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Reliability-based partial safety factors for dual performance level design of prestressed inner containment shells in Indian nuclear power plants

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HIGHLIGHTS

- We develop reliability based partial safety factors for design of prestressed containments.
- Two limit states – cracking and collapse – are considered and derived from first principles.
- The PSFs are optimized for all structural groups and explicitly satisfy target reliabilities.
- Detailed numerical example on design of a typical 220 MWe Indian PHWR is provided.

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ABSTRACT

Partial safety factors (PSFs) used in reliability-based design are intended to account for uncertainties in load, material and mathematical modeling while ensuring that the target reliability is satisfied for the relevant class of structural components in the given load combination and limit state. This paper describes the methodology in detail for developing a set of optimal reliability-based PSFs for the design of prestressed concrete inner containment shells in Indian NPPs under Main Steam Line Break (MSLB)/Loss of Coolant Accident (LOCA) conditions at two performance levels in flexure: cracking and collapse. The methodology follows current design practices in the country, accounts for uncertainties in loads and material properties and dependence among capacities and demands, develops the limit states from first principles, explicitly lays down the target reliabilities and criteria for PSF optimization. The optimization of the PSFs is based on reliability indices for each representative group of components obtained from importance sampling and a local linear response surface fit. A detailed numerical example on a typical 220 MWe Indian PHWR demonstrating the methodology is provided.

1. Introduction

Containment structures used in nuclear power plants constitute the ultimate barrier to the emission of radioactive elements in the case of an internal accident or an external hazard or hostile event. Containments can in general be either a single structure with a metallic liner, or a double walled structure with or without a metallic liner (the latter having evolved from the French design codes). Most of the recent containments are shell-type structures made of prestressed concrete.

The design of containment shells for Indian Pressurized Heavy Water Reactors (PHWRs) has evolved over the years, originating from a steel cylindrical shell capped with a steel dome (CIRUS Reactor, Trombay), followed by the use of reinforced concrete walls and pre-stressed concrete dome (Rajasthan Atomic Power Station) to the use of pre-stressed concrete for the entire shell (Madras Atomic Power Station) and pre-stressed concrete double containment shells (first employed in the Narora and Kakrapar Power Stations). The Kaiga and Rajasthan Atomic Power Plants marked a further improvement in the design philosophy with complete double containment shells having independent domes (Roy and Verma, 2004). The inner containment shells used in recent PHWRs are cylindrical structures of 63 m height, with prestressed concrete spherical domes containing 4 openings to facilitate the replacement of steam generators (Ray et al., 2003). Until recently, nuclear containment structures in India were designed using the French RCC-G code. The raft of the PWHR at Tarapur was designed using the ASME code and checked against RCC-G (Roy and Verma, 2004). There is yet no formal Indian design standard for containment

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structures. In 2007, the Atomic Energy Regulatory Board (AERB) of India released the CSE-3 codes (AERB, 2007) which is currently under review. The design philosophy adopted from RCC-G follows the limit state concept at two performance levels (serviceability and collapse). Structural analysis methodologies used to carry out the design procedure focus chiefly on membrane stresses acting on the shell structure of the cylindrical wall and spherical dome with due consideration given to stresses in the radial direction (along the thickness) due to sudden thickness changes and embedding of prestressing cables.

Significant uncertainties exist in the structural behavior of the IC Shells of PHWRs, arising out of the random nature of material, geometry, prestressing and loadings. As early as 1974, Shinozuka and Shao (1974) conducted a probabilistic assessment of prestressed concrete pressure vessels using the first order second moment approximation. Uncertainties in loads and in the material and geometry of the vessels were considered while short term accidental load effects were modeled as Poisson Processes. The uncertainty associated with the resistance of containment shells arises out of uncertainties in the strengths of concrete and steel, in long-term prestress losses and in other aging effects, as well as in the shell geometry. Uncertainty in the long term behavior of these structures is highly variable owing to material changes (for example, prestress loss in tendons and creep in concrete) and the occurrence of accidental events (Hwang et al., 1985; Pandey, 1997). While concrete strength has been found to be better controlled in the nuclear power plant industry than in the ordinary building industry, steel strength variability does not display a noticeable reduction. Variability in sectional dimensions is comparatively quite low and has negligible impact on the overall uncertainty in structural resistance (Hwang et al., 1985). Different loads have different degrees of randomness and may entail appropriate adjustments in the probabilistic framework, for example, the variability of dead load being substantially lower than that of an accidental pressurization load, the former can be treated as a deterministic quantity for simplification of analysis (Hwang et al., 1985) or neglected altogether if dead load magnitudes are insignificant compared to pressure loads (Hwang et al., 1985).

It is most rational to treat uncertainties associated with parameters governing the design and construction of a structure in a probabilistic format, specifically, to model the time-invariant quantities as random variables and the time-dependent ones as stochastic processes. Recognizing the existence of these uncertainties is an admission of the fact that the structure may not always satisfy its performance and safety objectives during its intended design life. The logical extension of this admission is to ensure that the likelihood of unsatisfactory performance be kept acceptably low during the life of the structure.

The subject of structural reliability provides the tools and methodologies to explicitly determine the probability of such failures ("failure" here in the sense of non-compliance or non-performance) by taking into account all relevant uncertainties. These techniques can be used to design new structures with specified (i.e., target) reliabilities, and to maintain existing structures at or above specified reliabilities. Formulation of the reliability problem and target reliabilities is discussed in the next section. Even though such computed probabilities of failure (reliability being 1 minus failure probability) may not have a frequentist or actuarial basis, structural reliability provides a neutral and non-denominational basis to compare different (and often disparate) designs and maintenance strategies on a common basis.

Structural reliability methods are also important in establishing successful performance of NPP structural components or systems during rare events for several reasons. The mechanical and electrical components of NPP's are easily and frequently tested in-service unlike structural components. Additionally, they are often identical for most NPP's in contrast to structural components that are usually unique and plant-specific. Structural systems also stay passive under most conditions, are often inaccessible to inspection and it may be impossible to replace them economically (Panel, 1997).

This paper describes the methodology in detail for developing reliability-based partial safety factors (PSFs) for the design of prestressed concrete inner containment shells in Indian NPPs under MSLB/LOCA conditions both in strength (i.e., collapse) and serviceability (i.e., cracking) limit states. These PSFs are "optimized" so as to be applicable for a range of structural components (differentiated by importance, location, load magnitudes, geometries, etc.) while ensuring the required target reliability for the given limit state. Basics of structural reliability formulation and mechanics of prestressed sections are described next. Numerical examples involving a typical 220 MWe Indian PHWR are provided.

2. Background

2.1. Formulation of structural reliability problems

A limit state function (or performance function), $g(\underline{X})$, for a structural component is defined in terms of the basic variables, $\underline{X}$, such that:

$$\begin{aligned} &g(\underline{X}) < 0 \text{ denotes failure} \\ &g(\underline{X}) > 0 \text{ denotes satisfactory performance} \end{aligned} \tag{1}$$

and the surface given by

$$g(\underline{X}) = 0 \tag{2}$$

is called the limit state equation or limit state surface. The performance function g is typically obtained from the mechanics of the problem at hand. For multiple failure modes or if there are multiple critical sections, Eq. (2) is generalized to an appropriate union of failure events.

The basic variable generally comprise of quantities like material properties, loads or load-effects, environmental parameters, geometric quantities, modeling uncertainties, etc. mentioned above. Those basic variables with negligible uncertainties may be treated as deterministic. The general expression of failure probability is

$$P_{\mathrm{f}} = 1 - Rel = P(g(\underline{X}) < 0) = \int_{g(\underline{x})<0} f_{\underline{X}}(\underline{x})\mathrm{d}\underline{x} \tag{3}$$

where $f_{\underline{X}}(\underline{x})$ is the joint probability density function for $\underline{X}$ and Rel is the reliability of the component.

Like any other design approach, reliability based design is an iterative process: the design is adjusted until adequate safety is achieved and cost and functional requirements are met. The final step of meeting the target reliability can either be direct where the computed structural reliability has to exactly satisfy the target reliability for each relevant limit state or it can be indirect as in partial safety factors (PSF) based design where the structure implicitly satisfies the target reliability within a certain tolerance (Bhattacharya et al., 2001). The term load and resistance factor design (LRFD) refers to the approach followed in the United States where the nominal resistance in the design equation is multiplied by an explicit "resistance factor" but the nominal material properties that go into determining the resistance are not factored. The term PSF based design implies the approach taken in Europe where there is no explicit resistance factor in design, but each material property generally has its own partial safety factor. The latter approach is taken in this work.

Closed-form solutions to Eq. (3) are generally unavailable. Two different approaches are widely in use: (i) analytical methods based on constrained optimization and normal probability approximations and (ii) simulation based algorithms with or without variation reduction techniques. Both can provide accurate and efficient solutions to the structural reliability problem. The first kind, grouped under First Order Reliability Methods (or FORM), holds an advantage over the simulation based methods in that the design point(s) and the sensitivity of each basic variable can be explicitly determined. However, FORM can prove to be costly or even infeasible if the size of the reliability problem goes up (in terms of basic variables and/or number of limit states) or if the limit state is not analytic in the basic variables, and FORM was not found suitable for this work. Monte Carlo simulations with Importance Sampling have been used here to compute failure probabilities.

