below is used for cargo and passenger transport. Thus, some clear span and height must be provided. The design life of the bridge has to be specified.

A concept design follows. Economic considerations play a part here. At some point in the design cycle, the owner has to justify if the bridge will be worth the expense.For the given geometry, location, design life and expected loading, the most economical material (among steel vs. prestressed vs. reinforced concrete) and form (box girder, truss, cable stayed etc.) are selected. Material properties are required. A finite element model of the structure is developed.

Loads have to be obtained. Is the bridge going to be in earthquake prone area? Will there be tidal waves? Scouring? Barge impact? What kind of wind forces are we looking at? What are the uncertainties in the loads and what load magnitudes shall we design for? How do we combine the loads? Do we collect data? How large should the data set be?

The cycle of design and analysis continues until a final form is obtained. Costs must be contained. Functional requirements have to be met. Safety must be ensured. How are failure criteria to be defined – both for collapse and functionality? And how safe is safe enough? Is an explicit dynamic analysis of the bridge structure necessary? Is the finite element model of the structure accurate enough? Does it account for nonlinearities near failure? Does it represent realistic boundary conditions? What are the uncertainties in the model?

Construction begins and quality must be maintained. Once construction is complete, the bridge is put into service. Then occasional maintenance needs to be performed, sometimes major maintenance. Is the bridge deteriorating? Is traffic becoming heavier? How often should we inspect the bridge, and how extensively should we repair? Can the bridge be kept closed to traffic – for a month, for a day, for 6 hrs? Should the bridge be made stronger or more durable at the construction stage so that maintenance actions can be minimized?

Finally the bridge may become too unfunctional or too costly to maintain and/or too risky to operate. It is then demolished and a new one takes its place. The cycle restarts.

In this chapter, we isolate the key concepts from the above discussion and treat them systematically.

2 Uncertainty in bridge modeling and assessment

2.1 Probabilistic modeling of uncertain phenomena

In the context of probability theory, the "sample space" is the universal set of all possible events. Probabilities are assigned to an appropriately defined collection of subsets or "events" (called a sigma algebra) of the sample space. A (random) experiment implies the occurence of an event. When the outcome of an experiment can be given in numerical terms, then we have a random variable (RV) in hand. Any possible outcome of a random variable is called a "realization." A random variable can be either discrete, or continuous. If a quantity varies randomly with time, we model it as a stochastic process. A stochastic process can be viewed as a family of random variables indexed by time. If a quantity varies randomly in space, we model it as a random field, which is the generalization of a stochastic process in two or more dimensions. It is assumed that the reader is familiar with the basic concepts of probability theory and random variables and processes, and may refer to standard texts [1-3] for a refresher.

2.1.1 Common random variables encountered in structural reliability

A random variable is governed by its probability laws. The probability law of a RV can be described by any of the following equivalent ways:CDF (cumulative distribution function), PDF/PMF (probability density function for continuous RVs, probability mass function for discrete RVs), CF (characteristic function), MGF (moment generating function) etc.

Although any non-decreasing function bounded by 0 and 1 can be a candidate cumulative distribution function for a random variable X, only a few models (e.g., Normal, Poisson, Geometric, Weibull etc.) are commonly used by the scientific and engineering community in their work. This is because the underlying process appears repeatedly in a wide class of problems. Deriving models solely from data, without basing it on the underlying physics, will be very expensive and often inconclusive.

The uniform distribution arises naturally when there is no reason to favour one outcome over another from the sample space – making all sample points equally likely. This distribution also corresponds to the state of maximum Shannon entropy.

The Bernoulli trial refers to a binary outcome: $X=0$ (often called "failure") that occurs with probabilty q and X = 1 (often called "success") that occurs with probability p, so that $p + q = 1$. A sequence of independent and identical Bernoulli trials can help model large classes of phenomena of engineering interest. The number of trials to the first success gives rise to the Geometric distribution. Generalizing, the number of trials to the r^{th} success gives rise to the Pascal (or negative binomial) distribution. The number of successes in a fixed number of Bernoulli trials, on the other hand, follows the Binomial distribution.

The concept of "mean (or average) return period" (also called "mean recurrence interval") arises from a sequence of IID Bernoulli trials. Let "success" in the Bernoulli trial refer to the occurrence of an event A (so that "failure" means non-occurrence of A) – typically a relatively rare phenomenon, like annual rainfall exceeding 50 inches, annual maximum wave height exceeding 20 m, annual maximum wind speed exceeding 150 km/hr, earthquake magnitude exceeding 7 on the Richter scale, etc. The time (or more literally the number of trials), T, between successive occurrences of the event A in a sequence of Bernoulli trials is a random variable. T follows the geometric distribution due to the IID (independent and identically distributed) nature of the trials, hence the mean of T is $1/p$ time units where p is the probability of occurence of A in each trial (or, time unit). In the continuous time scenario, the time between occurrences is exponentially distributed and the mean occurrence time is the reciprocal of the occurrence rate of the underlying Poisson process.

The Gaussian (or, normal) distribution appears as the limiting form for the sum of a number of random variables, subject to certain conditions [4], and is the most widely used model for continuous random variables. In structural engineering, dead loads are almost exclusively modeled as Gaussian. In fact, in the absence of evidence to the contrary, the Gaussian distribution is the default choice. The exponentiated Gaussian gives the lognormal random variable and is popular in the literature of structural reliability, especially for non-negative quantities. Extreme value theory has given rise to three limiting forms [5] – Gumbel, Frechet and Weibull – and are often adopted for the distribution of the maximum (like wind, wave, traffic etc.) or the minimum (like strength) of a process.

The typical loads considered in bridge analysis and design are dead load, live load (mostly traffic load [6-10], wind load and earthquake loads. Dead loads represent the gravity loads (i.e., self weight) of various components of the bridge starting from prefabricated elements and cast insitu members to wearing surfaces and fixtures. Depending on the location of the bridge, snow load, wave load, impact load etc. may also be considered. NBS 577 [11] lists the mean bias and coefficient of variation and distribution for common material resistance and load random variables and these are widely adopted in the structural reliability literature.

Uncertainties in material properties, and to a lesser extent those in geometries, lead to uncertainties in strength. Uncertainties in boundary condition, e.g., the extent of joint fixity, are typically listed under modeling uncertainties.

2.1.2 Common stochastic processes encountered in structural reliability

Various types of stochastic processes appear in the analysis of structural reliability. Mostly, they represent load processes. In some cases, however, strength degradation also need to be modelled as stochastic processes. Broadly, load processes are either sustained or intermittent. The sustained kind can be further subdivided into (approximately) time-invariant such as dead loads, and fluctuating such as occupant live loads. The sustained loads can be modeled as random variables. Intermittent loads are present during a very short duration compared to the life of the structure, such as seismic loads. In the limit, intermittent loads can be modeled as pulses with random magnitudes occuring at random instants of time. Wind loads and traffic loads can have both fluctuating components (low level continuous) and intermittent pulses (storms and heavy trucks). In many cases, it is only the life-time maximum of the fluctuating or intermittent load processes, rather than detailed temporal characteristics, that may be required in structural reliability analysis. In such cases, the said maximum is modelled as a random variable. The corresponding design quantity is a characteristic value of the distribution of the random variable which may be defined as n-year return period value or some other quantile.

Pulse processes, occuring randomly in time with random pulse magnitudes, are particularly suited for modeling the occurrence of heavy trucks, high winds, high waves and earthquakes on bridges – as long as the "within event variations" are not important. If the within event variations need to be considered, e.g., to determine the response history (of the order of a minute) of a bridge due to a strong motion earthquake, then the occurrence can still be modeled as a pulse, but the frequency content, envelope function etc. of the load time history will also be needed. The Poisson process is the most common model of pulse processes.

The stationary Gaussian process is the most common model for continuous stochastic processes. It can be fully defined in terms of the mean and the covariance functions. Non-stationary and/or non-Gaussian processes may be created through various transformations and filtering of the stationary Gaussian process [12-14].

2.2 Types of uncertainty

Uncertainties in engineering problems occur as a result of natural variability, incomplete information or imperfect knowledge. A traditional classification system for uncertainties is Type I vs. Type II. Type I uncertainties (also known as natural, inherent or aleatory uncertainties) cannot be reduced as they are intrinsically associated with the quantity. Type II uncertainties (also known as modeling or epistemic uncertainties), on the other hand, can be reduced with increased information or sophistication. A more modern classification of uncertainty is Statistical vs. Parameter vs. Modeling and is preferred since it gives a greater resolution to the analyst.

Regardless of classification, the mathematical representation of uncertainty must follow the laws of probability and is generally described by random variables. Apart from the type of distribution, two dimensionless constants are popularly used to describe a random variable: the mean bias, *B*, which is the ratio of the mean to the nominal (or predicted or handbook) value,

$$B = \frac{\mu}{X_n} \qquad [1]$$

(the *median bias* can also be defined similarly) and the coefficient of variation (COV) which is the normalized standard deviation,

$$V = \frac{\sigma}{\mu} \qquad [2]$$

2.2.1 Statistical uncertainty

Suppose the mathematical model of a phenomenon requires the use of a random variable X with the distribution function F_X. We do not wish to probe further where the uncertainty in X is coming from. We are happy with treating X as random and F_X as a black box. We can either use an "empirical" form for F_X , or assume a parametric form (e.g., Normal, Weibull etc.) and obtain its parameters from data. Most random variables used in structural reliability problems represent this kind of uncertainty.

2.2.2 Parameter uncertainty

Suppose we know from analytical, subjective or experimental considerations that a random variable X follows the distribution *g* that is governed by a set of parameters, $\underline{\theta}$. However, there may be uncertainties about the exact value of $\underline{\theta}$. In that case, for any fixed value of $\underline{\theta}$, we say that g is the *conditional* distribution of X. If the parameters are now expressed as a random variable, $\underline{\Theta}$, then we can write:

$$
\begin{aligned}
&P[X \le x \mid \underline{\Theta} = \underline{\theta}] = g(x;\underline{\theta}) \\
&F_X(x) = P[X \le x] = \int_{\text{all } \underline{\theta}} g(x;\underline{\theta})\, f_{\underline{\Theta}}(\underline{\theta})\, d\underline{\theta}
\end{aligned}
\qquad [3]
$$

to obtain the unconditional distribution of *X*.

Zio and Apostolakis [15] say that some model uncertainties can in fact be used as parameter uncertainties, e.g., when using a Monte Carlo simulation, a flag can be used to switch some models on or off – or just use a “switch case” kind of parameter that will select one model at a time in repeated simulations. It may also not always be possible to separate model uncertainties for parameter uncertainties – say, there are parameters whose values need to be estimated from available data, but how the estimates themselves are computed may depend on the model chosen in the first place.

Regardless of the classification, probability theory allows us to treat all uncertainties as random variables. For some, though, conditional distributions may be necessary.

2.2.3 Modeling uncertainty

Analysis tools for predicting global response, stress analysis etc. are commonly deterministic in nature. The model predictions deviate from the actual due to three broad classes of reasons: mathematical idealizations, numerical errors and ignorance. Mathematical idealization includes simplifications such as neglect of non-linearities, etc. Ignorance effectively leads one to neglect a group of variables. The difference between idealization and ignorance is that in the former one knows what is being left out, while in the latter, one does not. Gallegos and Bonano [16] named these respectively as: a) mathematical model uncertainty, b) conceptual uncertainty, and c) computer code uncertainty.

