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# Adaptive nonlinear Kalman filtering technique for parameter identification: an application to Bouc-Wen model

Subhamoy Sen[1] and Baidurya Bhattacharya[2]

[1,2]Indian Institute of Technology Kharagpur, WB, India, [1]PH +918900479405 [1]email: subhamoysen@civil.iitkgp.ernet.com [2]email: baidurya@iitkgp.ac.in

## ABSTRACT

Exactness of the Bouc-Wen hysteresis model entirely depends on the correctness of the model parameters. This paper applies Extended and Unscented Kalman filtering approach with adaptive process and measurement error covariance matrix to identify these parameters in an efficient way. We define time invariant model parameters as states of the process while the error in model output is defined through the measurement equation. In addition we compare two different methods of identification of the hysteresis parameters based on their computational cost and convergence criteria and their fields of applicability are discussed. First, in the "iterative" approach, parameter updating is done iteratively comparing model output to measured response for a fixed time span. While in the second ("sequential") approach, updating is performed in real time comparing true and model response in each time step. Two numerical cases are investigated: a SDOF system and a three story shear frame building for validation.

## INTRODUCTION

Recent research in structural health monitoring includes precise prediction about the response of existing structures subjected to loading beyond their prescribed design limit. However, this excess loading can cause nonlinear behaviour in the structure for which response cannot be predicted by its simplistic linear model. In order to have a reliable model researchers rely on nonlinear hysteretic material models and of them Bouc-Wen model [1, 6] is most acknowledged in recent research owing to the ease of implementation and broadband applicability [2] on different material types. The precision of this type of material model however depends entirely on the exactness of the model parameters which needs to be perfectly calibrated to obtain an accurate nonlinear predictor model. These parameters are therefore identified as solutions of an inverse problem using response history of the existing system. Gradient or Hessian based optimization or evolutionary algorithms can be found as common approaches to solve these inverse problems. However due to the inherent nonlinearity within the system performance of these algorithms are not always certain. In addition evolutionary algorithms [4, 7] are often impractical for complex expensive models owing to the cost constraint. In this paper we use filtering techniques to identify the parameters efficiently with an objective to keep model simulations as minimum as possible to develop a computationally inexpensive parameter identification algorithm.

## PARAMETER IDENTIFICATION

In this study we use first order extended Kalman filter (EKF) and unscented Kalman filter (UnKF) as they can handle nonlinearity in the equation. Nonlinearity in process

and measurement equations are dealt in EKF through 1st order Taylor series expansion to locally linearize the nonlinear equations whereas UnKF propagates uncertainty by using specially selected sigma points and their weights. Details of these filtering techniques can be found in much celebrated articles of Julier and Uhlmann [3] and Welch and Bishop [5]. In this study unlike existing filtering based techniques we define the process equation as the time evolution of the model parameter while the measurement equation deals with mismatch between measured and model predicted response. Thus process and measurement equations are:

$$x_{k+1} = x_k + u_k \;; \quad \varepsilon_k = \left\{ y_{observed} - FEM(x_k) \right\} + v_k \tag{1}$$

where $u_k$ and $v_k$ are process and measurement noise respectively with covariance Q and R. $\varepsilon_k$ is due to the error in model output as well as measurement noise $v_k$ .*k* signifies iteration step or time step depending on whether the approach is iterative (ITR) or sequential (SEQ) respectively. *FEM(*)* is the nonlinear finite element model which uses Bouc-Wen model to describe its material and $y_{observed}$ is the measured response. Filtering thus aims to find the optimum value of the parameters i.e. $x_k$ for which the error is minimum in least square sense. In this study we solve the same problem in two different approaches, namely iterative (ITR) and sequential (SEQ) which is discussed in detail in the following section.

## ITERATIVE AND SEQUENTIAL METHOD

In iterative method (ITR) in each iteration measurement equation evaluates the nonlinear model and compares the response for the whole time span with the measured response. This error measure is then used to calculate the gain matrix for that particular iteration step which is then used to predict states for the next iteration step. On the other hand in the sequential method (SEQ) parameter updating is done in real time. In each time step measurement equation evaluates the nonlinear model to predict response value for the next time step which is then compared with the measured response. Algorithm then proceeds to calculate the gain matrix to update states for the next time step. As nonlinear systems are sensitive towards initial condition these are kept unaltered during the model simulation. Both the algorithms are detailed below.