2.2. Target reliability

It has become increasingly common to express safety requirements, as well as some functionality requirements, in reliability based formats. A reliability based approach to design, by accounting for randomness in the different design variables and uncertainties in the mathematical models, provides tools for ensuring that the performance requirements are violated as rarely as considered acceptable.

The cause, reference period, and consequences of violation of different performance requirements may vary, and if a reliability approach is taken, the target reliability in each performance requirement must take such difference into account (ISO, 1998; Bhattacharya et al., 2001; JCSS, 2001a,b; Wen, 2001). For example, if the structure gives appropriate warning before collapse, the failure consequences reduce and that in turn can reduce the target reliability for that mode (DNV, 1992; JCSS, 2001a,b). Functionality target reliabilities may be developed exclusively from economic considerations. The safety target reliability levels required of a structure, on the other hand, cannot be left solely to the discretion of the owner, or be derived solely from a minimum total expected cost consideration, since structural collapse causing a large loss of human life and/or property, even if an "optimal" solution in some sense, may not be acceptable either to the society or the regulators. Design codes, therefore often place a lower limit on the reliability of safety related limit states (Galambos, 1992; Bhattacharya et al., 2001). For optimizing a structure with multiple performance requirements, Wen et al. (1996) suggested minimizing the weighted sum of the squared difference of the target and actual reliabilities.

ISO 2394 (1998), and later JCSS (2001a,b), proposed three levels of requirements with appropriate degrees of reliability: (i) serviceability (adequate performance under all expected actions), (ii) ultimate (ability to withstand extreme and/or frequently repeated actions during construction and anticipated use), (iii) structural integrity (i.e., progressive collapse in ISO 2394 and robustness in JCSS). Target reliability values were suggested based on the consequences of failure for ultimate limit states and relative cost of safety measure for serviceability limit states. The Canadian Standards Association (CSA, 1992) defines two safety classes and one serviceability class (and corresponding annual target reliabilities) for the verification of the safety of offshore structures (i) Safety class 1 – great risk to life or high potential for environmental pollution or damage, (2) Safety class 2 – small risk to life or low potential for environmental pollution or damage, and (3) Serviceability Impaired function and none of the other two safety classes being violated. Det Norske Veritas (DNV, 1992) specifies three types of structural failures for offshore structures and target reliabilities for each corresponding to the seriousness of the consequences of failure. The American Bureau of Shipping (ABS, 1999) identified four levels of failure consequences for various combinations of limit states and component class for the concept Mobile Offshore Base and assigned target reliabilities for each. Ghosn & Moses (1998) suggest three levels of performance to ensure adequate redundancy of bridge structures corresponding to functionality, ultimate and damaged condition limit states, while Nowak et al. (1997) recommend two different reliability levels for bridge structures corresponding to ultimate and serviceability limit states. Nuclear power plant containment structures are designed for earthquakes at two different levels of intensity and correspondingly to two different criteria for failure (USNRC, 1973; E.D.F., 1988; AERB, 2007). Damage, if any, caused by the Operating Basis Earthquake (OBE) must not lead to loss of functionality of the nuclear power plant, whereas the Safe Shutdown Earthquake (SSE) that has a higher intensity and longer recurrence interval than OBE, is allowed to cause the power plant to shutdown but must not cause any radioactive leakage to the environment or loss of structural integrity.

Given the inability to predict the occurrence or magnitude of earthquakes, the uncertainties involved from source to site, and the potential for massive damage, it is not surprising that performance based design (PBD) has been most enthusiastically espoused in the seismic engineering community, as evident in SEAOC (1995), ATC-40 (ATC, 1996) and FEMA 273 and 350 (FEMA, 1997, 2000). Perhaps the earliest work in which uncertainty estimates were used for both ground motion parameters and structural response for nuclear power plants was published in 1980 (Kennedy et al., 1980). Typical PBD procedures for seismic risk analysis of NPP's use response/ground based fragility curves for demand estimation, and time history analyses for capacity estimation, as demonstrated in Huang et al. (2011). Performance levels for seismic design are commonly defined in terms of increasing severities, e.g., (i) Immediate Occupancy (IO), the state of damage at which the building is safe to occupy without any significant repairs, (ii) Structural Damage (SD), an intermediate level of damage in which significant structural and non-structural damage has occurred without loss of global stability, and (iii) Collapse Prevention (CP), representing extensive structural damage that causes global instability (FEMA, 1997; Kinali and Ellingwood, 2007). A comparison of the performance of structures designed to one ultimate design earthquake vs. those designed to dual level performance levels indicated that the latter produces relatively stronger structures (Wen et al., 1996). A similar finding was echoed by Ghobarah (2001) who opined that the reason for the revision of the then design standards to more reliable performance based methods was that after severe earthquakes (such as Northridge and Kobe), while structures designed to the existing codes performed well with respect to safety, the extent of damage and the economic costs were unexpectedly high.

The fact that the consequences of a severe core damage accident are potentially catastrophic led to the concept of "inherently safe reactors" in the 1980s (Cave and Kastenberg, 1991). The maximum acceptable failure probability of such events is governed in part by the need to preserve public confidence in nuclear energy. Cave and Kastenberg (1991) suggested a maximum acceptable probability of the order of 10^{-8} per reactor year for large release of radioactive materials to the atmosphere (whereas the then USNRC limit was 10^{-6} per reactor year), with the limit for structural failure leading to such events being 1×10^{-6} per reactor year. In light of new types of emerging hazards such as terrorist attacks, Kostadinov (2011) has concluded that the severity of consequences of certain very low probability hazards for nuclear power plants has historically been underestimated. Taking into consideration these and other recommendations, we use target reliability levels of 3.5 for collapse and 2.5 for serviceability limit states in this work.

The capacity-demand model used in this work can in principle be expanded to include reliability assessment of nuclear power plants under seismic loads (akin to Cornell et al., 2002). In the context of the present work, it is assumed that failure sequences and acceptable failure probabilities are either known or can be evaluated separately. Uncertainties involved with hazard and fragility, the quantification of which is crucial for inclusion of seismic loads in the present framework, arise from the soil and seismic characteristics of the region, the properties of the structure and its behavior under seismic loads, and the mathematical models used to represent hazard and fragility (Kennedy and Ravindra, 1984; Ravindra, 1990; Baker and Cornell, 2008; Jalayer et al., 2010).

2.3. Reliability of prestressed concrete sections

The tensile strength of concrete is negligible compared to its compressive strength. In ordinary reinforced concrete, the reinforcing steel is used to carry the tensile stresses, and the concrete near the tensile face may crack. Prestressing is intended to artificially induce compressive stresses in the concrete to counteract the tensile stresses caused by external loads, such that the loaded section remains mostly if not entirely in compression (Raju, 2007).

Prestressed concrete (for shells, slabs, girders, etc.) is often adopted when in addition to satisfying strength requirements, the member is also required to be slender (e.g., from aesthetic or weight considerations) and/or to limit cracking (e.g., to satisfy leak-tightness). Prestressed concrete sections may fail in several possible ways (such as a combination of flexure, shear and torsion, bursting of end blocks, bearing, anchorage or connection failures, excessive deflections, etc.). Prestressed concrete members are relatively lightweight as they are built from high strength steel and high strength concrete, more resistant to shear, and can recover from effects of overloading. However, prestressed concrete structures are more expensive, have a smaller margin for error, and the design process of prestressed members is more complicated. Although the loss of prestress with time is built into the design, unintended loss of prestress arising from corrosion of the tendons, slippage, etc. can have catastrophic consequences.

Several reliability based studies on partially prestressed concrete sections have been conducted over the years. Al-Harthy and Frangopol (1994) studied prestressed beams designed to the 1989 ACI 318 standard considering 3 different limit states (ultimate flexure, cracking in flexure and permissible stresses) under random dead and live loads, material and geometric properties, prestressing forces and modeling uncertainty. Their studies concluded that the reliability indices implied by the 1989 ACI 318 standard are non-uniform over various ranges of loads, span lengths and limit states. Hamann and Bulleit (1987) examined the reliability of under reinforced high-strength concrete prestressed beams designed in accordance with the 1983 ACI-318 standard, considering only the ultimate flexural limit state of beams subjected to dead and snow loads. While Al-Harthy and Frangopol (1994) included all the material and geometric random variables in a FORM analysis, Hamann and Bulleit (1987) first estimated the moment capacity through Monte Carlo simulations, fitted the data to standard distributions, and then performed a first order second moment reliability analysis on the linear limit state.

Reliability for Class-1 structures, particularly concrete containment structures for nuclear power plants, is a much researched subject primarily due to the potentially dire failure consequences of the containment structure in terms of environmental impact, radiation effect on human health and other economic costs. Hwang et al. (1985) described an LRFD approach to determine the critical load combinations for design of concrete containment structures. The limit state, corresponding to ultimate strength of concrete, was defined in the 2-D space of membrane stress and bending moment in the shell, leading to an octagonal limit state surface. Han et al. (1991), Varpasuo (1996), Pandey (1997) and Han and Ang (1998) also worked on the reliability of concrete containments, their limit states forming sides of the octagonal limit state considered by Hwang et al. (1985).

