CHAPTER 20. RELIABILITY-BASED MAINTENANCE (OF NON REPAIRABLE SYSTEMS)

20.1 Types of maintenance

By definition, reliability is a non-increasing function of time. How to maintain reliability above target throughout design life? Consider the reliability function shown in Figure 20-1. If the target reliability ("acceptable limit") is 0.9 and the remaining life (t_L) is 10 years, then this item becomes unacceptable in around t_u = 8 years. Four options are available:

1) Make a stronger item, so that no repair becomes necessary.
2) Restrict loads
3) Replace item by new item at t_u
4) Repair item well before t_u (preventive maintenance)

Option 1 may be uneconomical or even impractical for many systems. Option 2 may be undesirable, and may render the system functionally compromised or useless. For non-repairable systems, which is the subject of this chapter, option 3 is not possible. This Chapter is about the fourth option.

Maintenance is of two types:

1. Preventive - system is not repairable (depends on failure mode & consequence)
2. Breakdown/ corrective / repair – assumes repairable system measured with availability

For non-repairable systems, waiting for failure is not an option, preventive maintenance is must. Hence breakdown maintenance is not relevant for non-repairable systems. Preventive maintenance can restore strength of an element to as-new or near-new condition. It also can detect *hidden failures* – i.e., failure of redundant elements.

20.2 Preventive maintenance

<u>This chapter is about preventive maintenance</u>. Reliability-based Maintenance of non-repairable systems is preventive in nature, as opposed to corrective maintenance in availability.

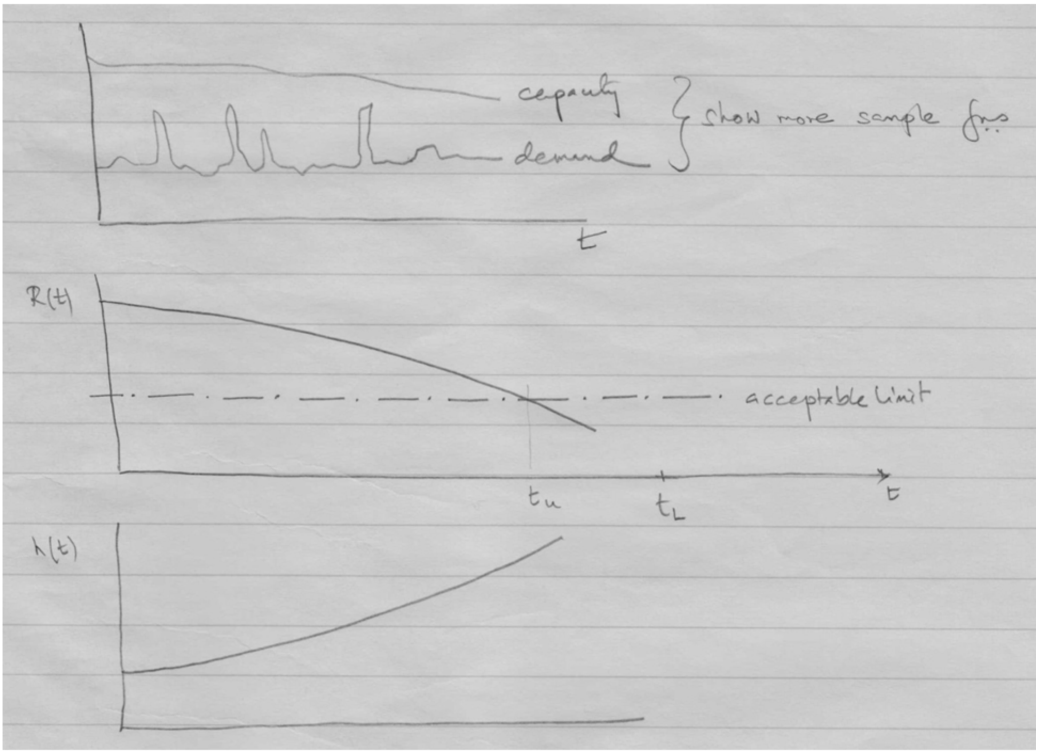


Figure 20-1 Time dependent reliability

We ask the following questions:

- how often to repair
- go for perfect repair vs. partial repair
- can repair harm the reliability fn?

The questions can be placed in the context of minimizing the total expected cost (eq ref???) subject to constraints like budget and reliability. Repair can be either perfect (in which the item is made as new), or partial (only a fraction of original strength is restored). The question is, how is the reliability function altered due to periodic maintenance? In other words, we are looking to describe the conditional reliability $\mathrm{Rel}(t \mid M_{0:t})$ given the maintenance plan, *M*, up to time *t*. Please note that we are still looking into the future when we are trying to predict $\mathrm{Rel}(t \mid M_{0:t})$, i.e., the analyst's position on the time axis is $t = 0$. Thus the conditional reliability function would still have the essential properties of the unconditional reliability, namely, it is a non-increasing function that drops from 1 to 0 with time.