All three introduce new random variables into the reliability problem, and hence into the limit state. Ditlevsen [17]incorporated modeling uncertainty in the limit state equation by transforming the vector of basic variables into another random vector of the same dimension, i.e., by substituting the basic variables $\underline{X}$ with $\underline{V}(\underline{X})$.

In the aggregate sense, the model uncertainty, *M*, in predicting some property or response may be expressed as:

$$M = \frac{\text{actual}}{\text{predicted (or nominal)}} \qquad (\text{predicted} \neq 0) \qquad [4]$$

M is a random variable because the exact deviation is unknown. Of course, in many situation, "actual" results may just not be available, e.g., failure pressure of an actual nuclear power plant containment under pressurization due to an actual core meltdown. For such phenomena, we can have only competing models, and in some cases, scaled model test results under idealized conditions.

Some modeling uncertainty distributions cannot be estimated from data. They can only be estimated from *subjective* probabilities given by a group of experts [18-20].

Examples of treatment of modeling uncertainty

(i) A simple example is the yield strength [11], Y, of a steel member, which is commonly modeled as the product of three random variables representing intrinsic and extrinsic uncertainties, and the nominal value, Y_n:

$$Y = B_P B_M B_F Y_n \qquad [5]$$

where B_P accounts for professional or modeling error, B_M is the material variability, and B_F is the fabrication error. For example, if these three factors are considered to be mutually statistically independent and Lognormally distributed, then the yield strength Y also is Lognormal.

(ii) Nikolaidis and Kaplan [21] performed a survey of uncertainties in FEA in marine and other industries (automobile, aerospace etc). Their conclusions and findings are the following: Depending on the loading case, the mean bias in FEA of containership ranged from 0.9 to 1.4, and the COV from 0.1 to 0.4. For aerospace structures, the stress modeling uncertainty is uniformly distributed with mean 1.0 and COV 0.12. In automobile industry, FEA underestimates flexibility of a car body. The error in predicting deflection due to bending or torsion ranges between 10 - 20%. For offshore structures, the uncertainty in modeling members forces has a mean bias between 0.8 and 1.1 with COV between 0.2 and 0.4.

(iii) In fatigue strength computation [22], the permissible stress range, $\Delta\sigma_P$, in a component may be determined by the use of several adjustment factors on the S-N curve-based reference stress range, $\Delta\sigma_R$, which corresponds to N=2x10^6

$$\Delta\sigma_{\max} \leq \Delta\sigma_P = \Delta\sigma_R f_n f_m f_R f_t f_s f_w f_c \qquad [6]$$

where the factors:

f_n takes into account the effect of the stress spectrum (compared to the constant amplitude assumption of the S-N curve); f_m accounts for material type; f_R accounts for the mean-stress effect; f_t accounts for the plate thickness effect; f_s accounts for imperfections; f_w accounts for weld shape improvements; and

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f_c accounts for corrosive environments. Of the above, the factors f_n , f_R and f_t may be considered as representing modeling uncertainty.

(iii) Similar approaches have been taken to derive strain-based limit states for nuclear power plant containments. [23] adopted the Hancock model in his fragility analysis of steel containments. They defined the equivalent plastic strain at failure, ε_{fail} , in terms of the uniaxial fracture ductility, $\varepsilon_{f,uni}$, and four correction factors (f_1, …, f_4) as follows:

$$\varepsilon_{fail} = \varepsilon_{f,uni} \times f_1 \times f_2 \times f_3 \times f_4 \qquad [7]$$

where f_1 =1.6 exp(-3σ_m/2σ_{von}) accounts for multiaxial stress state, f_2 accounts for model sophistication (i.e., modeling error), f_3 accounts for material variability, f_4 accounts for variability in corrosion degradation (reduction of ductility). This failure criteria can be applied locally in conjunction with a finite element analysis.

(iv) In seismic design, the target displacement, δ_t, of the control node at roof-top of a building may be calculated as [24]:

$$\delta_t = C_0 C_1 C_2 C_3 S_a \frac{T_e^2}{4\pi^2} \qquad [8]$$

C_0 = modification factor to relate spectral displacement and expected maximum inelastic displacement at the roof ; C_1 = modification factor to relate expected maximum inelastic displacements to displacements calculated for linear elastic response; C_2 = modification factor to represent the effects of stiffness degradation, strength deterioration, and pinching on the maximum displacement response; C_3 = modification factor to represent increased displacements due to dynamic second order effects; S_a = response spectrum acceleration at the effective fundamental period and damping ratio of the building; T_e = effective fundamental period of the building in the direction under consideration calculated using the secant stiffness at a base shear force equal to 60% of the yield force. The factors C_1 C_2 and C_3 serve to modify the relation between mean elastic and mean inelastic displacements where the inelastic displacements correspond to those of a bilinear elastic plastic system. The factors C_0 through C_3 may be considered as representing modeling error.

(iv) The environmental load-effect due to sea-load, wind, ice and earthquake is [25]:

$$S = KC_1C_2 \ldots E^{\alpha} \qquad [9]$$

where K = constant, C_1 = transfer from environmental condition to load, C_2 = transfer from load to load effect, E = characteristic environmental parameter and α is a constant. E usually follows an extreme value distribution, e.g., Gumbel. K, C_1,C_2, … are generally random, and may be assumed Lognormal. Each of the transfer functions above may be decomposed into a nominal value and a modeling error variable.

(v) In seismic code development, Cornell et al. [26] separated uncertainty in four parts: β_{DU} (*uncertainty in estimating* the median demand), β_{DR} (the *record to record* variability in demand), β_{CU} (the *uncertainty in estimating* the capacity), β_{CR} (the *randomness* in capacity) so that the total uncertainty (in the lognormal standard deviation sense) in demand and capacity are respectively:

$$\begin{aligned}\beta_D^2 &= \beta_{DU}^2 + \beta_{DR}^2 \\ \beta_C^2 &= \beta_{CU}^2 + \beta_{CR}^2\end{aligned} \qquad [10]$$

β_{DU} and β_{CU} correspond to modeling uncertainty type, while β_{DR} and β_{CR} correspond to the statistical uncertainty type. In a related work, Yun et al. [27] took into account β_{NTH}, the uncertainty in non-linear time history analysis and assumed it to be 0.15, 0.20, and 0.25 for 3-, 9-, and 20-story buildings, respectively.

3 Reliability of bridges

For a structure with several critical locations subject to time dependent loads and possessing time- and space- dependent material properties, the reliability function estimates the probability that the capacity, *C*, exceeds the demand, *Q*, at all locations at all times that the structure is in service:

$$\mathrm{Rel}(t) = 1 - P_f(t) = P\left[C^j(\underline{x},\tau) \geq Q^j(\underline{x},\tau), \forall\, \tau \in (0,t), \forall \underline{x} \in \Omega, j \in J\right] \qquad [11]$$

where Ω is the set of critical locations of the structure and t is total time horizon. Both capacity and demand of the structure are generally functions of space and time and constitute a multidimensional stochastic process. Capacity can change randomly due to aging or other time-dependent effects, and can recover due to maintenance operations. Further, there may be several modes of failure *j* (e.g., shear, flexural, deflection etc.), as indicated by the superscript to *C* and *Q*, associated with any given location. The demand represents the effect of all loads acting simultaneously on the structure (e.g., dead, live, wind etc.) and may be expressed either in load space or in load effect space:

$$Q^j(\underline{x},\tau) = DL^j(\underline{x},\tau) + LL^j(\underline{x},\tau) + WL^j(\underline{x},\tau) + \ldots \qquad [12]$$

The "+" sign indicates combination, not necessarily superposition, and thus may involve non-linear effects.

The structural reliability problem in its most general formulation is thus infinite dimensional both in time and space which of course makes it computationally intractable; hence engineering judgment and various simplifications and restrictions are adopted. For example, if there were only one critical location with only one failure mode, and demand and capacity were time invariant as well, Eq [11] would boil down to a time-independent element level reliability problem in two random variables:

$$\mathrm{Rel} = P[C_0 - Q_0 > 0] \qquad [13]$$

which could easily be computed with the help of the joint PDF of C_0 and Q_0:

$$\begin{aligned}\mathrm{Rel} &= \int_{-\infty}^{\infty}\int_{c=q}^{\infty} f_{C_0,Q_0}(c,q)\,dc\,dq \\ &\left.\begin{aligned}&= \int_{-\infty}^{\infty}[1-F_{C_0}(q)]\,f_{Q_0}(q)\,dq \\ &= \int_{-\infty}^{\infty}F_{Q_0}(c)\,f_{C_0}(c)\,dc\end{aligned}\right\}\text{if } \mathrm{C}_0 \text{ is indepdent of } Q_0\end{aligned} \qquad [14]$$

Thus, the way to making the general reliability problem [11] tractable is to identify only the key sets of (i) failure modes, (ii) load combinations, (iii) critical locations, and (iv) temporal statistics of important processes, so that only a manageable number of failure events need to be analyzed and checked against acceptance criteria.

Figure 1 shows the important steps involved in structural reliability analysis. The concept of limit states, various solution techniques for the reliability problem, and introduction of explicit time-dependence into the reliability problem are discussed next.