3. Reliability analysis and calibration of PSFs

3.1. Mechanics of pre-stressed concrete sections

As stated above, we look at two different limit states in this work: (1) collapse limit state defined as crushing of concrete in compression (reinforcements may yield), (2) cracking limit state defined by cracking of concrete up to a specified depth from the tensile face, e.g., the depth of cover. Bidirectional flexure on shell elements corresponding to nuclear power plant inner containment structures with voids has been considered. The material properties of concrete and steel and the mechanistic formulation of both the limit states are discussed next.

In Indian Standards such as IS 456 (BIS, 2000) the compressive stress-strain relationship for concrete is taken to be parabolic up to a strain of 0.002, and horizontal from that point on. The failure strain of concrete in bending compression is 0.0035. The nominal compressive strength of concrete is taken to be $f_{cn} = f_{ck}/1.5$ in collapse and $f_{cn} = f_{ck}/1.25$ in serviceability where f_{ck} is the 28-day characteristic cube compressive strength. The design compressive strength of concrete is $f_{cd} = f_{cn}/\gamma_c$, where γ_c is the material safety factor on concrete strength. The value of γ_c is usually taken to be 1.5 in strength limit state and 2.0 in serviceability limit state for both normal and abnormal design conditions (Roy and Verma, 2004). IS 1343(BIS, 2000) specifies the minimum grade of concrete as M30 for post-tensioning and M40 for pre-tensioning.

The stress–strain behavior of concrete in tension is linear (BIS, 2003) and the tensile strength is taken to be $f_{ct} = 0.7\sqrt{f_{ck}}$ and the modulus of elasticity of concrete in tension is assumed to be same as the secant modulus of concrete in compression which is $E_c = 5000\sqrt{f_{ck}}$. The maximum tensile strain in concrete is then $\varepsilon_t = f_{ct}/E_c = 0.00012$.

The design yield stress for reinforcing steel is f_{yn}/γ_s where f_{yn} is the nominal yield strength and γ_s is the material safety factor on yield strength of steel and is taken to be 1.15 in strength limit state and 1.8 in serviceability limit state for both normal and abnormal design conditions. The nominal modulus of elasticity of steel, E_n, is 200,000 N/mm^2 and is not factored.

The moment capacity of a partially prestressed concrete section, given the amount of prestressing force and the geometric and material properties can be obtained in the form of an interaction diagram using strain compatibility equations and force balance. Interaction diagrams are plots of normalized compressive force, $P \leftarrow P/(f_{ck}bD)$ and normalized moment capacity, $M \leftarrow M/(f_{ck}bD^2)$ where b and D are the width and the depth of the section, respectively.

Fig. 1 shows the strain and stress diagrams for an example section similar to the ones used in this work – with one set of prestressing tendons and two layers of ordinary reinforcement. In the figure, C is compressive force in concrete, f_{s1} is force in top reinforcement, f_{s2} is force in bottom reinforcement and P is prestressing force.

For given amount of prestress the position of the neutral axis is determined iteratively by balancing the tensile and compressive forces on the section. The moment capacity can then be found by taking the moment of the forces about any convenient point. In determining the collapse moment capacity, two cases are possible (Fig. 2): the neutral axis (NA) outside and the neutral axis inside the section. In the former, the entire section is in compression and in the latter, concrete has cracked and is assumed not to carry any load in the tensile zone.

The cracking limit state is reached when the tensile strain in concrete at depth equal to the cover exceeds ε_t, while the maximum compressive strain ε_c on the opposite edge can lie anywhere between 0 and 0.0035 (Fig. 3), which is determined iteratively, from which the cracking moment capacity is determined.

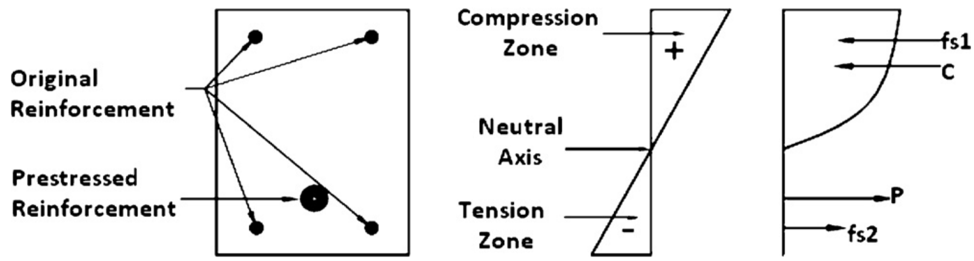

Fig. 1. Force balance and moment computations for partially prestressed section.

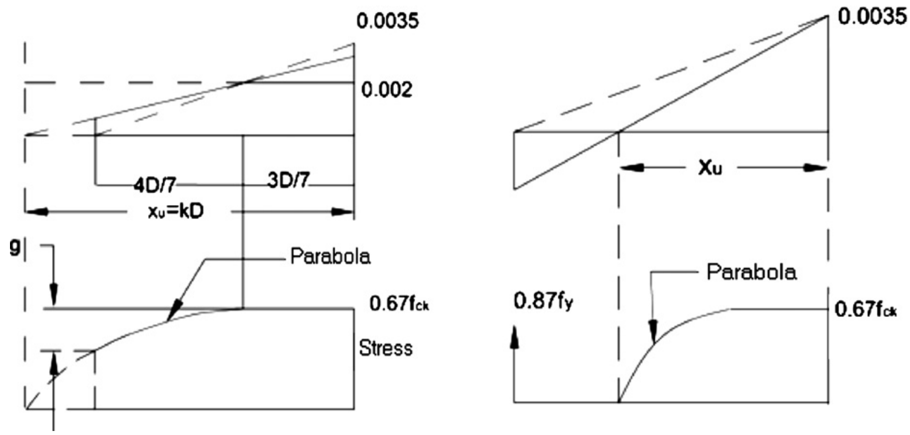

Fig. 2. Strain and stress distributions on section for neutral axis outside (left) and inside (right) the section for limit state of collapse.

Fig. 4 shows an example of the so-called "P–M interaction diagram" – the normalized moment capacity as function of the normalized net inplane compressive force, both for collapse (in red) and cracking (in black) for $p = 0.2\%$, $e/D = 0$, $c/D = d/D = 0.05$, $f_{yn} = 415$ MPa and $f_{ck} = 45$ MPa. The material safety factors are as described above. No voids due to prestressing cables have been considered. As can be expected, the cracking capacity curve is fully contained within the collapse capacity curve. It passes through the origin indicating that in the absence of any compressive force, the section cannot resist any bending moment without cracking. With increasing compressive force the section's cracking moment capacity increases up to a limit, starts decreasing, and then quickly drops to zero. The limiting point marked $e_r = 0.002$ on it corresponds to the situation where the compressive strain on the right face reaches 0.002 (i.e., the stress on the right face reaches its maximum value and that in the right reinforcement comes close to its maximum). Thus the "desirable" range in the cracking P–M curve is clearly well below yield. The collapse moment capacity on the other hand starts with a non-zero value in the absence of any compressive force; the knee of the curve is the balance point where the entire section is used efficiently, the point marked $k = 1$ indicates the instant the neutral axis goes out of the section. Clearly, the "desirable" range in the collapse P–M curve involves substantial cracking of the section.

Fig. 5 shows an example prestressed concrete element corresponding to the shell structure of nuclear power plant inner containment structures. Two layers of ordinary reinforcement top and bottom can be seen and JI and JO correspond to prestressing cables in the North–South and East–West directions, respectively. In the co-ordinate system adopted, these two are considered as the x and y directions.

When calculating the flexural strength of an element such as this, the space taken by the prestressing cable JI has to be considered as a void in concrete, i.e., while calculating the contribution of concrete to the strength the area considered is the total area minus

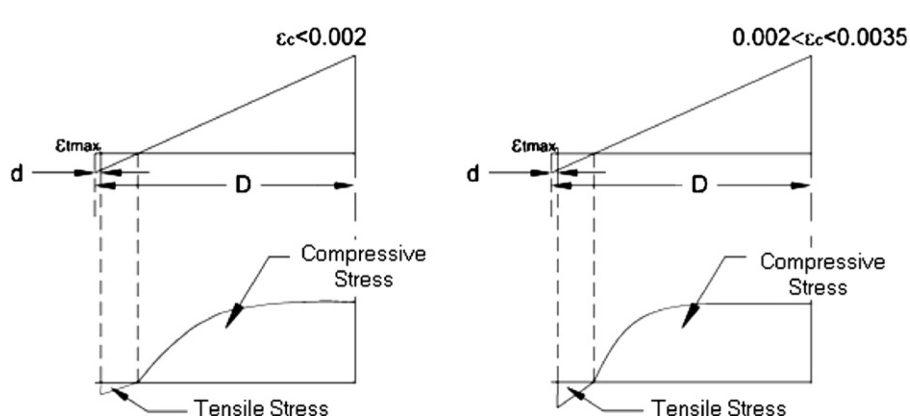

Fig. 3. Strain and stress distributions on section for $\varepsilon_c < 0.002$ (left) and $\varepsilon_c > 0.002$ (right) for limit state of cracking.