**Table 1: Iterative and Sequential algorithms**

| ***Algorithm 1:*** | ***Algorithm 2:*** |
|---|---|
| 1. *Initialize $x_k$ which is the parameter estimate at iteration step k.* | 1. *Initialize x(t) which is parameter estimate at time instant t .* |
| 2. *Go to: measurement equation.* | 2. *Go to: measurement equation* |
| 3. *Simulate the nonlinear model for time span 0 to t and obtain $y^k_{estimate}$.* | 3. *Simulate the nonlinear model to obtain $y_{estimate}(t+dt)$.* |
| 4. *Calculate the error measure at $k^{th}$ iteration step i.e. $(y^k_{actual} - y^k_{estimate})$.* | 4. *Calculate the error measure $y_{actual}(t+dt) - y_{estimate}(t+dt)$.* |
| 5. *Go to: Process equation* | 5. *Go to: Process equation* |

| | |
|---|---|
| 6. *Calculate gain.* | 6. *Calculate gain.* |
| 7. *Predict parameters for next iteration step $x_{k+1}$.* | 7. *Predict parameter estimate x(t+dt).* |
| 8. *Repeat step 2 and 3 till tolerance is achieved.* | 8. *Repeat step 2 and 3 till signal length exceeds.* |

Thus while with iterative algorithm filtering is done in pseudo-time, where it is represented by iteration steps, sequential algorithm is based in real time and updating is performed only when new set of data is available. Iterative scheme analyses the system for complete duration in each iteration step making the algorithm computationally demanding. On the other hand, with sequential scheme the model does not run for the whole time span making it computationally inexpensive. However, in this algorithm the horizon to reach the optimum solution is limited by length of measured time signal and therefore it may or may not satisfy tolerance criteria before the signal ends which is not a problem for iterative algorithm as it can achieve required precision in expense of computation time.

## ADAPTIVE SCHEME FOR Q AND R

It is observed that for parameter identification using Kalman filtering fast convergence depends on the proper selection of the state and measurement error covariance matrix i.e. Q and R. Kalman filter is generally used to filter out the noise from the signal with the underlying assumption that process or measurement noise is a zero mean white noise sequence with constant covariance. However use of Kalman filtering technique in parameter identification is characteristically different. For example the measurement mismatch, which in case of filtering used to be attributed to only noise in the signal, is actually a combined effect of wrong parameter selection as well as noise in the measured signal. Thus instead of representing R by a constant entity an adaptive selection depending on the measurement error would be practical. We can further observe that the gain is a function of Q and R and therefore theoretically we can control the gain by choosing proper Q and R.

$$K_k = \left(F_{k-1} P_{k-1|k-1} F^T{}_{k-1} + Q_{k-1}\right) H_k^T \left(H_k P_{k|k-1} H_k^T + R_k\right)^{-1} \tag{2}$$

It is evident from the above equation that higher values for Q or lower value for R will cause larger updating in each step but upon reaching near to the actual solution it may oscillate around the actual value. On the other hand smaller Q and higher R will cause small updating in each step rendering the method to be time taking and computationally demanding. It is observed that at the initial phase of algorithm generally larger updating of parameters is needed which should get damped as the parameter values converges towards their respective true values. Therefore instead of using constant Q and R value we use an adaptive scheme for Q and R which can ensure rapid yet smooth convergence. This is a trivial issue for complex time consuming nonlinear FE model updating as rapid along with smooth convergence reduces simulations required to achieve the optimum solution. The philosophy behind the adaptive scheme for Q is that at the initial steps when prior states are less reliable we can assume a high value for Q causing high gain but as the measurement error decreases Q should decrease as well. State covariance matrix P holds the

information about the variance in error between true and estimated states. We therefore use this information to define an adaptive scheme for Q, which forces Q to take a higher value when P is high and as P decreases it forces Q to decrease as well. This approach is given in the following equation as:

$$Q_k = \delta_1 * Q_{k-1} + Q_{noise} \quad where \quad \delta_1 = \frac{diag(P_k)}{diag(P_{k-1})} .$$

The time invariant part $Q_{noise}$ is due to true process noise which is dominant only when the error dependent part diminishes to some lower values than $Q_{noise}$.