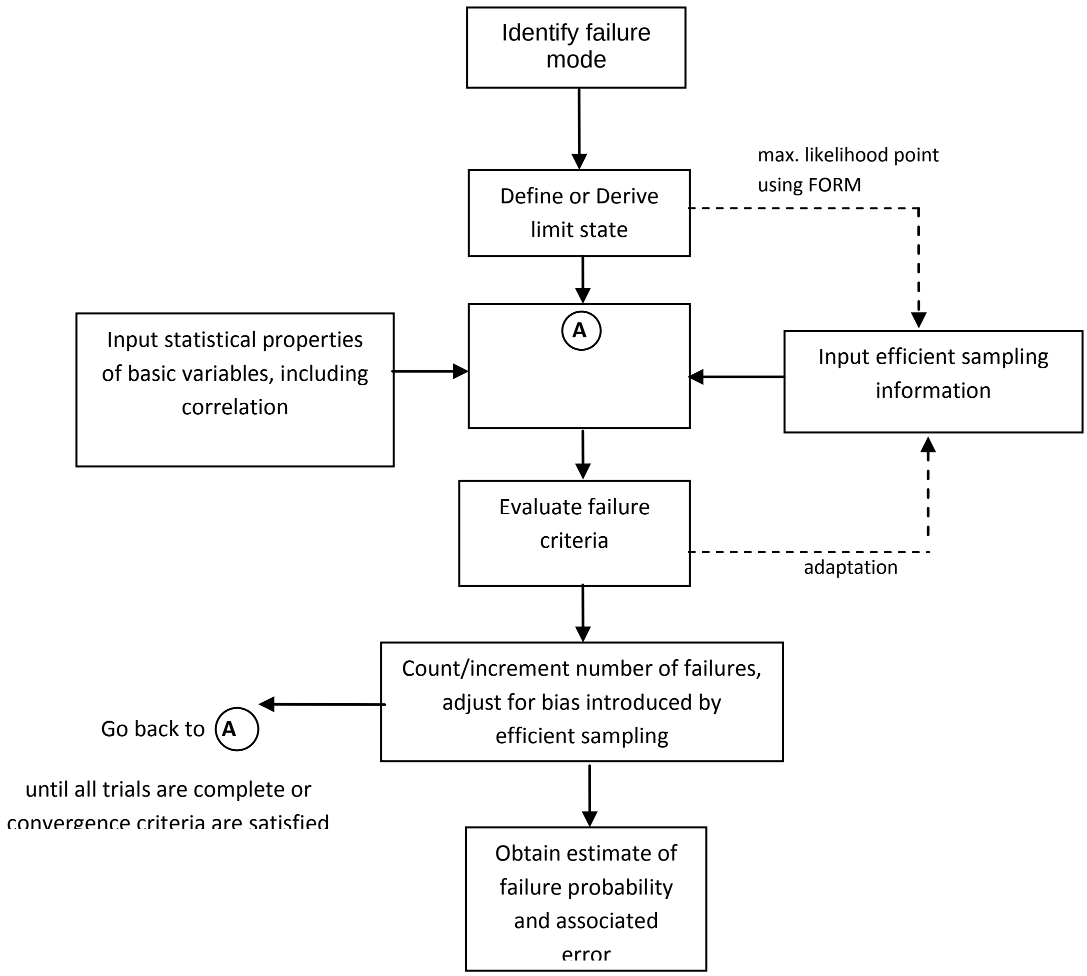


Figure 1: General scheme for reliability analysis

3.1 Limit states

One of the first steps in a structural reliability analysis is to identify the failure modes (or, more generally, non-performance modes) of the structure. A limit state is the boundary between the *safe* (or acceptable) and *failed* (or unacceptable) domains of structural performance in the failure mode under consideration. It is represented with the help of the limit state function (also called "performance function"), $g(\underline{X})$, in the following manner:

$$\begin{aligned} g(\underline{X}) &< 0 \text{ , unacceptable or failed domain} \\ g(\underline{X}) &\geq 0 \text{ , acceptable or safe domain} \end{aligned} \qquad [15]$$

so that, $g(\underline{X}) = 0$ is the limit state equation

The boundary between the two regions, $g(\underline{X})=0$, is called the limit state equation. $\underline{X}$ is the set of basic variables which consist of the complete set of quantities used to describe structural performance in the failure mode under consideration. They may include material properties, loads or load-effects, environmental parameters, geometric quantities, modeling uncertainties, etc. Basic variables in a limit state are usually modeled as random variables, however, those with negligible uncertainties may be treated as deterministic.

Limit states may be defined for elements as well as the system. The difference between "element" and "system" in reliability analysis has less to do with the scale and complexity of the participating component/assembly/sub-structure, and more to do with the form of the limit state function and whether one needs to undertake Boolean combinations, discussed next.

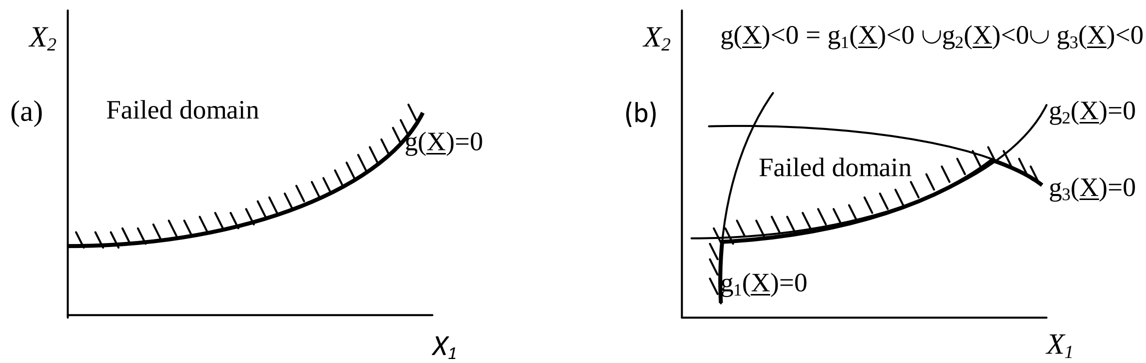


Figure 2: Limit state functions (a) for an element, (b) for a series system. The failure domain is indicated by the hashing.

3.1.1 Structural limit states and load combinations used in bridges

There are broadly two kinds of failure for a structure: *irreversible* and *reversible*. Irreversible failure can be divided into two types:

(i) Overload – e.g., ultimate failure that happens under a single high loading event. Design codes refer to these as strength/extreme/accidental limit states. This type of failure is irreversible in nature. The structure needs to be replaced/repaired after such failure. Consequences of such failure is serious, even catastrophic.

(ii) (ii) Cumulative damage – e.g., fatigue cracking. It too is irreversible in nature and the structure needs to be replaced/repaired after such failure. Consequence of such failure can be serious. However, this damage proceeds gradually and can be detected through inspection before failure occurs.

Reversible structural damage is temporary in nature and have typically to do with functional requirements of the bridge (e.g., deflection, vibration etc.). There is no lasting damage, but the structure is not available for the duration of this kind of failure. Consequence of such failure is usually minor. Most serviceability limit states listed in design codes come under this category.

Strength and serviceability limit states can be formulated both as element and system level limit states depending on the objective and available information. For each of these, several load combinations need to be evaluated (Eq [12]). For example, the AASHTO bridge LRFD code [28] specifies five *strength* type load combinations (involving dead loads and various live loads with or without wind loads), two extreme event load combinations (involving dead loads and reduced live loads with earthquake or ice/collision/flood etc. loads), four service load combinations (involving dead loads, various live loads, wind loads etc.) and two fatigue load combinations (involving only live loads).

Eq [11] therefore simplifies to checking the following groups of limit states one at a time:

$$\begin{array}{ll} \text{overload:} & C^{j}(\underline{x}_i) \geq Q^{j}(\underline{x}_i; \mathrm{LC}_k(t)) \\ \vdots & \\ \text{cumulative:} & C^{j'}(\underline{x}_{i'}) \geq Q^{j'}(\underline{x}_{i'}, \mathrm{LC}_{k'}(t)) \\ \vdots & \\ \text{reversible:} & C^{j''}(\underline{x}_{i''}) \geq Q^{j''}(\underline{x}_{i''}; \mathrm{LC}_{k''}) \end{array} \qquad [16]$$

3.1.2 *Element level limit states*

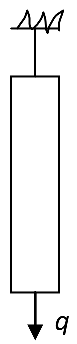
If it is possible to define a single differentiable performance function $g(\underline{X})$ of the basic variables for a given failure mode, then we have what is known as an element reliability problem at hand. An element reliability problem is most naturally realized in the case of a *single* critical cross-section of one structural component in a *single* failure mode (such as flexural failure); in such cases the function *g* is commonly derived from analytical/mechanistic modeling. It can be the same function used in a corresponding deterministic analysis, with some or all of the variables now treated as random variables. However, it is entirely possible that the performance function corresponding to the roof displacement of a tall building under wind loading can be derived in the form of a *single* response surface given in terms of a relevant set of basic variables (obtained from a set of finite element analyses of the structure); in this case, the reliability problem of excessive roof displacement for the building too will qualify as an element reliability problem.

The simple two-variable linear problem as in Eq [13] is an element reliability problem, the basic variables are $\underline{X} = [C_0, Q_0]^T$, and the limit state equation is:

$$g(\underline{X}) = C_0 - Q_0 \qquad [17]$$

Typical modes of failures in bridge structures that give rise to element reliability problem include yielding, crushing, buckling, fatigue failure etc. Element reliability problems are easy are to formulate and inexpensive to compute.

Example 1: A small structural design problem.



Consider a cable (8 inch diameter) in a suspension bridge made of A36 steel with random yield strength *Y* (time invariant). Let *Y* be Weibull distributed with COV 15%. It is a one RV problem. No modelling uncertainty is considered. The axial load q = 1600 kip and the cross sectional area a = 50.3 in^2 are deterministic. Let cable failure be defined as yield of the gross section. Find the failure probability of the cable. The target failure probability is 0.001. Redesign if necessary.

Figure 3: A cable in tension

Given *Y* is Weibull distributed with COV 15%. The mean yield strength of A36 steel is 38 ksi. The shape and location parameters of *Y* are therefore, $V_Y = 15\% \Rightarrow k = 8$ and , $u = \frac{\mu}{\Gamma(1+1/8)} = \frac{38}{.94} = 40.4 \text{ ksi}$.

The failure event is:

$$\{\text{Failure}\} = \left\{ \frac{q}{a} > Y \right\} \qquad [18]$$

The probability of failure,

$$\begin{aligned} P_f &= P[\text{failure}] = P\left[Y < \frac{1600\,\text{kip}}{50.3\,\text{in}^2} \right] \\ &= P[Y < 31.8] = 1 - e^{-\left(\frac{31.8}{40.4}\right)^8} = 0.14 \end{aligned} \qquad [19]$$

is solved using the Weibull CDF.

Since it is required that $P[\text{failure}] < .001$, the cable is inadequate. Reliability can be increased in four ways for this problem: increasing the area, reducing the load, increasing the mean strength, and decreasing the variability of strength. Of these, the second is not possible without restricting traffic, and the third and fourth would require a different material and possibly be very expensive. Thus, we decide to first try to increase the cross-sectional area.

The revised cross-sectional area can be found by finding the inverse of the CDF at the target P_f:

$$\therefore P\left[Y < \frac{q}{a_{new}}\right] = .001 \Rightarrow 1 - e^{-\left(\frac{q}{a_{new}40.4}\right)^8} = .001. \qquad [20]$$

which yields,

$$a_{new} = \frac{1600}{40.4 \times .4217} = 93.9\,\mathrm{in}^2 \qquad [21]$$

Suppose the resultant diameter, about 11 inches, proves to be impractical. The next option is to try a different grade of steel without changing the diameter. Assume the distribution of Y_{new} remains Weibull and its COV remains 15%. The approach now is to select a new mean. The target probability of failure remains 0.001:

$$P\left[Y_{new} < \frac{q}{a}\right] = .001 \qquad [22]$$

which yields:

$$\begin{aligned} &\exp\left[-\left(\frac{31.8}{u_{new}}\right)^8\right] = .999 \\ &\Rightarrow u_{new} = 75.4 \\ &\Rightarrow \mu_{new} = 75.4\Gamma\left(1+1/8\right) = 70.9 \text{ ksi} \end{aligned} \qquad [23]$$

The new mean strength is acceptable provided this new grade of steel has sufficient ductility, corrosion resistance and other desirable properties. Otherwise, a totally new design may need to be adopted.

3.1.3 System level limit states

As should be clear by now, the difference between an *element* and a *system* in a reliability analysis context is somewhat arbitrary and largely dependent on the available information and scale of interest. Indeed, a problem of tensile failure of a prismatic rod made of a brittle material that can be treated as a simple element reliability problem from a continuum viewpoint may amount to an intractable system reliability problem from micro-structural considerations. For practical purposes, it is mostly the availability of a single, differentiable and closed-form performance function that separates an element reliability problem from a systems one.