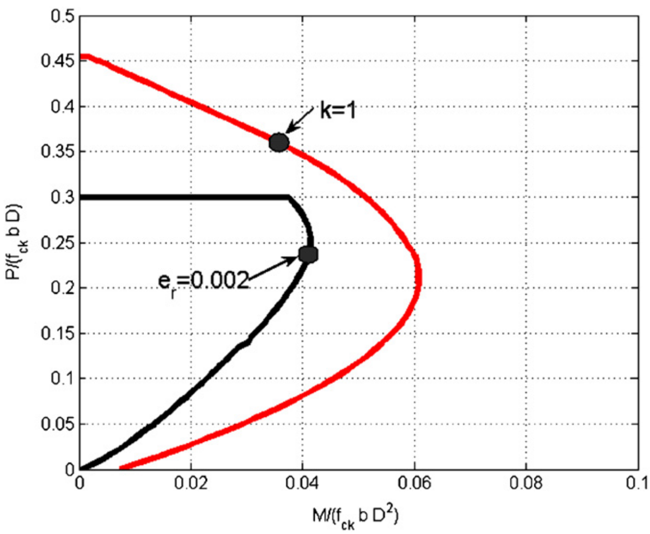

Fig. 4. P–M interaction diagram between inplane compressive force and moment capacity – black (inner) = cracking, red (outer) = collapse. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

the area of the void. For the typical prestressed containment shell, the void depth is approximately 0.15 times the total depth of the element.

The forces and moments acting on the section have two normal/inplane components (*xx* and *yy*) and one shearing/torsional (*xy*) component. Additionally, areas occupied by pre stressing cables are considered as voids in concrete. The section is thus under bi-directional flexural loading.

The load effects in terms of stress resultants for the membrane action (N_{xx} and N_{yy}) are generally obtained along element local coordinates. The moment capacity of the reinforced concrete section depends on this membrane action (in-plane) through P–M interaction. At the same time, the applied moments (M_{xx} and M_{yy}) on the section occur due to the same set of loads. Thus both the capacity of the system and the loading on the system are affected by a common source and therefore the applied moment and the moment capacity will show some degree of correlation.

The moment-based design check is carried out in the principal directions 1 and 2 (principal with respect to the in-plane forces). Applied moments are converted to these two directions according to the basic rules of tensorial transformation. The moment capacities of the section in each of the two principal directions are computed from interaction relations described above. The applied moments in directions 1 and 2 are obtained using Wood (1968) criteria which enhances the induced moments (M_{xx} and M_{yy}) with equivalent components of the torsional moment component M_{xy}.

Since the structural analysis used to compute stress resultants is completely linear in nature, the different load-effect components (N_{xx}, N_{yy}, N_{xy}, M_{xx}, M_{xy}, M_{xy}) in a given load case (e.g., in dead/live/pre-stress, etc. load case) are statistically fully dependent on one another. Additionally, stress resultants due to different load cases are completely independent. The correlation matrix that is developed for analysis is based on these two assumptions.

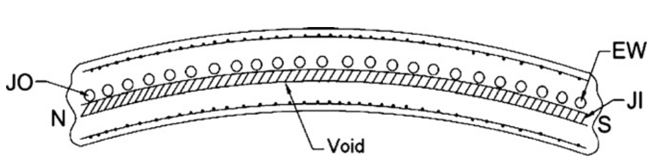

Fig. 5. Prestressed concrete shell.

3.2. Limit states and basic variables

From this point forward, unless otherwise mentioned, all moments are normalized by $f_{ck}bD^2$ and all forces by $f_{ck}bD$. Since this work concerns the reliability of prestressed concrete shells in biaxial flexure, the limit states in principal directions 1 and 2 can be written respectively as

$$g_1 = M_{cap,1} - M_{app,1} = 0 \tag{4}$$

$$g_2 = M_{cap,2} - M_{app,2} = 0 \tag{5}$$

so that failure of the section is given by

$$\{\text{Failure}\} = \{g_1 < 0\} \cup \{g_2 < 0\} \tag{6}$$

and the failure probability can be written as

$$P_f = \int_{\underline{x} \in \{\text{Failure}\}} f_{\underline{X}}(\underline{x})\, d\underline{x} = \int_{\text{all}\,\bar{x}} I\left[\left\{\text{Failure}\right\}\right] f_{\underline{X}}(\underline{x})\, d\underline{x} \tag{7}$$

The indicator function, *I*, evaluates the expression within brackets so that:

$$\text{II}[\bullet] = \begin{cases} 1, & \text{if}\,[\bullet]\ \text{is true} \\ 0, & \text{if}\,[\bullet]\ \text{is false} \end{cases} \tag{8}$$

and is a convenient way to convert the domain of integration from the failure region to the entire range of $\underline{x}$ which is useful in simulation based estimates as described subsequently.

$M_{cap,1}$ and $M_{cap,2}$ are the moment capacities in *x* and *y* directions, respectively. Likewise, $M_{app,1}$ and $M_{app,2}$ are the applied moments. Depending on the limit state in question, the moment capacity corresponds to either cracking or collapse of the section. Although in general the applied moments too can have different specifications at two different performance levels (e.g., the earthquake load in cracking limit state may correspond to an operating basis while that in collapse limit state may correspond to the safe shut down level), we have taken the same definition for the applied moment (and hence the same statistics and same nominal value) in either limit state for each load case (e.g., dead, live, etc.).

Before going into the details of the individual terms above, it is important to recall that the moment capacities and applied moments are mutually statistically dependent since the capacities are functions of the axial loads which in turn are linearly related to the applied moments in each load case. In addition, the capacities in directions 1 and 2 are strongly correlated as they are functions of the same material properties and some of the same axial loads.

The applied moments in the two principal directions, $M_{app,1}$ and $M_{app,2}$, are functions of the applied moments M_{xx}, M_{yy} and M_{xy} caused by all load cases in the load combination at hand. For example, if we have the load combination Dead (D) + Prestressing (P_s) + Ordinary Live (L_o) + Temperature (T) + Accidental Pressure (P_a), the total moments are:

$$\left.\begin{aligned} M_{xx} &= M_{xx}^{D} + M_{xx}^{P_s} + M_{xx}^{L_o} + M_{xx}^{T} + M_{xx}^{P_a} \\ M_{yy} &= M_{yy}^{D} + M_{yy}^{P_s} + M_{yy}^{L_o} + M_{yy}^{T} + M_{yy}^{P_a} \\ M_{xy} &= M_{xy}^{D} + M_{xy}^{P_s} + M_{xy}^{L_o} + M_{xy}^{T} + M_{xy}^{P_a} \end{aligned}\right\} \tag{9}$$

Likewise, the total forces for the same load combination are

$$\left.\begin{aligned} N_{xx} &= N_{xx}^{D} + N_{xx}^{P_s} + N_{xx}^{L_o} + N_{xx}^{T} + N_{xx}^{P_a} \\ N_{yy} &= N_{yy}^{D} + N_{yy}^{P_s} + N_{yy}^{L_o} + N_{yy}^{T} + N_{yy}^{P_a} \\ N_{xy} &= N_{xy}^{D} + N_{xy}^{P_s} + N_{xy}^{L_o} + N_{xy}^{T} + N_{xy}^{P_a} \end{aligned}\right\} \tag{10}$$

It may be noted that the moments and forces in Eqs. (9) and (10) are random variables and partial safety factors are not used for random load combinations. PSFs are multiplied with the corresponding nominal moments to obtain the combined nominal applied moments; however, PSFs are not used with the nominal forces to obtain the nominal moment capacities.

The normalized moment capacity, $M_{\mathrm{cap},j}$, whether in cracking or collapse, in given direction j ($j = 1, 2$), is a function of the principal values (N_1, N_2) of the applied in-plane compression arising from the components of Eq. (10), material properties (f_c, f_y, E, ε_c, ε_t) and geometric quantities (p/f_{ck}, d/D, e/D, t_{void}/D):

$$M_{\mathrm{cap},j} = M_{\mathrm{cap}}\left(N_j, f_c, f_y, E, \varepsilon_c, \varepsilon_t, \frac{p_j}{f_{ck}}, \frac{d}{D}, \frac{e_j}{D}, \frac{t_{\mathrm{void},j}}{D}\right), \quad j = 1, 2 \tag{11}$$

Of these, the random terms are: the applied in-plane compressive forces, N_1 and N_2, the compressive strength of concrete, f_c, the yield strength, f_y, and the Young's modulus, E, of the reinforcing steel. The compressive forces are obtained from a combination of all load cases as explained in the previous section. The nominal or design values of the moment capacities, to be used in design equations discussed below, can be obtained by substituting the random quantities in Eq. (12) by their design values:

$$M^n_{\mathrm{cap},j} = M_{\mathrm{cap}}\left(N_j, \frac{f_{cn}}{\gamma_c}, \frac{f_{yn}}{\gamma_s}, E_n, \varepsilon_c, \varepsilon_t, \frac{p_j}{f_{ck}}, \frac{d}{D}, \frac{e_j}{D}, \frac{t_{\mathrm{void},j}}{D}\right), \quad j = 1, 2 \tag{12}$$