In this parameter identification problem, measurement error, which is due to model error owing to incorrect parameter values and as well as measurement noise, is considered together as noise in the measurement equation. Thus it is practical to define measurement noise covariance matrix R as an inversely proportional function of measurement error itself. As the error decreases R increases reducing gain in each step. The adaptive scheme for R is described below:

$$R_k = \min(diag\left[\left|\delta_2 * e^{-\delta_3 \log_{10}(\varepsilon_k)}\right|\right], R_{noise})$$

Where $\delta_2$ and $\delta_3$ are scaling parameter defining the increase in the value of R as error converges. The error invariant part of the equation $R_{noise}$ describes the true measurement noise covariance which will be dominant when error dependent part becomes larger than $R_{noise}$ allowing the algorithm to incur noise effects as well.

## RESULTS AND DISCUSSION

Two sets of numerical experiment are performed and the details of modelling, assumed parameters values, simulation related information is furnished concisely in Table 2. For each problem the response signal is contaminated with 2% noise to better represent field measurement scenario where noise contamination is obvious. Iterative and sequential algorithms are then applied on the noisy signal to identify the parameters. Comparisons on the computational demand for both the problems and for both the proposed methods i.e. iterative and sequential are given in Table 3. Table 3 also lists total time taken by each algorithm with EKF and UNKF and required iteration steps (IP1 And IP2) and corresponding time (TP1 and TP2) to reach two pre-set precision point P1 and P2 (1E-3 and 1E-5). 7th column of Table 3 lists required run time for model only ($T_{FEM}$) which tracks how much time the algorithm uses to evaluate the model only. This data is important in the sense that as the nonlinear model becomes complicated and computationally expensive $T_{FEM}$ increases significantly whereas remaining part termed here as $T_{ID}$ depends only on the order of calculation and therefore does not increase in that extent. This helps to set the primary objective for the proposed algorithm to keep number for model simulation minimum in order to achieve faster convergence. Based on the results a general suggestion is recommended on the applicability of the proposed methods on different category of problems.

From the result we observe that for a small inexpensive model like the SDOF system iterative algorithm outperforms its sequential counterpart. Although considering $T_{FEM}$ we obtain a better understanding about the efficiency of these algorithms. $T_{FEM}$

**Table 2: Details of SDOF and MDOF numerical experiments**

| Case | SDOF | | | | MDOF | | | |
|---|---|---|---|---|---|---|---|---|
| Problem | Single spring-mass-damper | | | | Three story shear frame building | | | |
| Parameters | $A$ | $\alpha$ | $\beta$ | $\gamma$ | $\alpha$ | $\beta$ | $\gamma$ | $n$ |
| | 0.5 | 0.4 | 4 | 2.1 | 0.4 | 4 | 2.1 | 2 |
| Sampling | 100 Hz | | | | 50 Hz | | | |
| Time span | 5 Sec. (ITR) and 20 Sec. (SEQ) | | | | 31 Sec. for both | | | |
| Noise | 2% | | | | 2% | | | |
| Force | Sinusoidal forcing | | | | El- Centro ground motion | | | |
| Simulation time for: | For full simulation : $1.568\times10^{-2}$<br>For each time step : $2.384\times10^{-4}$ | | | | For full simulation : $4.732\times10^{-2}$<br>For each time step : $9.312\times10^{-4}$ | | | |
| Precision | $P1=1\times10^{-3}$ and $P2=1\times10^{-5}$ | | | | $P1=1\times10^{-3}$ and $P2=1\times10^{-5}$ | | | |