It would be highly desirable, then, to somehow cast the performance of a structural *system* in terms on a single limit state (perhaps using approximate numerical techniques such as a response function fit) and thereby take advantage of the speed, elegance and accuracy of element reliability solution techniques; such formulation unfortunately remains elusive more often than not. Needless to add that structural system failure events are thankfully so rare (and in any case, structural systems can hardly be deemed to constitute a nominally identical sample) that the other alternative – a frequentist interpretation of structural system reliability – is not feasible. The usual systems reliability formulation therefore is presented in terms of Boolean combinations of element limit states depending on the logical construct of the system in terms of its components and the definition of *failure* at the systems level[29].

If the system failure event can be cast as an intersection of *m* element failure events (i.e., a classical parallel system), then the system failure probability is:

$$\text{Parellel system:} \quad P_{f,sys} = P\left[\bigcap_{i=1}^{m} g_i \leq 0\right] \qquad [24]$$

Where g_i's are the element limit state surfaces in the basic variable space ($\underline{X}$). For a series system type configuration, the system failure probability is:

$$\text{Series system:} \quad P_{f,sys} = P\left[\bigcup_{i=1}^{m} g_i \leq 0\right] \qquad [25]$$

For systems more general than the simple series and parallel organizations, the greatest challenge is to identify the minimal cut sets (at least the dominant ones), particularly in light of the circumstances peculiar to structural systems mentioned above. A set of elements of a system is a *cut set* if the failure of all members of the cut set causes system failure [30]. A *minimal cut set* is one that if any element is removed from it, the subset no longer remains a cut set.

If the cut sets $C_i, i = 1, \ldots, n_c$ can be identified for the system, the system failure probability becomes:

$$\text{Series-parallel system:} \quad P_{f,sys} = P\left[\bigcup_{i=1}^{n_c} C_i\right] = P\left[\bigcup_{i=1}^{n_c}\left\{\bigcap_{j=1}^{n_i} g_{ij} \leq 0\right\}\right] \qquad [26]$$

where g_{ij} is the *j*th limit state in cut set *i*. Exact solution of Eq. [26] may be impossible, thus bounds on system reliability, based on marginal events[31], pairs of joint events[32] or triplets of joint events [33] are available. Cut sets, without regard to an ordering of element failure events, are possible to be determined for elastic-perfectly plastic structures.

The binary nature of elements (being either in failed or safe states), though not a necessity, facilitates the use of standard methods such as fault or event trees (or a combination of the two), including their variants to suit the peculiarities of structural systems, to describe system failure in terms of component failure events, and hence to identify minimal cut and/or minimal path sets of the system. System reliability computation for structures is not straightforward since the component failures are not mutually independent events on account of (i) active redundancy in the structure leading to load sharing, (ii) load path dependence in case of successively applied multiple yet sustained loads, (iii) load redistribution after initial member failures for redundant structures, (iv) non-linear behavior and non-brittle failure of the components, (v) failure sequences of different probabilities for the same cut set in a progressive collapse or incremental loading situation, and (vi) possible statistical dependence among the basic variables.

3.2 Computation of reliability

The failure probability corresponding to Eq [15] is given by the multidimensional integral in the basic variable space:

$$P_f = P(g(\underline{X}) < 0) = \int_{g(\underline{x})<0} f_{\underline{X}}(\underline{x}) d\underline{x} \qquad [27]$$

where $f_X(x)$ is the joint probability density function for $\underline{X}$. The reliability of the structure would then be defined as $Rel=1-P_f$.

Closed-form solutions to Eq [27] are generally unavailable. Two different approaches are widely in use: (i) analytic methods based on constrained optimization and normal probability approximations, and (ii) simulation based algorithms with or without variation reduction techniques and both can provide accurate and efficient solutions to the structural reliability problem. The first kind, grouped under First Order Reliability Methods (or FORM), holds a distinct advantage over the simulation based methods in that the design point(s) and the sensitivity of each basic variable can be explicitly determined.

3.2.1 First order reliability method

FORM calculates the reliability of a system by mapping the failure surface onto the standard normal space and then by approximating it with a tangent hyperplane at the design point (defined as the point on the limit state surface in the standard normal space that is closest to the origin) [34]. Provided the limit state surface is well-behaved, the solutions obtained by FORM are reasonably close to that obtained by the relatively expensive simulation based solutions.

The two important steps of FORM are described in detail in the following.

1. Map the basic variables $\underline{X}$ on to the independent standard normal space $\underline{Y}$ and hence $g(\underline{X})$ to $g_1(\underline{Y})$. Several mappings are possible, such as(i) Hasofer-Lind [35]or second moment transformation which uses information only on the first two moments of each X, (ii) Nataf transformation [36]which uses marginal distribution of each X and the correlation matrix of the $\underline{X}$ vector, (iii) Rosenblatt transformation [36]which uses nth order joint distribution information, a special case of which is the so-called full distribution transformation valid when the $\underline{X}$ are mutually independent, (iv) the Rackwitz-Fiessler[37] transformation which converts each X point-by-point into an equivalent normal U through a marginal distribution and density

equivalence, and then the vector $\underline{U}$ into the independent standard normal vector $\underline{Y}$ through a Nataf type transformation.

2. Locate on g_1 the point $\underline{y}^*$ closest to the origin,

$$\begin{aligned} &\min F = \underline{y}^T \underline{y} \\ &\text{subject to } G = g_1(\underline{y}) = 0 \end{aligned} \qquad [28]$$

Let the solution to this optimization problem be $\underline{y}^*$ and let β be the distance of this optimal point from the origin. This minimum norm point $\underline{y}^*$, is known as the checking or the design point. The limit state surface g1 can be approximated by a tangent hyperplane at y*, yielding the approximate probability of failure as

$$P_f = \Phi\left(-\beta \operatorname{sgn}\left[g_1(\underline{0})\right]\right) \qquad [29]$$

The signum function determines whether the origin is in the safe domain or not. The drawback of FORM is that it provides the exact solution only if the original limit state is linear and the basic variables are normally distributed. Otherwise, the extent of error depends on the curvature of the limit state and the method of mapping of $\underline{X}$ onto $\underline{Y}$.

After performing a FORM analysis, the design point $\underline{y}^*$ can be transformed back into the basic variable space, yielding the "checking point", $\underline{x}^*$ which cannot be obtained from simulation based solutions. It is implied that if the structural element in question is designed using this combination $\underline{x}^*$, the reliability of the component would be β(within the approximations of FORM). This, in fact is the basis of load and resistance factor design, discussed subsequently.

The gradient projection method, originally developed by Rosen [38], is well-suited to tackle the constrained non-linear optimization problem in Eq [28].

3.2.2 Monte Carlo simulations

Except in very special situations, closed form solution to the structural reliability problem (Eq. [27]) does not exist and numerical approximations are needed. The true probability of failure, P_f ,

$$P_f = \int_{\text{all } \underline{x}} \mathbb{I}\left[\{\text{Failure}\}\right] f_{\underline{X}}(\underline{x})\, d\underline{x} = \int_{\text{all } \underline{u}} \mathbb{I}\left[\{\text{Failure}\}\right] f_{\underline{U}}(\underline{u})\, d\underline{u} \qquad [30]$$

can be estimated using basic (or "brute-force" or "crude") Monte Carlo simulations (MCS) in practice as:

$$\hat{P}_f = \frac{1}{N} \sum_{i=1}^{N} \mathbb{I}\left[g\left(T\left(\underline{U}_i\right)\right) < 0 \right] \qquad [31]$$

where a zero-mean normal vector $\underline{U}$ with the same correlation matrix $\boldsymbol{\rho}$ as the basic variables is generated first and then transformed element by element according to the full distribution transformation:

$$T(\underline{u}) = \underline{x} \;\Rightarrow F_{X_i}(x_i) = \Phi(u_i) \qquad [32]$$

The use of the same $\boldsymbol{\rho}$ for $\underline{U}$ as for $\underline{X}$ results in error, but the error is generally small [39]. N is the total number of times the random vector $\underline{U}$ is generated, and $\underline{U}_i$ is the i^{th} realization of the vector. It is well-known that the basic Monte-Carlo simulation-based estimate of P_f has a relatively slow and inefficient rate of convergence. The coefficient of variation (COV) of the estimate is :

$$\hat{V}(\hat{P}_f) = \sqrt{(1-P_f)/(NP_f)} \approx \sqrt{1/(NP_f)} \qquad [33]$$

which is proportional to $1/\sqrt{N}$ and points to an inefficient relation between sample size and accuracy (and stability) of the estimate. Such limitations of the basic Monte Carlo simulation technique have led to several "variance reducing" refinements. Notable among them are Latin hypercube sampling [40], importance sampling along with its variants [41, 42], subset simulations [43]**,** which, if performed carefully, can significantly reduce the required sampling size. Nevertheless, importance sampling and other variance reducing techniques should be performed with care, as their results may be quite sensitive to the type and the point of maximum likelihood of the sampling distribution, and an improper choice can produce erroneous results [44].

3.2.3 System reliability computation

An ordered sequence of failure events from a cut set is variously termed in structural systems reliability analyses, sometimes with subtle differences among them, as *failure sequence* or *failure path*. To be specific, a *failure sequence* under incremental loading accounts for load redistribution after each component failure while a *failure path* does not, and leads to different events whenever load redistribution occurs after each successive component failure[45]. The terms *failure mode* and *collapse mode*, unfortunately, have been used in the literature to denote a cut set both with and without regard to ordering of failure events and have led to confusion in some cases. We prefer "collapse mode" to imply a cut set without reference to failure order, and "failure sequence" to imply an ordered sequence from a cut set. A path set is sometimes referred to as a *stable configuration* [46] although this approach is rarely taken in structural problems.

Depending on the structural complexity and desired accuracy of the solution, the dominant failure sequences (or collapse modes) can be found in a variety of ways: some of these involve only a deterministic analysis of the structural system while some employ a fully probabilistic analysis, and still others that use some limited probabilistic information. The assumption of rigid perfectly plastic material behavior is fairly popular in structural system reliability analysis as it eliminates load history dependence. It is well-known that deterministic plastic mechanism analysis can lead to collapse mode identification in case of rigid-plastic framed structures, although the number of modes generated quickly becomes huge [47, 48]. Such deterministic rules have been variously adapted to search for the probabilistically dominant collapse modes by (i) creating linear combinations of those basic mechanisms that have the lowest reliability indices (the beta-unzipping method [49]), (ii) using linear programming[50], (iii) stochastic programming [51], (iv) genetic algorithms [52] etc. The probabilistically dominant failure *sequences* can be searched using truncated enumeration schemes that include the branch and bound method [49] and, importantly, the incremental loading method[53, 54]. The incremental loading method is particularly useful (and often the only way out) when component failure is multistate instead of the usual binary [55], material behavior is brittle, semi-brittle or non-linear instead of ideal plastic [56], and system failure occurs not due to formation of a mechanism, but due to excessive deformation or a specified drop in structural stiffness with regard to specified degrees of freedom. Nevertheless, one potential drawback of the incremental analysis method is its quasi-static assumption of structural behavior: the load duration needs to be sufficiently long to allow potential redistribution of load effects throughout the system.