3.3. Monte Carlo simulations and importance sampling

Except in very special situations, closed form solution to the structural reliability problem (Eq. (7)) does not exist and numerical approximations are needed. The true probability of failure, P_f,

$$P_f = \int_{\mathrm{all}\,\underline{x}} I\left[\{\mathrm{Failure}\}\right] f_{\underline{X}}(\underline{x})\,d\underline{x} = \int_{\mathrm{all}\,\bar{u}} I\left[\{\mathrm{Failure}\}\right] f_{\underline{U}}(\underline{u})\,d\underline{u} \tag{13}$$

can be estimated using basic (or "brute-force" or "crude") Monte Carlo simulations (MCS) in practice as

$$\hat{P}_f = \frac{1}{N}\sum_{i=1}^{N} I\left[g_1(T(\underline{U}_i)) < 0 \cup g_2(T(\underline{U}_i)) < 0\right] \tag{14}$$

where a zero-mean normal vector $\underline{U}$ with the same correlation matrix $\boldsymbol{\rho}$ as the basic variables is generated first and then transformed element by element according to the full distribution transformation:

$$T(\underline{u}) = \underline{x} \Rightarrow F_{X_i}(x_i) = \Phi(u_i) \tag{15}$$

The use of the same $\boldsymbol{\rho}$ for $\underline{U}$ as for $\underline{X}$ results in error, but the error is generally small (der Kiureghian and Liu, 1986). N is the total number of times the random vector $\underline{U}$ is generated, and $\underline{U}_i$ is the ith realization of the vector. It is well known that the basic Monte-Carlo simulation-based estimate of P_f has a relatively slow and inefficient rate of convergence. The coefficient of variation (COV) of the estimate is

$$\mathrm{c.o.v.}(\hat{P}_f) = \sqrt{\frac{(1-P_f)}{(NP_f)}} \approx \sqrt{\frac{1}{(NP_f)}} \tag{16}$$

which is proportional to $1/\sqrt{N}$ and points to an inefficient relation between sample size and accuracy (and stability) of the estimate.

Such limitations of the basic Monte Carlo simulation (MCS) technique have led to several "variance reducing" refinements. Notable among them are Latin hypercube sampling (LHS), importance sampling (IS)) along with its variants (e.g., Melchers, 1989; Ayyub and McCuen, 1995; Melchers, 1990), subset simulations (Au and Beck, 2001), which, if performed carefully, can significantly reduce the required sampling size. Olsson et al. (2003) have suggested that importance sampling performed with LHS can potentially be more efficient than IS involving basic Monte Carlo trials. Nevertheless, importance sampling and other variance reducing techniques should be performed with care, as their results may be quite sensitive to the type and the point of maximum likelihood of the sampling distribution, and an improper choice can produce erroneous results. In this work, we have adopted importance sampling to estimate the failure probability in Eq. (13).

The mathematical formulation of importance sampling is simply obtained by modifying the basic expression of failure probability (Eq. (7)) as

$$P_f = \int_{\underline{x}\in\{\mathrm{Failure}\}} f_{\underline{X}}(\underline{x})\,d\underline{x} = \int_{\underline{x}\in\{\mathrm{Failure}\}} \frac{f_{\underline{X}}(\underline{x})}{f_{\underline{H}}(\underline{x})} f_{\underline{H}}(\underline{x})\,d\underline{x} \tag{17}$$

where $f_{\underline{H}}$ is any PDF not equal to zero in the region of interest. A judicious choice of $f_{\underline{H}}$ can ensure low variance of the estimated failure probability. By a simple change of the variable of integration, the failure probability estimate is as before the computation of the expectation of the indicator function but now modified with a correction factor ($f_{\underline{U}}/f_{\underline{H}}$):

$$\hat{P}_f = \frac{1}{N}\sum_{i=1}^{N} I\left[g_1(T(\underline{h}_i)) < 0 \cup g_2(T(\underline{h}_i)) < 0\right] \; \frac{f_{\underline{U}}(\underline{h}_i)}{f_{\underline{H}}(\underline{h}_i)} \tag{18}$$

It is important to note that this expectation as computed with respect to the sampling density $f_{\underline{H}}$ and the estimate of failure probability is obtained by simulating vectors of $\underline{H}$. The choice of $f_{\underline{H}}$ is extremely important, and depending on the limit state function, an improper choice may lead to errors in the estimate of P_f.

In this work, $\underline{H}$ has been taken as a jointly Normal random vector with the same correlation matrix $\boldsymbol{\rho}$ as $\underline{U}$, but with a mean vector that is closer to the failure region. This mean vector is chosen carefully by comparing the IS results with basic MCS results for the range of problems encountered. The variance of the estimate in Eq. (18) is

$$\mathrm{var}(\hat{P}_f) = \frac{1}{N^2}\sum var\left(I_i \frac{f_{\underline{U}}(\underline{h}_i)}{f_{\underline{H}}(\underline{h}_i)}\right) \tag{19}$$

which can be estimated during the sampling as

$$\hat{S}^2(\hat{P}_f) = \frac{\sum I^2 f/f_H^2}{N^3} - \frac{1}{N^2}\left(\frac{\sum I f_U/f_H}{N}\right)^2 \tag{20}$$

giving the coefficient of variation (COV) of the failure probability estimated through importance sampling as

$$\hat{V}(\hat{P}_f) = \frac{\hat{S}(\hat{P}_f)}{\hat{P}_f} \tag{21}$$

One of our stopping criteria for the Importance Sampling simulation in this work involves an upper limit on the COV of the estimated failure probability.

3.4. Partial safety factors and their optimization

Reliability based partial safety factor (PSF) design is intended to ensure a nearly uniform level of reliability across a given category of structural components for a given class of limit state under a particular load combination (Ellingwood, 2000). We approach the topic of optimizing PSFs by noting that any arbitrary point, $\underline{x}^a$, on the limit state surface, by definition, satisfies

$$g(\underline{x}^a) = 0 \tag{22}$$

We can, for example, choose each member of $\underline{x}^a$ to correspond to a particular quantile of the respective element of the random vector

$\underline{X}$, such that Eq. (22) defines a functional relation among these quantiles. By choosing different values for $\underline{x}^a$, we can effectively "move" the joint density function of $\underline{X}$ with respect to the limit state surface. Clearly, this relative "movement" of the limit state surface in the basic variable space will affect the limit state probability. In other words, by specifying a functional relation among quantiles (or some other statistics) of the basic variables $\underline{X}$ we can affect the reliability of the structure.

Extending this idea, a "design point" $\underline{x}^d$ on the limit state surface can be carefully chosen so that it "locates" the limit state in the space of basic variables such that a desired target reliability is ensured for the design. The ensuing design equation:

$$g(\underline{x}^d) = 0 \tag{23}$$

is essentially a relationship among the parameters of the basic variables and gives a minimum requirement type of tool in the hand of the design engineer to ensure target reliability for the design in an indirect manner. Since nominal or characteristic values of basic variables are typically used in design, Eq. (23) may be rewritten as

$$g\left(\frac{x_1^n}{\gamma_1}, \ldots, \frac{x_k^n}{\gamma_k}, \gamma_{k+1}x_{k+1}^n, \ldots, \gamma_m x_m^n\right) \geq 0 \tag{24}$$

where the superscript n indicates the nominal value of the variable. We have partitioned the vector of basic variables into k resistance type and m–k action type quantities. The partial safety factors, γ_i, are typically greater than one: for resistance type variables they divide the nominal values while for action type variables they multiply the nominal values to obtain the design point:

$$\begin{aligned} &\text{resistance PSFs}: \gamma_i = \frac{x_i^n}{x_i^d}, \quad i = 1, \ldots, k \\ &\text{action PSFs}: \gamma_i = \frac{x_i^d}{x_i^n}, \quad i = k+1, \ldots, m \end{aligned} \tag{25}$$

If the design equation (23) can be separated into a strength term and a combination of load-effect terms, the following safety checking scheme may be adopted for design:

$$R_n\left(\frac{S_i^n}{\gamma_i^s}, i = 1, \ldots, k\right) \geq l\left(\sum_{i=1}^{m-k}\gamma_i^q Q_i^n\right) \tag{26}$$

where $R_{\rm n}$ is the nominal resistance and a function of factored strength parameters, l is load-effect function, S_i^n is nominal value of ith strength/material parameter, γ_i^s is ith strength/material factor, Q_i^n is the nominal value of the ith load and γ_i^q is ith load factor. Note that there is no separate resistance factor multiplying the nominal resistance (as in LRFD) since material partial safety factors have already been incorporated in computing the strength.

The nominal values generally are fixed by professional practice and thus are inflexible. Some of the m partial safety factors (often those associated with material properties) can also be fixed in advance. The remaining PSFs can be chosen by the code developer so as to locate the design point, and hence locate the limit state as alluded to above, and hence achieve a desired reliability for the structure. Such an exercise for finding these remaining PSFs can be conveniently performed if, after the strength and load effect terms are separated as above, the limit state equation can be normalized by the design equation (for each principal direction j):

$$\frac{M_{\text{cap},j}}{M_{\text{cap},j}^n} - \frac{M_{\text{app},j}}{M_{\text{app},j}^n} = 0, \quad j = 1, 2 \tag{27}$$

The reliability problem now becomes

Find $\gamma_1^s, \ldots, \gamma_k^s, \gamma_1^q, \ldots, \gamma_{m-k}^q$ such that

$$P\left[\bigcap_{j=1,2}\frac{M_{\text{cap},j}}{M_{\text{cap},j}^n(\gamma_1^s, \ldots, \gamma_k^s)} - \frac{M_{\text{app},j}}{M_{\text{app},j}^n(\gamma_1^q, \ldots, \gamma_{m\text{-}k}^q)} \leq 0\right] = \Phi(-\beta_T) \tag{28}$$

where β_T is the target reliability index. Of course, this is an underdefined problem and even though some of the PSFs may be fixed in advance as stated above, it has an infinite number of solutions. Additional considerations are needed to improve the problem definition. Such considerations naturally arise when PSFs are needed to be "optimized" for a class of structures and are discussed next.