**Table 3: Results for both SDOF and MDOF systems. ITR-Iterative, SEQ=Sequential, IP1, IP2= Iteration to reach P1 and P2, TP1, TP2=Time to reach P1 and P2**

| | Total time | IP1 | TP1 | IP2 | TP2 | $T_{FEM}$ |
|---|---|---|---|---|---|---|
| | | | **Case SDOF** | | | |
| ITR-EKF | 6.398 | 29 | 4.526 | 41 | 6.398 | 3.075 |
| ITR-UnKF | 10.656 | 45 | 6.850 | 70 | 10.656 | 9.450 |
| SEQ-EKF | 65.815 | 458 | 7.535 | 1571 | 25.848 | 1.806 |
| SEQ-UnKF | 91.589 | 642 | 14.706 | 1882 | 43.092 | 3.895 |
| | | | **Case MDOF** | | | |
| ITR-EKF | 46.170 | 98 | 29.004 | 156 | 46.170 | 36.663 |
| ITR-UnKF | 68.914 | 92 | 30.599 | 117 | 68.914 | 49.491 |
| SEQ-EKF | 126.315 | 369 | 29.878 | 921 | 74.574 | 4.1445 |
| SEQ-UnKF | 156.699 | 502 | 50.425 | 1282 | 128.775 | 10.384 |

depends on the computational demand of the model which is bound to increase with the model order whereas $T_{ID}$ doesn't increase in that extent. This guides us to the conclusion that in order to achieve an inexpensive algorithm required model simulation should be maintained at its minimum possible level. From the result we find that although sequential approach takes more time to reach convergence it actually ran the model for lesser time than its iterative counterpart. The iterative algorithm outperforms only because $T_{FEM}$ is very small compared to $T_{ID}$ for this simple SDOF model. This claim can be verified from the second numerical experiment with a MDOF system. Although based on the total required time consideration iterative algorithm comes out to be the best. However, for iterative scheme the total time increased almost 6-8 times compared to the SDOF system whereas the same is increased lesser than 2 times for sequential scheme. This difference will be more prominent with the increasing time demand of the model.

However, considering the precision restriction for both approaches iterative scheme offers flexibility to choose the precision level whereas in the case of sequential

algorithm requires signal of sufficient length to achieve desired precision. In the SDOF problem we achieved the required precision with a 5 sec. measured response history while to achieve the same precision level sequential scheme needed response signal of 20 sec. Thus, this algorithm is not suitable for the cases with limited data points.

In a nutshell, we observe that the adaptive scheme for Q and R helps the algorithm to enhance its time efficiency to achieve convergence. Since these methods are computationally inexpensive they are capable of handling complex and costly system models. Iterative scheme is found to be most suitable for simpler system with less time demand, whereas for costly system models sequential scheme is best suited as it does not require evaluating the complete system in successive iterations. Furthermore, while the iterative algorithm demands availability of the complete signal prior to the commencement of the algorithm which restricts its online applications, sequential algorithm uses new sets of data only when they are available. Thus iterative algorithm is suitable for simpler model with limited length of signal but sequential algorithm holds promise as an efficient online identification algorithm.

## CONCLUSION

Proposed algorithms focus on developing a filtering based parameter identification technique which ensures rapid yet smooth convergence reducing simulation demand of the expensive FE model. To achieve this reduced simulation demand adaptive scheme for Q and R is employed which achieved optimum solution much faster than conventional technique of using constant covariance matrices. This study also compares the pros and cons of sequential and iterative approaches and classifies the problems based on the field of applicability criteria for these two approaches. It is found that while iterative algorithm can be used for model with limited data sequential scheme can be used online and is also suitable for costly models.

## REFERENCES

Bouc, R. (1967) "Forced vibration of mechanical systems with hysteresis." *Proceedings of the fourth conference on non-linear oscillation, Prague, Czechoslovakia*.

Dominguez, A., and Sedaghati, R. (2008) "Modelling and application of MR dampers in semi-adaptive structures." *Computers & structures* 86.3 407-415.

Julier, Simon J. and Jeffrey K. Uhlmann. (2004) "Unscented filtering and nonlinear estimation." *Proceedings of the IEEE* 92.3 :401-422.

Kwok, N. M., et al. (2007) "Bouc–Wen model parameter identification for a MR fluid damper using computationally efficient GA." *ISA transactions* 46.2 : 167-179.

Welch, G. and Bishop, G. (1995) "An introduction to the Kalman filter."

Wen, Y. (1976) "Method for random vibration of hysteretic systems." *Journal of the Engineering Mechanics Division* 102.2 (1976): 249-263.

Ye, M. and Xiaodong W. (2007) "Parameter estimation of the Bouc–Wen hysteresis model using particle swarm optimization." *Smart Materials and Structures* 16.6 : 2341.