3.3 Specifying target reliabilities for design and assessment

It has become increasingly common to express safety requirements, as well as some functionality requirements, in reliability based formats. A reliability based approach to design, by accounting for randomness in the different design variables and uncertainties in the mathematical models, provides tools for ensuring that the performance requirements are violated as rarely as considered acceptable. Such an approach comes under the broad classification of performance design (PBD). In structural engieering, PBD has most enthusiastically been espoused in the seismic engineering community as evident in SEAOC [57], ATC-40[58], FEMA 273 [59], FEMA 350 [60] etc.

Mathematically, we go back to Eq [16], and set a lower limit to the reliability, or equivalently, an upper limit to the failure probability for each limit state:

$$1 - \mathrm{Rel}(t) = P_f(t) \le P_f^* = \Phi^{-1}(-\beta_T) \qquad [34]$$

where P_f^* is the maximum permissible failure probability and β_T is the equivalent target reliability index. The cause, reference period, and consequences of violation of different limit states may vary, and if a reliability approach is taken, the target reliability for each limit state must take such difference into account [61-64]. For example, If the structure gives appropriate warning before collapse, the failure consequences reduce and that in turn can reduce the target reliability for that mode [62, 65]. Functionality target reliabilities may be developed exclusively from economic considerations. The safety target reliability levels required of a structure (i.e., in strength or ultimate type limit states), on the other hand, cannot be left solely to the discretion of the owner, or be derived solely from a minimum total expected cost consideration, since structural collapse causing a large loss of human life and/or property, even if an “optimal” solution in some sense, may not be acceptable either to the society or the regulators. Design codes, therefore often place a lower limit on the reliability of safety related limit states [63, 66].

3.3.1 *Code specified target reliabilities*

Conventional structures that have a history of successful service, such as concrete buildings, highway bridges and steel vessels, can be deemed sufficiently safe, and their calculated reliability levels may be used as the targets for new structures of the same kind. This, in principle, is done when a new reliability-based code is developed for a given class of structures having a successful history of use and a wide knowledge-base about their performance [67]. The objective is to produce more uniform levels of safety and more optimal structures. ISO 2394 [61], and later JCSS [62], proposed three levels of requirements with appropriate degrees of reliability: (i) serviceability (adequate performance under all expected actions), (ii) ultimate (ability to withstand extreme and/or frequently repeated actions during construction and anticipated use), (iii) structural integrity (i.e., progressive collapse in ISO 2394 and robustness in JCSS). Target reliability values were suggested based on the consequences of failure (C) and relative cost of safety measure (S) [62]. In ultimate limit state, these ranged from 10^{-3}/ year for minor C and large S, to 10^{-5}/year for moderate C and normal S, down to 10^{-6}/year for large C and small S. In serviceability limit state, the maximum annual failure probability ranged from 0.1 (high S) to 0.01 (low S).

The Canadian Standards Association[68] defines two safety classes and one serviceability class (and corresponding annual target reliabilities) for the verification of the safety of offshore structures (i) Safety class 1- great risk to life or high potential for environmental pollution or damage, 2) Safety class 2-small risk to life or low potential for environmental pollution or damage, and 3) Serviceability Impaired function and none of the other two safety classes being violated. Det Norske Veritas [65] specifies three types of structural failures for offshore structures and target reliabilities for each corresponding to the seriousness of the consequences of failure. The American Bureau of Shipping [69] identified four levels of failure consequences for various combinations of limit states and component class for the concept Mobile Offshore Base and assigned target reliabilities for each.

3.3.2 *Bridge Structures*

Ghosn & Moses [70] suggest three levels of performance to ensure adequate redundancy of bridge structures corresponding to functionality, ultimate and damaged condition limit states, while Nowak et al.[(71)] recommend two different reliability levels for bridge structures corresponding to ultimate and serviceability limit states.

Nowak et al. [71] recommend a (life-time) target component reliability index of 3.5 and a target system reliability index of 5.5 in the ultimate limit states for bridge structures. For serviceability limit states, they recommend a target component (i.e., girder) reliability index of 1.0 in tension and 3.0 in compression. They also compute component reliabilities of different kind of bridges (reinforced concrete, prestressed concrete and steel built to AASHTO 1992 and BS 5400 specifications) in bending, shear and serviceability limit states.

Ghosn and Moses [70] suggest the following reliability requirements to ensure adequate redundancy of a highway bridge structure:

$$\beta_u - \beta_1 \geq 0.85, \quad \beta_f - \beta_1 \geq 0.25, \quad \beta_d - \beta_1 \geq -2.7 \qquad [35]$$

The subscripts 1*, f, u* and *d* refer, respectively, to first member failure, functionality limit state, ultimate state and damaged condition limit state.

The design of the Confederation Bridge (Northumberland, Canada) required that load and resistance factors be calibrated to "a β of 4.0 for ultimate limit states, for a 100 year life" [72]. Sarveswaran and Roberts [73] chose an acceptable annual failure probability of bridge collapse in UK equal to 2x10^{-5} which corresponded to an FAR of 2 (FAR is discussed in Section 3.3.4).

3.3.3 Loss based approaches

The risk of an undesirable event is commonly defined as:

$$\text{Risk } = p \times C \qquad [36]$$

where, *p* = probability of occurrence of the event and *C* = Consequence of event (lives lost, lost revenue, monetary compensation, lost utility etc.). Eq. [36] is valid when there is only one level of undesirable consequence. A more general expression would be:

$$\text{Risk } = \sum p_i \times C_i \qquad [37]$$

The term "risk" is also used in the sense of an individual's probability of death in the public health and actuarial literature. Definition of risk and what constitutes consequences of failure depend on whose risk is it – the public's, a corporation's or an individual's. Once the tolerable risk *R** is known, and C can be quantified, the maximum permissible failure probability can be set:

$$P_f^* = \frac{R^*}{C} \qquad [38]$$

The actual risk from an activity may be markedly different from the risk perceived by the public. Society's general reaction to hazards of different levels can range from indifference, to rational to dread. If exposure to an activity is voluntary, the acceptable level of risk is generally higher. Involuntary activities on the other hand have a much lesser acceptable risk to an individual. In the absence of proper information about a perceived hazardous activity, the public may have a "dread risk." Appreciating this, the maximum tolerable risk suggested in the Netherlands for existing situations is 10^{-5}/person/year while for new situations it is 10^{-6}/ person/year[74]. However, it needs to be underlined that a society's sense of tolerable risk for a given activity may change with time.

3.3.4 Fatality based approaches

When the loss from failure is measured in terms of human lives lost, there are several fatality based approaches to setting target reliabilities. It is controversial to put a monetary value to human life.

Various agencies and researchers have investigated levels of probability that are acceptable to society for events causing fatalities, as described in the following. The acceptable probabilities depend on the nature of the hazard, and decrease with increasing number of fatalities.

As reported in MSC 72/16 [75], UK's HSE suggests 10^{-4}/person/year as the limit of fatality risk to members of the general public. In a CIRIA [76] report, Flint developed an empirical formula for setting the annual target failure probability as:

$$P_f = \frac{K_s}{n_r} p' \,/\, yr \qquad [39]$$

where: p' = basic annual probability of death accepted by an individual member of society. K_s accounts for the voluntary nature of hazardous activity (a person may be willing to increase his exposure by a factor of K_s) and its typical value is 5. n_r = aversion factor defined as the number of lives involved. Public aversion to an accident is assumed to be directly proportional to the number of lives involved. However, other non-linear relations have also been proposed. For example, Allen [77] proposed a somewhat different formula for annual target failure probability that incorporated the nature of warning available for the impending failure:

$$P_f = \frac{A}{W\sqrt{n_r}} 10^{-5} \,/\, yr \qquad [40]$$

where, n_r = aversion factor, A = activity factor, and W = warning factor. The factor 10^{-5} was ascertained from data on building collapse in Canada. For normal activities, A ranges from 1 (buildings) to 10 (high exposure structures like offshore structures) and equals 3.0 for bridges. W ranges from 0.01 (fail safe condition) to 1.0 (for failure without warning). Note that Eq [40] uses $\sqrt{n_r}$ rather than n_r in the denominator implying that the rate of growth in risk aversion decreases with the number of fatalities. Later, ISO [61] tied the acceptable failure probability to the square of the number of lives involved, signifying perhaps a decrease in the public's sense of tolerable risk in engineered systems.

A somewhat different measure of hazardous activities that accounts for exposure time is the fatal accident rate (FAR). The FAR for an activity is the number of fatalities per 100 million hours of exposure to that activity (i.e., 1000 people working 2500 hours a year and having working lives of 40 years each):

$$\text{FAR} = 10^8 P[F] / T_h \qquad [41]$$

where $P[F]$ is probability of fatality, and T_h is the exposure time in person-hours. Typical values of FAR in the UK [78] range from 5 (chemical processing industry) to 67 (construction industry). FARs for various activities in Japan[79] range from 0.2 for fires, 4.3 for railway travel to 46.3 for civil aviation.

4 Reliability based design codes of bridges

4.1 Partial safety factors

Reliability based partial safety factor (PSF) design is intended to ensure a nearly uniform level of reliability across a given category of structural components for a given class of limit state under a particular load combination[80]. We approach the topic of optimizing PSFs by noting that any arbitrary point, $\underline{x}^a$, on the limit state surface, by definition, satisfies:

$$g(\underline{x}^a) = 0 \qquad [42]$$

We can, for example, choose each member of $\underline{x}^a$ to correspond to a particular quantile of the respective element of the random vector $\underline{X}$, such that Eq [42] defines a functional relation among these quantiles. By choosing different values for $\underline{x}^a$, we can effectively "move" the joint density function of $\underline{X}$ with respect to the limit state surface. Clearly, this relative "movement" in the basic variable space affects the limit state probability. In other words, by specifying a functional relation among quantiles (or some other statistics) of the basic variables $\underline{X}$ we can affect the reliability of the structure.

Extending this idea, a "design point" $\underline{x}^d$ on the limit state surface can be carefully chosen so that it "locates" the limit state in the space of basic variables such that a desired target reliability is ensured for the design. The ensuing design equation:

$$g(\underline{x}^d) = 0 \qquad [43]$$

is essentially a relationship among the parameters of the basic variables and gives a minimum requirement type of tool in the hand of the design engineer to ensure target reliability for the design in an indirect manner. Since nominal or characteristic values of basic variables are typically used in design, Eq. [43] may be rewritten as:

$$g\left(\frac{x_1^n}{\gamma_1}, \ldots, \frac{x_k^n}{\gamma_k}, \gamma_{k+1} x_{k+1}^n, \ldots, \gamma_m x_m^n\right) \geq 0 \qquad [44]$$

where the superscript n indicates the nominal value of the variable. We have partitioned the vector of basic variables into k resistance type and $m - k$ action type quantities. The partial safety factors, γ_i, are typically greater than one: for resistance type variables they divide the nominal values while for action type variables they multiply the nominal values to obtain the design point:

$$\begin{aligned}&\text{resistance PSFs}: \gamma_i = \frac{x_i^n}{x_i^d}, \quad i = 1,\ldots,k \\ &\text{action PSFs}: \gamma_i = \frac{x_i^d}{x_i^n}, \quad i = k+1,\ldots,m\end{aligned} \qquad [45]$$

If the design equation ([44]) can be separated into a strength term and a combination of load-effect terms, the following safety checking scheme may be adopted for design:

$$R_n\left(\frac{S_i^n}{\gamma_i^s}, i = 1,\ldots,k\right) \geq l\left(\sum_{i=1}^{m-k} \gamma_i^q Q_i^n\right) \qquad [46]$$

where R_n = the nominal resistance and a function of factored strength parameters, l = load-effect function, S_i^n =nominal value of i^{th} strength/material parameter, γ_i^s = i^{th} strength/material factor, Q_i^n = the nominal value of the i^{th} load and γ_i^q = i^{th} load factor. Note that there is no separate resistance factor multiplying the nominal resistance (as in LRFD) since material partial safety factors have already been incorporated in computing the strength.

