

Common discrete distributions
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| Distribution | PMF | CDF | Relation between parameters and moments |
|-------------------|---|--|--|
| Discrete uniform | $p_X(x) = \frac{1}{n}, x = x_1, x_2, \dots, x_n$ | Step function of height $1/n$ | $\mu = \frac{1}{n} \sum_{i=1}^n x_i,$ $\sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2$ |
| Bernoulli | $p_X(x) = px + q(1-x), x = 0, 1$ where, $p + q = 1$ | Steps of height q and p at 0 and 1 respectively. | $\mu = p,$ $\sigma^2 = pq$ |
| Geometric | $p_X(x) = q^{x-1}p, x = 1, 2, 3, \dots$ where, $p + q = 1$ | $F_X(x) = 1 - q^x, i = 1, 2, 3, \dots$ | $\mu = 1/p,$ $\sigma^2 = (1-p)/p^2$ |
| Binomial | $p_X(x) = \binom{n}{x} p^x q^{n-x}, x = 0, 1, 2, \dots, n$ where, $p + q = 1$ | Not available in closed form | $\mu = np,$ $\sigma^2 = npq$ |
| Multinomial | $p(x_1, x_2, \dots, x_k; p_1, p_2, \dots, p_k, n) = \frac{n!}{x_1! x_2! \dots x_k!} p_1^{x_1} p_2^{x_2} \dots p_k^{x_k}$ $\sum_{i=1}^k p_i = 1, \sum_{i=1}^k x_i = n$ | Not available in closed form | $\mu_i = np_i,$ $\sigma_i^2 = np_i q_i$ |
| Negative binomial | $p_X(x) = \binom{x-1}{r-1} p^r q^{x-r}, x = r, r+1, \dots$ where, $p + q = 1$ | Can be given in terms of binomial CDF | $\mu = r/p,$ $\sigma^2 = rq/p^2$ |
| Hyper-geometric | $p_X(x) = \frac{\binom{d}{x} \binom{N-d}{n-x}}{\binom{N}{n}}, x = 0, 1, 2, \dots, \min(d, n)$ | Not available in closed form | $\mu = nd/N,$ $\sigma^2 = \frac{nd(N-d)}{N^2} \left(\frac{N-n}{N-1} \right)$ |
| Poisson | $p_X(x) = e^{-\mu} \frac{\mu^x}{x!}, x = 0, 1, 2, 3, \dots$ | Not available in closed form | $\mu = \mu,$ $\sigma^2 = \mu$ |
| Zeta or Pareto | $p_X(x) = \frac{c}{x^{\alpha+1}}, x = 1, 2, 3, \dots, \alpha > 0$ such that $c = 1/\zeta(\alpha+1)$ where $\zeta(s) = \text{Reimann zeta fn.} = \sum_{k=1}^{\infty} 1/k^s, s > 1$ | Not available in closed form | $\mu = \frac{\zeta(\alpha)}{\zeta(\alpha+1)}$ |