It is common to expect that the design equation be valid for r representative structural components (or groups). Let w_i be the weight (i.e., relative importance or relative frequency) assigned to the ith such component (or group). These r representative components may differ from each other on account of different locations, geometric dimensions, nominal loads, material grades etc. For a given set of PSFs, let the reliability of the ith group be β_i. Choosing a new set of PSFs gives us a new design, a new design point, and consequently, a different reliability index. If there has to be one design equation, i.e., one set of PSFs, for all the r representative components, the deviations of all β_i's from β_T must in some sense be minimized. The design equation (Eq. (24) or Eq. (26)), when using the optimal PSFs obtained this way, can ensure a nearly uniform reliability for the range of components. Several constraints may be introduced to the optimization problem to satisfy engineering and policy considerations (as summarized in Agrawal and Bhattacharya, 2010). Moreover, some partial safety factors, such as those on material strengths, may be fixed in advance as stated above. The PSF optimization exercise adopted in this paper has the following form:

$$\begin{aligned} &\min\left[\sum_{i=1}^{r} w_i\left(\beta_i\left(\gamma_1^q, \ldots, \gamma_{m-k}^q\right) - \beta_T\right)^2\right] \quad \text{where} \sum_{i=1}^{r} w_i = 1 \\ &\text{subject to}: \min(\beta_i) > \beta_T - \Delta\beta, \quad i = 1, \ldots, r \\ &\gamma_i^{\min} \leq \gamma_i^q \leq \gamma_i^{\max}, i = 1, \ldots, m-k \\ &\gamma_i^s = m_i, i = 1, \ldots, k \end{aligned} \tag{29}$$

The weighted squared error from the target reliability index over all groups is minimized while ensuring that the lowest reliability among all the groups does not drop by more than $\Delta\beta$ below the target. The material PSFs are fixed while the load PSFs have upper and lower limits.

4. Numerical example

We now describe a detailed example of a generic prestressed IC shell found in recently built 220 MWe Indian PHWRs in order to demonstrate the methodology developed in this paper. We emphasize that this numerical example is for demonstrative purposes only and does not represent the design of any current or future Indian nuclear power plant. The shell comprises of a cylindrical wall of height 44.1 m, a ring beam of height 4.3 m and a spherical dome whose highest point is 9 m above the top of the ring beam. The inner diameter of the cylindrical wall is 43 m while the radius of curvature of the spherical dome is 33.5 m.

Two different limit states corresponding to cracking of concrete up to the depth of cover and flexural collapse through crushing of section are considered. The load combination involves five load cases (LCs): Dead Load (D), Pre-Stressing Load ($P_{\rm s}$), Ordinary Live Load ($L_{\rm o}$), Accidental Temperature Load (T) and

Accidental Pressure Load (P_a). For each load case, sets of six load effects ($N_{xx}^{LC}, N_{yy}^{LC}, N_{xy}^{LC}, M_{xx}^{LC}, M_{yy}^{LC}, M_{xy}^{LC}$) are obtained from linear elastic finite element analyses. The FE model consists of about 2500 elements which represent one half of the symmetrical IC shell. Most of the cylindrical wall and dome are modeled using shell elements while solid elements are used to model certain critical parts of the structure. Since the load factors derived in this example depend on the overall load acting on the structure, their values are valid only for the given load combination. Inclusion of the seismic load case, or any other load case, gives rise to a new problem with a different solution.

Four structural groups of the IC Shell have been selected for finding optimal PSFs (Group 1: dome general area between two SG openings, Group 2: SG opening, Group 3: dome general area between SG opening and ring beam, Group 4: IC wall). The section depths (D) are respectively 500, 1200, 500 and 610 mm. For each finite element, the combined nominal forces (in the given load combination) $\sum_{LC} N_{xx}^{LC}, \sum_{LC} N_{yy}^{LC}, \sum_{LC} N_{xy}^{LC}$ are transformed into the principal axes and nominal moment capacities in the two principal directions are determined through the interaction diagrams. The nominal moment demands $\sum_{LC} M_{xx}^{LC}, \sum_{LC} M_{yy}^{LC}, \sum_{LC} M_{xy}^{LC}$ are also transformed along the principal axes. For each structural group the critical element is identified as the one having the lowest nominal capacity to nominal demand ratio for the given load combination. Table 1 lists the nominal load effects in all five load cases for the critical element in each structural group. Note that these forces and moments are *not* normalized by $f_{ck}bD^2$ and $f_{ck}bD$. We assume that the critical element in each group is the same in both limit states since nominal loads acting on the elements are identical in both. The objective of this section is to obtain cracking and collapse PSFs for the five applied moments optimized so as to be applicable to all four structural groups. The optimality criteria are as in Eq. (29) and the numerical values are described subsequently. Fig. 6 summarizes the algorithm for PSF optimization that has been described in detail above.

The basic variables defining the reliability problem are as follows and their statistics and descriptions are provided in Table 2. The distribution types and statistics (bias and COV) have been assumed based on available literature (Hwang et al., 1985; Hamann and Bulleit, 1987; Varpasuo, 1996; Al-Harthy and Frangopol, 1997; Pandey, 1997; Barakat et al., 2004; Agrawal and Bhattacharya, 2010). In the strength category, the random variables are concrete crushing strength, reinforcement yield strength and elastic modulus. Properties of prestressing cables do not enter the problem explicitly, their combined effect shows up on the load side as random prestressing loads (forces and moments). In the load category there are five load cases (LC = D, P_s, L_o, T, P_a), each giving rise to three applied moments and three applied forces, bringing the total number of load random variables to 30. However, due to the linear elastic assumption made about structural behavior and due to their common origin in each load case, the six components ($N_{xx}^{LC}, N_{yy}^{LC}, N_{xy}^{LC}, M_{xx}^{LC}, M_{yy}^{LC}, M_{xy}^{LC}$) in a given load case (LC) are mutually fully dependent in this formulation. Thus we have five independent load random variables (one per load case, for example, N_{xx}^{LC}, LC = D, P_s, L_o, T, P_a) and for each of them we have five more (the remaining five out of a total of six components, that is, $N_{yy}^{LC}, N_{xy}^{LC}, M_{xx}^{LC}, M_{yy}^{LC}, M_{xy}^{LC}$) which are linearly scaled. The two moment capacities are assumed to follow the lognormal distribution; their bias and COVs have been obtained through Monte Carlo simulations.

The deterministic parameters and various nominal values adopted in the problem are listed in Table 3.

As mentioned earlier, the moment capacities of the prestressed shell depend in part on the applied in-plane forces. These in-plane forces, in turn, are functionally related to the applied moments in each load case. The moment capacities and applied moments therefore are statistically dependent and since the limit states are formulated in terms of moments, this dependence must be taken into account in reliability analyses and PSF computations. Table 4 shows the correlation coefficients among the basic variables for the Group 1 critical element in cracking limit state (only the upper triangle is shown due to symmetry). These values are typical of all groups in either limit state and have been estimated by Monte Carlo simulations. The moment capacities in x and y directions are almost fully correlated. This results from the fact that both depend on the same material properties of the section and the in-plane loads in the two orthogonal directions are functionally fully dependent. Noticeable also is the high positive correlation between moment capacity and prestressing moment and the negative correlation between the moment capacity and the accidental pressurization moment.

Table 1
Nominal load effects for the critical element in each group.

Load case	Load effects					
	N_{xx} (ton/m)	N_{yy} (ton/m)	N_{xy} (ton/m)	M_{xx} (ton-m/m)	M_{yy} (ton-m/m)	M_{xy} (ton-m/m)
	Group 1					
D	−9.61	−12.2	9.64	−0.214	−0.227	0.0391
P_s	−444	−477	125	1.53	1.90	−1.39
L_o	−0.961	−1.03	0.282	−0.0337	−0.0363	0.0103
T	3.12	2.75	1.34	3.38	3.42	−0.127
P_a	208	226	−70.9	5.96	6.29	−1.30
	Group 2					
D	−54.1	−15.4	2.80	−8.11	−1.31	2.74
P_s	−1020	−620	−56.3	35.5	−5.00	6.20
L_o	−2.58	−0.615	0.318	0.0120	0.0127	0.0693
T	1.56	−0.128	−0.486	11.7	2.78	−1.90
P_a	580	150	−70.1	16.4	−3.10	−256
	Group 3					
D	−26.5	2.12	6.12	0.672	−0.124	−0.0442
P_s	−523	−243	24.2	18.5	8.95	−2.07
L_o	−1.04	−0.324	0.0612	−0.0236	−0.0136	0
T	−0.109	5.88	1.86	4.56	3.66	−0.0839
P_a	246	62.3	−19.1	−0.147	1.98	0.585
	Group 4					
D	3.73	−41.0	0.224	−0.134	−0.623	−0.0118
P_s	−670	−532	−18.9	1.17	−13.9	−0.189
L_o	0.255	−0.837	−0.00262	−0.00452	−0.0228	0
T	7.88	0.0593	0.0148	5.01	5.61	−0.0106
P_a	285	180	1.77	2.91	12.3	−0.0857