The nominal values generally are fixed by professional practice and thus are inflexible. Some of the *m* partial safety factors (often those associated with material properties) can also be fixed in advance. The remaining PSFs can be chosen by the code developer so as to locate the design point, and hence locate the limit state as alluded to above, and hence achieve a desired reliability for the structure.

4.2 Calibration of partial safety factors

By normalizing the limit state with the design equation in a two variable problem, the reliability problem can be written as:

$$\begin{aligned}&\text{Find } \gamma_1^s,\ldots,\gamma_k^s,\gamma_1^q,\ldots,\gamma_{m-k}^q \text{ such that} \\ &P\left[\frac{C}{C^n(\gamma_1^s,\ldots,\gamma_k^s)} - \frac{Q}{Q^n(\gamma_1^q,\ldots,\gamma_{m-k}^q)} \leq 0\right] = \Phi(-\beta_T)\end{aligned} \qquad [47]$$

where β_T is the target reliability index, C is the random capacity and C^n is its nominal value. Of course, this is an under-defined problem and even though some of the PSFs may be fixed in advance as stated above, it has an infinite number of solutions. Additional considerations are needed to improve the problem definition. Such considerations naturally arise when PSFs are needed to be "optimized" for a class of structures and are discussed next.

It is common to expect that the design equation be valid for *r* representative structural components (or groups). Let w_i be the weight (i.e., relative importance or relative frequency) assigned to the i^{th} such component (or group). These *r* representative components may differ from each other on account of different locations, geometric dimensions, nominal loads, material grades etc. For a given set of PSFs, let the reliability of the i^{th} group be β_i. Choosing a new set of PSFs gives us a new design, a new design point, and consequently, a different reliability index. If there has to be one design equation, i.e., one set of PSFs, for all the *r* representative components, the deviations of all β_i's from β_T must in some sense be minimized. The design equation [44], when using the optimal PSFs obtained this way, can ensure a nearly uniform reliability for the range of components. Several constraints may be introduced to the optimization problem to satisfy engineering and policy considerations (as summarized in [81]). Moreover, some partial safety factors, such as those on material strengths, may be fixed in advance as stated above. The PSF optimization exercise has the following form:

$$\begin{aligned}
&\min\left[\sum_{i=1}^{r} w_i\left(\beta_i\left(\gamma_1^q,\ldots,\gamma_{m-k}^q\right)-\beta_T\right)^2\right] \text{ where } \sum_{i=1}^{r} w_i = 1\\
&\text{subject to: } \min(\beta_i) > \beta_T - \Delta\beta, \qquad i = 1,\ldots,r\\
&\gamma_i^{\min} \le \gamma_i^q \le \gamma_i^{\max}, \quad i = 1,\ldots,m-k\\
&\gamma_i^s = m_i, i = 1,\ldots,k
\end{aligned} \qquad [48]$$

The weighted squared error from the target reliability index over all groups is minimized while ensuring that the lowest reliability among all the groups does not drop by more than $\Delta\beta$ below the target. The material PSFs are fixed while the load PSFs have upper and lower limits.

5 Bridge life cycle cost and optimization

In life cycle cost analysis of bridges, cost to owners ("agency") as well as the public ("users") need to be taken into account[82]. Agency costs include design, construction, maintenance, repair and replacement (less salvage value). If failure occurs, then costs may include compensation, clean up etc. For users, costs arise from accidents, delay and detours. Since some costs are fixed (i.e., deterministic) while some are outcome dependent (i.e., random), the total cost (i.e., life cycle cost),

$$C_T = C_I + \sum_{n_i} C_M(t_i) + \sum_{n_j} C_U(t_j) + C_F \qquad [49]$$

is probabilistic in nature. C_F is either the replacement cost (C_{rep}) at the end of life, or the failure cost (C_f) which occurs at some random instant T_f. The maintenance and user costs, C_M and C_U, are also uncertain as they depend on future loading and aging effects and whether the bridge fails before design life or not. Hence, the total expected cost can be written as:

$$E[C_T] = C_I + \sum_{n_i} E[C_M(t_i)I(t_i)] + \sum_{n_j} E[C_U(t_j)I(t_j)] + (1-P_f)C_{rep} + P_f C_f \qquad [50]$$

The indicator function I verifies if the bridge has survived up to the indicated time or not. Discounting of future costs can also be included[83]. In a decision context, the total expected cost is minimized, subjected to constraints like available budget, target reliability etc.

Decisions regarding new design as well as maintenance therefore require explicit determination of the bridge's time-dependent reliability function which is discussed next.

5.1 Time dependent structural reliability

5.1.1 Descriptors of the time to failure

Let *T* denote the random time to failure (or TTF, also known as failure-free operating time, or life-time) of an item. The reliability function Rel(*t*) evaluated at time t is the probability that the item survives beyond *t* (cf. Eq. [11]):

$$\mathrm{Rel}(t) = P[T > t] = \int_t^\infty f_T(\tau)d\tau \qquad [51]$$

where f_T is the probability density function of *T.* The hazard function, *h*(*t*), which is the conditional density of the TTF, presents the same information differently and can be very useful in revealing unsafe conditions:

$$h(t) = \frac{f_T(t)}{\mathrm{Rel}(t)} \text{ so that } \mathrm{Rel}(t) = \exp\left[-\int_0^t h(\tau)d\tau\right] \qquad [52]$$

Statistics of *T* are routinely obtained for electronic/electrical components through (accelerated) testing programs. This is possible because (i) an abundant number of nominally identical specimens can be obtained, (ii) a large number of test data can be generated in a relatively short time, (iii) tests can be performed in near actual conditions, (iv) tests are not hazardous and (v) tests are relatively inexpensive.

For civil engineering structures, very seldom are all five points above satisfied. Actual failure data are also, thankfully, rare. In the parlance of system reliability, structures constitute active redundant systems with load sharing and dependence – the most difficult type of system to model for reliability analysis.

Nevertheless, time dependent reliability functions are useful for civil engineering systems not only at the new design stage, but also for scheduling future maintenance, for posting load restrictions and for managing life cycle costs as explained above. The reliability function is obtained from the mechanics of the problem where time varying behaviour of some of the basic variables is now brought into the picture explicitly.

5.1.2 Capacity and demand both vary non-randomly in time

Without loss of generality, we look at only one critical location and one failure mode of the structure in Eq. [11]. For multiple critical locations and failure modes, the limit state below can be augmented by unions of individual failure events.

At a given location and for a given failure mode, let the capacity and demand vary deterministically in time:

$$\begin{aligned} C(\tau) &= C_0 \ d(\tau) \\ Q(\tau) &= Q_0 \ h(\tau) \end{aligned} \qquad [53]$$

C_0 and D_0 are random variables, and *d*, *h* are non-random functions of time, $d > 0, h > 0$. That is, if the process $C(\tau)$ is known at any instant t_1, its value can be known precisely at all other instants of time; likewise for $Q(\tau)$. Due to the non-random nature of *d* and *h*, the reliability function,

$$\text{Rel}(t) = P[C_0 \ d(\tau) - Q_0 \ h(\tau) > 0, \text{ for } \ \text{ all } \ \tau \in (0,t]] \qquad [54]$$

can be written as:

$$\text{Rel}(t) = P[C_0 - Q_0 \max_{0<\tau\le t} \frac{h(\tau)}{d(\tau)} > 0] \qquad [55]$$

d is commonly the "aging" function. Its form can be derived from the mechanics of damage growth (e.g., corrosion loss [84], fatigue crack growth[85] etc.) and the loading history. *d*=1 implies the capacity does not degrade with time, and *h*=1 implies the load is sustained in time. The above approach will be still valid for several simultaneously occurring loads (cf. Eq. [12]) if :

$$Q_0 \ h(\tau) = Q_0^{(1)} \cdot h_1(\tau) + Q_0^{(2)} \cdot h_2(\tau) + Q_0^{(3)} \cdot h_3(\tau) + ... \qquad [56]$$

in which the *h*'s are non-random functions of time and the initial load magnitudes $Q_0^{(i)}$ are random variables.

Example 2: We define a time-dependent problem based on Example 1. The cable is subject to uniform corrosion causing its radius, whose initial value r_0 = 4 in, to deteriorate as: $\Delta r(t) = b_1 t^{b_2}$ where $b_1 = 0.1\text{in} / \text{yr}^{b_2}$, $b_2 = 0.9$ are the corrosion law constants. The cross-sectional area thus deteriorates according to: $a(t) = \pi(r_0 - \Delta r)^2$. The cable is made of A36 steel, whose yield strength *Y* is now assumed to be normally distributed with mean μ_Y = 38 ksi and COV V_Y = 15%. The load, Q_0, is invariant and sustained in time, and is now considered a normal random variable. Its mean is μ_Q = 1000 kip and the

COV is $V_Q = 20\%$. In the context of Example 1, the mean bias of the load is then 1000/1600= 0.625. The load and capacity are independent. The reliability function (Eq. [55]) for this problem can be simplified as follows:

$$\begin{aligned}\mathrm{Rel}(t) &= P[Y - Q_0 \max_{0<\tau\le t} \frac{1}{\pi(r_0 - b_1\tau^{b_2})^2} > 0] \\ &= P[Y - Q_0 \frac{1}{\pi(r_0 - b_1 t^{b_2})^2} > 0] \\ &= P[\pi(r_0 - b_1 t^{b_2})^2 Y - Q_0 > 0] \\ &= P[M(t) > 0]\end{aligned} \qquad [57]$$

Note that due to the monotonically decreasing nature of $d(\tau)$, the limit state is evaluated *only at the right end point* of the interval (0,t]. In any other situation this simplification would be wrong and would lead to dangerous overprediction of reliability.

The margin process *M* is normally distributed being a linear combination of normals. Its mean and variance at time *t* are:

$$\begin{aligned}\mu_M(t) &= a(t)\mu_Y - \mu_Q \\ \sigma_M^2(t) &= a^2(t)\sigma_Y^2 + \sigma_Q^2\end{aligned} \qquad [58]$$

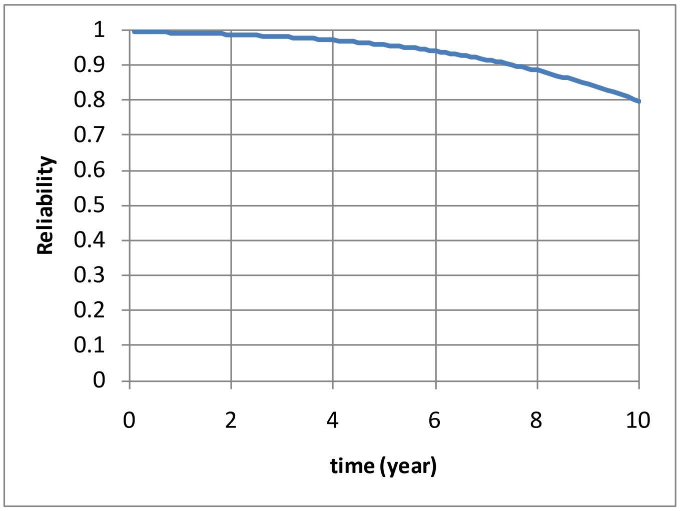
The reliability function therefore can be expressed as the normal CDF:

$$\mathrm{Rel}(t) = \Phi\left(\frac{\mu_M(t)}{\sigma_M(t)}\right) \qquad [59]$$

Differentiating the reliability function leads to the hazard function:

$$h(t) = -\frac{\phi\left(\frac{\mu_M(t)}{\sigma_M(t)}\right)}{\Phi\left(\frac{\mu_M(t)}{\sigma_M(t)}\right)} \frac{\dot{\mu}_M(t)\sigma_M(t) - \mu_M(t)\dot{\sigma}_M(t)}{\sigma_M^2(t)} \qquad [60]$$

These two functions are plotted in Figure 4. The choice of normal distribution for both random variables in the problem led to the closed form expressions for reliability and hazard functions above. For other distributions, FORM or Monte Carlo simulations may be adopted.



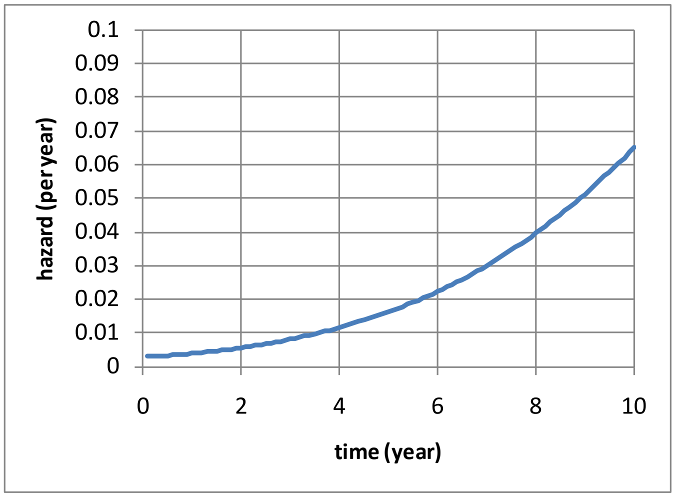


Figure 4: Reliability and hazard functions of corroding cable

5.1.3 Load occurs as a pulsed sequence with random magnitudes

5.1.3.1 Known number of load pulses and no aging

We first consider the case when *C* is time invariant (i.e., $d \equiv 1$ in Eq. [53]) and the load occurs as pulses of random magnitude $Q_1, Q_2, ..., Q_{n(t)}$ with the number of load pulses *n* in time *t* being known. We assume that the loads are IID, that is Q_i's are mutually independent and each Q_i has the same distribution F_Q. Further, the loads are independent of the capacity. The reliability function,

$$\mathrm{Rel}(t) = P[Q_1 < C_0, Q_2 < C_0, Q_3 < C_0, ..., Q_{n(t)} < C_0] \qquad [61]$$

can be simplified by first conditioning it on an arbitrary value of C_0, and using the IID property of the Q_i's:

$$\mathrm{Rel}(t \mid C_0 = c) = [F_Q(c)]^{n(t)} \qquad [62]$$

T he total probability theorem is then applied to yield:

$$\mathrm{Rel}(t) = \int_0^\infty [F_Q(c)]^{n(t)} f_{C_0}(c)\, dc \qquad [63]$$

5.1.3.2 Q is a Poisson pulse process and no aging

A *point process* $N(t)$ on the line $\mathbb{R}^+ = [0,\infty)$ is a set of randomly occuring points such that (i) any finite interval contains a finite number of points with probability 1, and (ii) the number of points in disjoint intervals is the sum of the individual counts[86]. The points are commonly designated as "arrival times": $T_1, T_2, \ldots, T_i \geq 0$. The inter arrival times are $\tau_1 = T_1, \tau_2 = T_2 - T_1, \ldots$ so that $T_n = \tau_1 + \tau_2 + \cdots + \tau_n$. The point process can be described by the joint distribution of (i) the arrival times, (ii) the interarrival times, or (iii) the increments in disjoint intervals. N(t) is a renewal process if the interarrival times are mutually independent and identically distributed. A renewal process is Poisson if the interarrival times are exponentially distributed, or equivalently, if the increments in disjoint intervals are independent.

A Poission process $N(t)$ is completely defined by its rate of occurrence, λ. The Poisson random variable, N_t, with its mean being equal to λt, represents the number of arrivals in the Poisson process N(t) in the interval (0,t]. The joint distribution of the interarrival times T_1, T_2, …T_n given $N(t) = n$ is [87]:

$$f_{T_1,T_2,\ldots,T_n|N(t)=n}(t_1,t_2\cdots t_n)=\begin{cases}\dfrac{n!}{t^n} & 0<t_1<\ldots<t_n<t \\ 0, & \text{otherwise}\end{cases} \qquad [64]$$

We generalize the above situation and consider the loads to occur according to a Poisson pulse process (with rate λ). As before, the magnitude of the pulses are IID and independent of capacity. No aging is considered. Since the number of pulses in time interval (0,t] is random, the reliability function is expressed as:

$$\begin{aligned}\mathrm{Rel}(t) &= \sum_{n=0}^{\infty} P\left[\bigcap_{i=1}^{n} Q_i < C_0 \middle| N(t)=n\right] P[N(t)=n] \\ &= \int_{c=0}^{\infty} \sum_{n=0}^{\infty} P\left[\bigcap_{i=1}^{n} Q_i < c \middle| N(t)=n, C_0=c\right] P[N(t)=n] f_{C_0}(c)dc\end{aligned} \qquad [65]$$

By using the form of the Poisson PMF, the reliability function simplifies to:

$$\mathrm{Rel}(t)=\int_0^{\infty} e^{-\lambda t(1-F_Q(c))} f_{C_0}(c)\,dc \qquad [66]$$

5.1.3.3 Q is a Poisson pulse process and structure ages deterministically

We now introduce aging, as in Eq. [53]. Figure 5 shows a schematic of this situation. Since the loads occur as a Poisson pulse, the occurrence times, T_i, are random in nature, and the individual limit states are evaluated at these random instants of time:

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$$\mathrm{Rel}(t) = \sum_{n=0}^{\infty} P\left[\bigcap_{i=1}^{n} Q_i < C_0\, d(T_i) \middle| N(t)=n\right] P[N(t)=n] \qquad [67]$$

Since these random occurrence times are ordered, $T_1 < T_2 < \ldots < T_i < T_{i+1} < \ldots$, their conditional joint PDF given that *n* pulses occurred in (0,t] is $1/t^n$ (cf. Eq [64]). The reliability function, conditioned on a fixed value of C_0, then can be written as:

$$\mathrm{Rel}\ (t \mid C_0 = c) = \sum_{n=0}^{\infty} \iiint_{\text{all}\ \tau_i} P\left[\bigcap_{i=1}^{n} Q_i < c\, d(\tau_i) \middle| N(t)=n, T_i=\tau_i, T_i<T_j, 1\le i<j\le n\right] \times f_{\underline{T}}(\underline{\tau})\, d\underline{\tau}\, P[N(t)=n] \qquad [68]$$

$$= \sum_{n=0}^{\infty}\left[\frac{1}{t}\int_{\tau=0}^{t} F_Q\left[c\, d(\tau)\right] d\tau\right]^n P[N(t)=n]$$

By using the form of the Poisson PMF, and removing the conditioning on C_0, the reliability function simplifies to:

$$\mathrm{Rel}(t) = \int_0^{\infty} e^{-\lambda t\left(1-\frac{1}{t}\int_{\tau=0}^{t} F_Q\left[c\, d(\tau)\right] d\tau\right)} f_{C_0}(c)\, dc \qquad [69]$$

Note that Eq [69] reduces to Eq [66] when *d* is identically equal to 1.

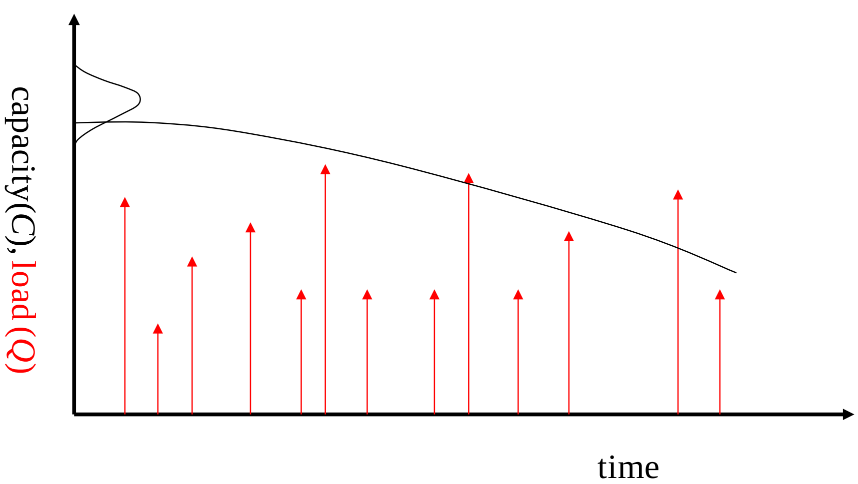

Figure 5: Deteriorating capacity and Poisson pulse loads with random magnitudes

5.1.4 Load and capacity both vary randomly in time

This is the most general case and constitutes a first passage problem [88, 89]. The rate at which the margin process $M(\tau) = C(\tau) - Q(\tau)$ crosses the zero barrier (i.e., enters or leaves the "safe" domain) at an arbitrary time *t* is given by the joint PDF of the process and its derivative, $\dot{M}$ at that instant:

$$\overline{\nu}_0(t) = \int_{-\infty}^{\infty} |\dot{m}(t)| \, f_{M(t)\dot{M}(t)}(0, \dot{m}) d\dot{m} \qquad [70]$$

If the margin process is statistically stationary, the passages into the unsafe domain becomes asymptotically Poisson, so that the reliability function represents the probability of the first passage into the unsafe domain beyond time *t*:

$$\mathrm{R}(t) = \left(1 - F_T(0)\right) e^{-\nu_0^- t} \qquad [71]$$

where $F_T(0)$ is the probability that the margin is negative at *t* = 0. In this stationary case, the constant rate of downcrossing (into the unsafe domain) is:

$$\nu_0^- = \int_0^{\infty} \dot{m} f_{M\dot{M}}(0, \dot{m}) d\dot{m} \qquad [72]$$

Further, if the margin is stationary Gaussian, it is independent of its derivative at the same instant, and the downcrossing rate becomes:

$$\nu_0^- = \frac{\sigma_{\dot{M}}}{\sqrt{2\pi}} \frac{1}{\sigma_M} \phi\left(\frac{\mu_M}{\sigma_M}\right) \text{ if } M \text{ is stationary Gaussian} \qquad [73]$$

5.2 Reliability based maintenance of bridges

Reliability-based maintenance of non-repairable systems is preventive in nature, as opposed to corrective maintenance performed to maintain availability of repairable systems. Consider the reliability function shown in Figure 4. If the target reliability is 0.9 and the remaining life is 10 years, then this item becomes unacceptable in around t_u = 8 years. Four options are available:

1) Replace item by new item at t_u
2) Repair item will before t_u (preventive maintenance)
3) Make a stronger item, so that no repair becomes necessary.
4) Or, restrict loads.