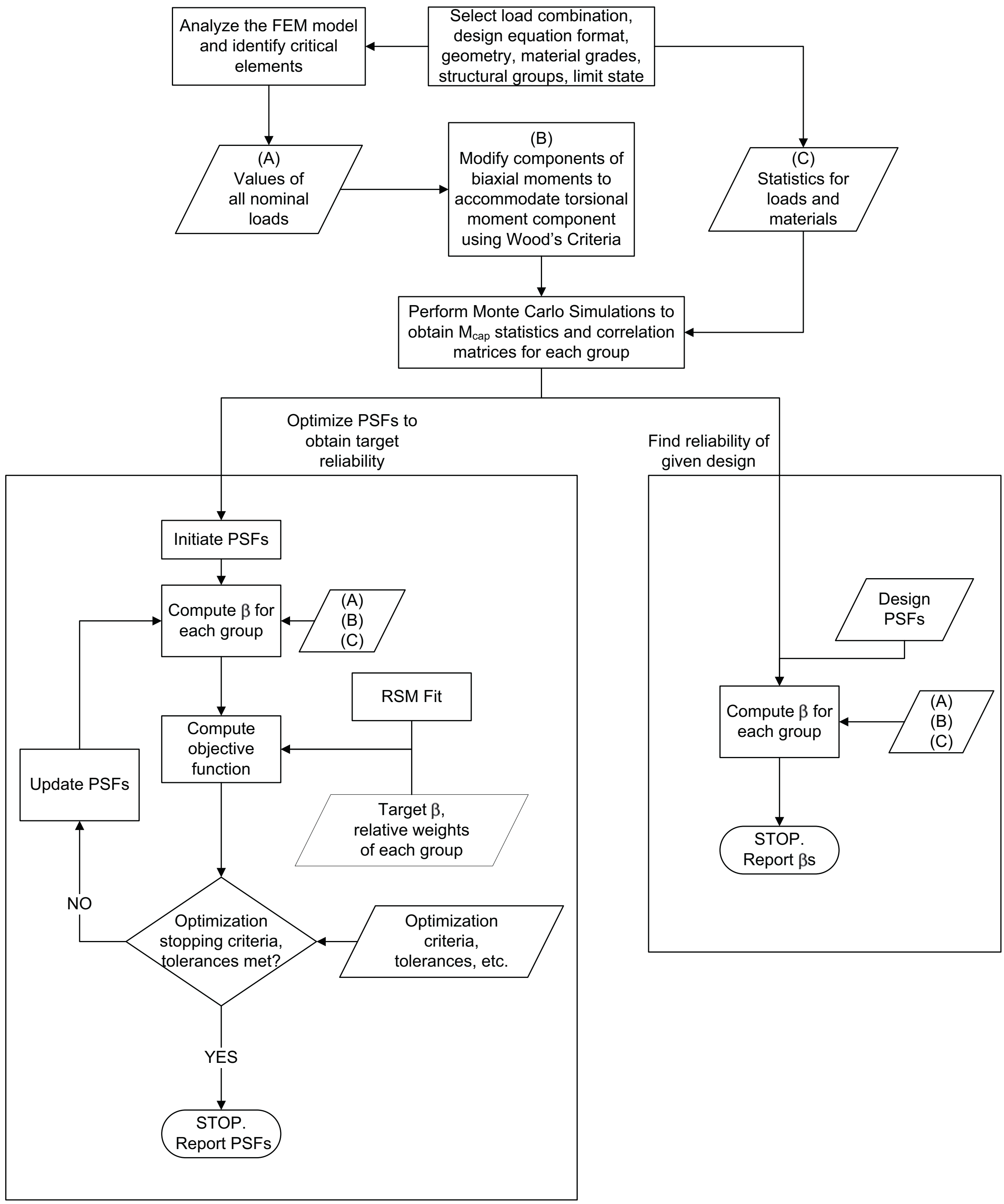

Fig. 6. Algorithm for PSF optimization.

The PSF optimization problem (Eq. (29)) in the present context has five decision variables: $\underline{\gamma} = (\gamma_D, \gamma_{P_s}, \gamma_{L_o}, \gamma_T, \gamma_{P_a})$. The optimization problem is non-linear in nature: it has a non-linear objective function:

$$f(\underline{\gamma}) = \sum_{i=1}^{r} w_i \left(\beta_i\left(\underline{\gamma}\right) - \beta_T\right)^2 \tag{30}$$

with non-linear constraints $\min(\beta_i) > \beta_T - \Delta\beta$, $i = 1, \ldots, r$ in addition to having upper and lower bounds on the decision variables. We employ a Hessian based algorithm to find the optimal solution (Coleman and Li, 1996). Each $\beta_i(\underline{\gamma})$ (reliability index for the ith structural group) is a non-linear function of the decision variables:

$$\beta(\underline{\gamma}) = \Phi^{-1}\left(1 - P\left[\frac{M_{\text{cap},1}}{M^n_{\text{cap},1}} - \frac{M_{\text{app},1}}{M^n_{\text{app},1}(\underline{\gamma})} \le 0 \cup \frac{M_{\text{cap},2}}{M^n_{\text{cap},2}} - \frac{M_{\text{app},2}}{M^n_{\text{app},2}(\underline{\gamma})} \le 0\right]\right) \tag{31}$$

Table 2
Statistics of basic variables.

Random variable	Description	Statistical properties				
M_{xx}^{LC} M_{yy}^{LC} M_{xy}^{LC}	Applied moments are combined: $\sum_{LC} M_{xx}^{LC}$, $\sum_{LC} M_{yy}^{LC}$ and $\sum_{LC} M_{xy}^{LC}$ are transformed to principal planes, and then combined according to Wood's criteria to yield $M_{app,1}$ and $M_{app,2}$	**LC**	**Distribution**	**COV**	**Bias**	
		D	Normal	0.1	1.0	
		P_s	Lognormal	0.15	1.2	
		L_o	Lognormal	0.15	1.0	
		T	Gumbel	0.15	0.9	
		P_a	Gumbel	0.15	0.8	
N_{xx}^{LC} N_{yy}^{LC} N_{xy}^{LC}	Applied forces $\sum_{LC} N_{xx}^{LC}$, $\sum_{LC} N_{yy}^{LC}$ and $\sum_{LC} N_{xy}^{LC}$ are transformed to principal planes to yield N_1 and N_2 that are then used to obtain capacities $M_{cap,1}$ and $M_{cap,2}$ in the principal directions.	Distribution type, COV and Bias same as above in respective load cases				
$M_{cap,1}$	Moment capacity in directions 1 and 2. Moment capacity is function of $\sum_{LC} N_{xx}^{LC}$, $\sum_{LC} N_{yy}^{LC}$, $\sum_{LC} N_{xy}^{LC}$ (through interaction diagram), f_c, f_y and E. First two moments obtained through Monte Carlo simulations. Nominal value obtained by fixing each basic variable equal to its nominal value. Distribution type assumed lognormal. Nominal values obtained from FEM Analysis of IC.	**Group**	**Cracking limit state**		**Collapse limit state**	
			Bias	**COV**	**Bias**	**COV**
		1	1.99	0.144	1.94	0.160
		2	1.80	0.110	1.72	0.133
		3	1.58	0.111	1.47	0.138
		4	1.67	0.131	1.67	0.135
$M_{cap,2}$		**Group**	**Cracking limit state**		**Collapse limit state**	
			Bias	**COV**	**Bias**	**COV**
		1	1.52	0.146	1.54	0.158
		2	1.55	0.151	1.53	0.171
		3	1.70	0.147	1.72	0.149
		4	1.64	0.147	1.66	0.147
f_c	Compressive strength of concrete	Normal, $(\max(f_{ck}+0.825s_c, f_{ck}+4), s_c)$[a]				
f_y	Yield strength of steel	Lognormal $(1.1133f_{yn}, 0.09)$				
E	Young's modulus	Normal $(1.001103E_n, 0.01)$				

[a] s_c, standard deviation for characteristic strength (in MPa) of concrete as given in IS 1343(BIS, 2003).
Note: All moments are normalized by $f_{ck}bD^2$ and all forces are normalized by $f_{ck}bD$.
LC = D, P_s, L_o, T or P_a.
Bias = mean/nominal.
COV = coefficient of variation = std. dev./mean.

Table 3
Deterministic parameters.