This Section is about the second option. It can be placed in the context of minimizing the total expected cost (Eq. [50]) subject to constraints like budget and reliability. Repair can be either perfect (in which the item is made as new), or partial (only a fraction of original strength is restored). The question is, how is the reliability function altered due to periodic maintenance? In other words, we are looking to describe the conditional reliability $\mathrm{Rel}(t \mid M_{0:t})$ given the maintenance plan, *M*, up to time *t*. Please note that we are still looking into the future when we are trying to predict $\mathrm{Rel}(t \mid M_{0:t})$, i.e., the analyst's position on the time axis is *t* = 0. Thus the conditional reliability function would still have the essential properties of the unconditional reliability, namely, it is a non-increasing function that drops from 1 to 0 with time.

Although not recommended, but as some authors do, one could also add the survival history up to time *t* and repeat the question. The difference is subtle but important. This would happen if the analyst were placed at some point in time in the future, say at t_0, and asked how the reliability function would behave henceforth. That is, one would estimate the conditional reliability $\mathrm{Rel}(t \mid M_{0:t}, S_{0:t})$ where S gives the survival information up to time *t*. The plot of $\mathrm{Rel}(t \mid M_{0:t}, S_{0:t})$ would no longer behave monotonically, but would jump to 1 at each discontinuous point t_0 where the structure is known to have survived. It is easy to show that this jump would happen even in the absence of any maintenance operation, but just due to the fact that the structure survived up to t_0. $\mathrm{Rel}(t \mid M_{0:t}, S_{0:t})$ is not a reliability function in the strict sense, rather it is a piecewise juxtaposition of several reliability functions, and must be interpreted cautiously.

To illustrate, we assume that only one maintenance operation is performed on the structure, which occurs at time t_R. It is convenient to start with the hazard function. It is altered due to the maintenance operation:

$$h(t) = \begin{cases} h_0(t), \ t < t_R \\ h_1(t), \ t \geq t_R \end{cases} \qquad [74]$$

The reliability function (Eq [52]) then becomes:

$$\mathrm{Rel}(t) = \begin{cases} \mathrm{Rel}(t), & t < t_R \\ \mathrm{Rel}(t_R)\ \exp\left[-\int_{t_R}^{t} h_1(\tau) d\tau\right], & t \geq t_R \end{cases} \qquad [75]$$

If perfect repair is undertaken at t_R, then the hazard function undergoes a time shift:

$$\text{Perfect repair at } t_R: \qquad h_1(t) = h_0(t - t_R), \quad t \geq t_R \qquad [76]$$

and the reliability function is repeated as a scaled version of itself:

$$\text{Perfect repair at } t_R: \quad \mathrm{Rel}(t \mid M_{t_R}^{100\%}) = \begin{cases} \mathrm{Rel}(t), & t < t_R \\ \mathrm{Rel}(t_R) \cdot \mathrm{Rel}(t - t_R), & t \geq t_R \end{cases} \quad [77]$$

The event $M_{t_R}^{100\%}$ signifies 100% repair at time t_R. We repeat example 2 above with 100% repair performed at 5 years (the red lines in Figure 6). It is clear that due to the repair, the reliability function stays above 0.9 at the end of the 10 year life as required. However, note that the reliability function never increases with time: at t_R, its slope changes to a more benign value due to repair.

Generalizing, if the repair is imperfect, we start with the second factor in Eq. [75] for $t \geq t_R$ and rewrite it as:

$$\begin{aligned} \exp\left[-\int_{t_R}^{t} h_1(\tau) d\tau\right] &= \exp\left[-\int_{0}^{t-t_R} h_1(\tau + t_R) d\tau\right] \\ &= \exp\left[-\int_{0}^{t-t_R} h_1^{'}(\tau) d\tau\right] \\ &= \mathrm{Rel}^{'}(t - t_R) \end{aligned} \quad [78]$$

where $h_1^{'}$ is a legitimate hazard function (generally different from h_0 due to the imperfect nature of the repair) and Rel'(t) is the corresponding reliability function which is generally different from (and less benign than) Rel(t). The reliability function due to imperfect repair can then be written as:

$$\text{Imperfect repair at } t_R: \quad \mathrm{Rel}(t \mid M_{t_R}^{\alpha\%}) = \begin{cases} \mathrm{Rel}(t), & t < t_R \\ \mathrm{Rel}(t_R) \cdot \mathrm{Rel}^{'}(t - t_R), & t \geq t_R \end{cases} \quad [79]$$

$M_{t_R}^{\alpha\%}$ represents imperfect repair at time t_R in which the strength is restored to α% of the initial value. The green lines in Figure 6 correspond to α = 90. The effect is not as good as perfect repair, as can be expected.

If in addition, the condition is imposed that the structure is found to survive at t_R, then the conditional reliability starts from 1 at t_R as stated before, and all past information is erased:

$$\begin{aligned} \mathrm{Rel}(t \mid T > t_R, M_{t_R}^{\alpha\%}) &= \frac{P[T > t \mid M_{t_R}^{\alpha\%}]}{P[T > t_R \mid M_{t_R}^{\alpha\%}]}, \quad t \geq t_R \\ &= \frac{\mathrm{Rel}(t \mid M_{t_R}^{\alpha\%})}{\mathrm{Rel}(t_R)}, \quad t \geq t_R \\ &= \mathrm{Rel}^{'}(t - t_R), \quad t \geq t_R \end{aligned} \quad [80]$$

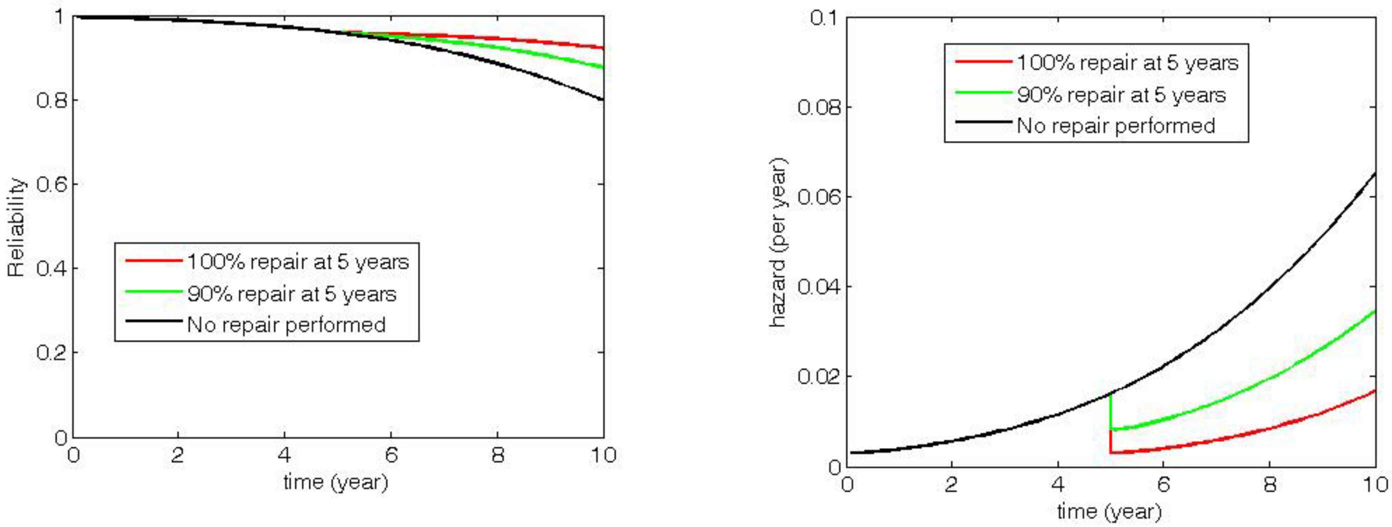

Figure 6: Effect of perfect and partial repair on reliability and hazard functions

6 Load and resistance factor rating methodology

As part of periodic inspection, a bridge may need to be rated for load carrying capacity. Load rating a bridge gains urgency in the face of changed traffic pattern, or due to any change in health of the bridge. When load rating a bridge, the best model is the bridge itself. By monitoring the bridge, one can gather in-service traffic and performance data and conduct in-service evaluations.

NCHRP Project 12-46 [90] led to the development of a reliability based bridge rating Manual [91] that was consistent with AASHTO's reliability-based LRFD approach for design of new bridges. The method was termed load and resistance factor rating (LRFR), and like LRFD, LRFR specifications were still based on design parameters and non-site-specific data. Nevertheless, they did open the door for using site specific information to load rate bridges, e.g., by using weigh-in-motion data and obtaining site specific live load factors. The recent AASHTO Manual for Bridge Evaluation [92] includes the older deterministic allowable stress and load factor rating methodologies, in addition to the modern LRFR approach for condition evaluation of bridges.

The load rating equation for existing bridges is of the general form [93]:

$$RF = \frac{\phi C_n - \gamma_D D_n}{\gamma_L (L_n + I_n)} \qquad [81]$$

where C_n is the nominal capacity, Dn is the nominal dead load, L_n is the nominal live load and I_n is the nominal impact. ϕ, γ_D, γ_L are respectively the capacity, dead load and live load factors. The rating may be performed at various live load levels – inventory, operating etc. The factors may be derieved from probabilistic considerations.

It may be relatively time consuming and expensive to inspect and instrument every bridge in a jurisdiction's inventory [94]. If in-service response from a limited number of sites can be deemed representative of a larger suite of bridges, the rating factors can be "optimized" for the entire suite of bridges (similar to the principle applied in LRFD and LRFR), and bridge owners may determine the safety of bridges in their inventory using such optimized rating equations. The factors can be adjusted to take care of aging [95] and system[96] effects.

7 Summary

Various sources of uncertainty affect a bridge structure during its life: first at the design stage, then during construction, and then throughout its useful life after it has been put into service. These uncertainties are modeled as random variables, random processes or random fields as appropriate. Performance requirements of a bridge are described in terms of limit state functions. The exceedance probabilities of these limit states, i.e., the probabilities of non-performance, need to be kept within acceptable limits. Reliability-based design and maintenance, whether through first principles or by using codes of practice, can ensure compliance. There are various methods of deciding acceptable failure probabilities (or, equivalently, target reliabilities). Once the bridge is put into service, its load characteristics may change and the structure may be subjected to various forms of (generally random) deterioration. Time dependent reliability analyses of an aging bridge, coupled with preventive maintenance, can ensure that reliability does not fall below acceptable limits. A suit of bridges can be rated in service by optimized site-specific partial safety factors.

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