Parameter	Description	Value
p	Percent ordinary reinforcement	0.2%
f_{ck}	Characteristic 28 day cube compressive strength of concrete	45 MPa
ε_c	Compressive strain of concrete at failure (crushing)	0.0035
ε_t	Tensile strain of concrete at failure (cracking)	0.00012
$f_{cn}=f_{ck}/1.5$ (collapse) $=f_{ck}/1.25$ (cracking)	Nominal compressive strength of concrete	30 MPa (collapse)/36 MPa (cracking)
$f_{cd}=f_{cn}/1.5$ (collapse) $=f_{cn}/2.0$ (cracking)	Design compressive strength of concrete	20 MPa (collapse)/18 MPa (cracking)
f_{yn}	Nominal strength of reinforcing steel	415 MPa
$f_{yd}=f_{yn}/1.15$ (collapse) $=f_{yn}/1.80$ (cracking)	Design yield strength of reinforcing steel	361 MPa (collapse)/231 MPa (cracking)
E_n	Nominal Young's modulus of reinforcing steel	200 GPa
b	Width of section	1000 mm
D	Depth of section	500, 1200, 500 and 610 mm, respectively, for groups 1, 2, 3 and 4.
e/D	Eccentricity of prestressing force	0
d/D	Cover depth	0.05
Void range	Location of transverse prestressing cables manifesting as void	$0.425D$ to $0.575D$

Table 4
Correlation matrix for group 1 in cracking limit state.

	Dead	Prestress	Live	Temperature	Pressure	$M_{cap,1}$	$M_{cap,2}$
Dead	1.00	0	0	0	0	0.02	0.02
Prestress		1.00	0	0	0	0.92	0.92
Live			1.00	0	0	0.01	0.01
Temperature				1.00	0	0.01	0.01
Pressure					1.00	−0.29	−0.29
$M_{cap,1}$						1.00	0.99
$M_{cap,2}$							1.00

Table 5
Optimization parameters and results.

Parameter	Cracking limit state	Collapse limit state
Target reliability, β_T	2.5	3.5
Tolerance on target reliability, $\Delta\beta$	1.0	1.0
Weights on Groups 1–4, w_i	0.25, 0.25, 0.25, 0.25	0.25, 0.25, 0.25, 0.25
Material PSF on nominal concrete strength, γ_c	2.0	1.5
Material PSF on steel strength, γ_s	1.8	1.15
Lower bounds on load PSFs	1.0, 0.6, 0.6, 0.6, 0.6	1.0, 1.0, 1.0, 1.0, 1.0
Upper bounds on load PSFs	1.0, 1.4, 1.4, 1.4, 1.4	1.4, 1.4, 1.4, 1.4, 1.8
Grid size for response surface fit, $\Delta\gamma$	0.05	0.05
Beta values at optimum (Groups 1–4, respectively)	1.46, 1.68, 3.75, 3.62	2.6, 3.87, 4.66, 3.97
Objective value at optimum (weighted squared deviation from β_T)	1.36	0.85
Optimal PSFs (D, P_s, L_o, T, P_a)	1.00, 1.06, 1.10, 1.16, 0.71	1.00, 1.02, 1.16, 1.19, 1.44

and as stated above, is estimated using importance sampling. Due to finite size of the random sampling, the estimated $\beta_i(\underline{\gamma})$, and hence the gradients and Hessian of the objective function $f(\underline{\gamma})$ suffer from sampling related noise which comes in the way of convergence of the optimization algorithm. We therefore fit a local linear response surface to f around the given point $\underline{\gamma}$ on every call to evaluate the objective function and estimate the objective and its gradients from the linear fit:

$$\tilde{f}(\underline{\gamma}) = \alpha_0 + \sum_{i=1}^{5} \alpha_i \gamma_i \tag{32}$$

The parameters α_i are estimated in each call to the objective function by obtaining values of f (through Eqs. (30) and (31)) at 2^5 points in a rectangular grid $\gamma_1 \pm \Delta\gamma$, $\gamma_2 \pm \Delta\gamma$, ..., $\gamma_5 \pm \Delta\gamma$ around the given point $\underline{\gamma}$. Table 5 lists the parameters used to define the PSF optimization problem for either limit state following which the optimal solutions are described. The target reliability index is 2.5 in cracking and 3.5 in collapse implying that the failure consequence in collapse limit state is expected to be about 25 times larger than in cracking if a constant risk criterion is maintained. The four structural groups described earlier have the same importance in the PSF optimization scheme which is why the weights have been assigned equal values. The material PSFs have been fixed based on the current professional practice for nuclear structures in India and are not subject to optimization. The dead load PSF has been fixed at 1.0 in the cracking limit state to conform to existing practices.

At the optimal point, the objective value i.e., the weighted squared deviation from the target reliability is 1.36 for the cracking limit state, and 0.85 in collapse. Group 1 (dome general area between two SG openings) is the most demanding in both limit states, having the lowest reliability index among the four groups (1.46 and 2.60, respectively). The optimal values of the first four partial safety factors are close to unity in both limit states – the difference is made by the accidental pressure PSF which is high at 1.44 in collapse and benign at 0.71 in cracking.

The optimal point obtained depends on the value of $\Delta\beta$. A change in $\Delta\beta$ changes the constraint set. For instance, for the problem described in Table 5 (cracking limit state), the average value of the objective function over 10 runs for the PSF set – {1.0, 1.0, 1.1, 1.2, 0.8} is 1.14 (more desirable than the optimal obtained). However, this set of PSFs is unacceptable for a $\Delta\beta$ of 1.0 since they produce a mean β of 0.78 for group 4, which is 1.72 less than the target of 2.5.

Conclusions

Partial safety factors (PSFs) used in reliability-based design are intended to account for uncertainties in load, material and mathematical modeling while ensuring that the target reliability is satisfied for the relevant class of structural components in the given load combination and limit state. This paper summarized past works on reliability of prestressed sections in general and prestressed containments in particular, discussed target reliabilities, Monte Carlo simulations, Importance Sampling and the principle behind PSF-based design, and described the methodology in detail for developing a set of optimal reliability-based PSFs for the design of prestressed concrete inner containment shells in Indian Nuclear Power Plants (NPPs) at collapse limit state under MSLB/LOCA conditions.

Two sets of optimal partial safety factors (one for cracking and another for collapse limit state) corresponding to two target reliabilities across 4 groups of structural elements in a typical IC Shell of an Indian PHWR were obtained. Correlations between demand and capacity terms owing to the structural mechanics underlying the problem were taken into account. Analysis of the structural behavior of prestressed concrete section was formulated using recommendations provided in IS 1343 and SP 16. Monte Carlo simulations using (1) Importance Sampling and (2) a linear response surface fit for variance reduction was used to compute probabilities of failure. The load PSFs obtained in this example problem for either limit state were in agreement with design practices from around the world, except that the temperature load factor typically have lower values than those found here since thermal loads are categorized as secondary loads caused by geometric constraints, so that local yielding and micro-cracking eventually result in redistribution of forces.

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Appendix A. Appendix – List of symbols

Symbol	Definition
ε_c	Maximum compressive strain at most compressed edge of section
ε_t	Maximum allowable tensile strain in concrete
e_r	Compressive strain at most compressed edge of section
f_{cd}	Design compressive strength of concrete
f_{ck}	28-Day characteristic cube compressive strength of concrete
f_{cn}	Nominal compressive strength of concrete
f_{ct}	Tensile strength of concrete
f_{yd}	Design yield strength of reinforcing steel
f_{yn}	Nominal yield strength of reinforcing steel
E_c	Secant modulus of concrete in compression, assumed to be same as modulus of elasticity of concrete in tension
γ_c	Material safety factor on concrete strength

E_{n}	Modulus of elasticity of steel
γ_{s}	Material safety factor on yield strength of steel
N_{xx}^{LC}, N_{yy}^{LC}, N_{xy}^{LC}	Applied loads
M_{xx}^{LC}, M_{yy}^{LC}, M_{xy}^{LC}	Applied moments
N_1, N_2	Combined applied loads in principal directions
$M_{\mathrm{app},1}$, $M_{\mathrm{app},2}$	Combined applied moments in principal directions
$M_{\mathrm{cap},1}$, $M_{\mathrm{cap},2}$	Moment capacities in principal directions
$M_{\mathrm{app},1}^{n}$, $M_{\mathrm{app},2}^{n}$	Nominal applied moments in principal directions
$M_{\mathrm{cap},1}^{n}$, $M_{\mathrm{cap},2}^{n}$	Nominal moment capacities in principal directions
F_X	CDF of random vector X
$g(\underline{X})$	General limit state function in terms of basic variables $\underline{X}$
Φ	Standard Normal CDF
$f_{\underline{H}}$	Importance sampling PDF
$f_{\underline{U}}$	PDF of transformed random vector $\underline{U}$
$f_{\underline{X}}$	PDF of basic random vector $\underline{X}$
β_i	Reliability index obtained for the ith structural group
β_t	Target reliability index
$\Delta\beta$	Tolerance on target reliability
$\underline{\gamma}$	Vector of partial safety factors on applied loads
b	Width of section
c	
d	Depth of cover
D	Depth of section
e	Eccentricity of prestressing cables
t_{void}	Area of void due to prestressing cables
j	Index indicating one of two mutually orthogonal principal directions with respect to applied loads
LC	Index denoting load case. Varies among D (Dead Load), P_{s} (PreStressing Load), L_{o} (Ordinary Live Load), T (Temperature Load) and P_{a} (Accidental Pressurization Load)
N	Total number of simulations
p	Percentage of total area of section occupied by reinforcing steel
P_{f}	Probability of failure
Q_{n}	Nominal load type variables
r	Total number of structural groups
R^{n}	Nominal resistance type variables
S^{n}	Nominal strength/material parameter
w_i	Weight given to the ith structural group during optimization
$\Delta\gamma$	Grid size for response surface fit
α_i	Coefficients of the fitted response surface
ρ	Correlation matrix

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