Although not recommended, but as some authors do, one could also add the survival history up to time *t* and repeat the question. The difference is subtle but important. This would happen if the analyst were placed at some point in time in the future, say at t_0, and asked how the reliability function would behave henceforth. That is, one would estimate the conditional reliability $\mathrm{Rel}(t \mid M_{0:t}, S_{0:t})$ where S gives the survival information up to time *t*. The plot of $\text{Rel}(t \mid M_{0:t}, S_{0:t})$ would no longer behave monotonically, but would jump to 1 at each discontinous point t_0 where the structure is known to have survived. It is easy to show that this jump would happen even in the absence of any maintenance operation, but just due to the fact that the structure survived up to t_0. $\text{Rel}(t \mid M_{0:t}, S_{0:t})$ is not a reliability function in the strict sense, rather it is a piecewise juxtaposition of several reliability functions, and must be interpreted cautiously.

20.2.1 Example 1:

Let us start with an example of time dependent reliability where no preventive maintenance is performed. In this example, the load is time invariant and normally distributed: $D(t) = D_0 \sim N(10, 30\%),\ t \geq 0$. The capacity degrades deterministically with time as: $C(t) = C_0 \exp(-t/150)$. The initial capacity is normally distributed: $C_o \sim N(20, 10\%)$. The time dependent capacity therefore is normally distributed as well: $C(t) \sim N(20e^{-t/150}, 10\%)$, $t \geq 0$. The design life is $t_L = 100$ years.

Because the load is time-invariant and the capacity degrades monotonically with time, the time dependent reliability function,

$$\text{Reliability: } R(t) = P\left[C(\tau) > D(\tau),\ \forall\, \tau \in (0, t_L]\right] \tag{20.1}$$

simplifies to:

$$R(t) = P\left[\min C(\tau) > D_0, 0 < \tau \leq t\right] = P\left[C(t) > D_0\right] \tag{20.2}$$

which can be estimated with the help of the normal distribution function:

$$R(t) = P\left[M(t) > 0\right] = 1 - \Phi\left(-\frac{\mu_M(t)}{\sigma_M(t)}\right) \tag{20.3}$$

since the safety margin, $M(t) = C(t) - D(t)$, is normally distributed with mean $\mu_M(t) = 20\exp(-t/150) - 10$ and variance $\sigma_M^2(t) = [0.1\mu(t)]^2 + 3^2$.

Table 20-1: Reliability computation for no degradation

t	muD	sigmaD	muC	sigmaC	muM	sigmaM	R(t)=P[M(t)>0]
10							$1-\Phi\left(\frac{-8.71}{3.54}\right)=1-\Phi\left(-2.46\right)=.992$
20							$\Phi\left(2.16\right)=.984$
30							(1.86)=.969
40							(1.58) .943
50							(1.30) .903
60							
70							
80							
90							
100							

20.2.2 Example 2:

Consider a cable (8 inch diameter, deterministic) in a suspension bridge made of A36 steel with random yield strength *Y* (time invariant). *Y* is normally distributed with mean $\mu_Y = 38$ ksi and COV $V_Y =$ 15%. The axial load, Q_0, is invariant and sustained in time, and is now considered a normal random variable. Its mean is $\mu_Q = 1000$ kip and the COV is $V_Q =$20%.

The cable is subject to uniform corrosion causing its radius, whose initial value r_0 = 4 in, to deteriorate as: $\Delta r(t) = b_1 t^{b_2}$ where $b_1 = 0.1\,\text{in / yr}^{b_2}$, $b_2 = 0.9$ are the corrosion law constants. The cross-sectional area thus deteriorates according to: $a(t) = \pi(r_0 - \Delta r)^2$.

Let cable failure be defined as yield of the gross section. The load and capacity are independent. Find the time dependent reliability of the cable.

The reliability function for this problem can be written as (cf. Eq.(18.15)):

$$\begin{aligned}\text{Rel}(t) &= P[Y - Q_0 \max_{0<\tau\le t} \frac{1}{\pi(r_0 - b_1\tau^{b_2})^2} > 0] \\ &= P[Y - Q_0 \frac{1}{\pi(r_0 - b_1 t^{b_2})^2} > 0] \\ &= P[\pi(r_0 - b_1 t^{b_2})^2 Y - Q_0 > 0] \\ &= P[M(t) > 0]\end{aligned} \tag{20.4}$$

Note that due to the monotonically decreasing nature of $d(\tau)$, the limit state is evaluated *only at the right end point* of the interval (0,t]. In any other situation this simplification would be wrong and would lead to dangerous overprediction of reliability.

The margin process *M* is normally distributed being a linear combination of normals. Its mean and variance at time *t* are:

$$\begin{aligned}\mu_M(t) &= a(t)\mu_Y - \mu_Q \\ \sigma_M^2(t) &= a^2(t)\sigma_Y^2 + \sigma_Q^2\end{aligned} \tag{20.5}$$

The reliability function therefore can be expressed as the normal CDF:

$$\text{Rel}(t) = \Phi\left(\frac{\mu_M(t)}{\sigma_M(t)}\right) \tag{20.6}$$

Differentiating the reliability function leads to the hazard function:

$$h(t) = -\frac{\phi\left(\frac{\mu_M(t)}{\sigma_M(t)}\right)}{\Phi\left(\frac{\mu_M(t)}{\sigma_M(t)}\right)} \frac{\dot{\mu}_M(t)\sigma_M(t) - \mu_M(t)\dot{\sigma}_M(t)}{\sigma_M^2(t)} \tag{20.7}$$

These two functions are plotted in Figure 20-2. The choice of normal distribution for both random variables in the problem led to the closed form expressions for reliability and hazard functions above. For other distributions, FORM or Monte Carlo simulations may be adopted.

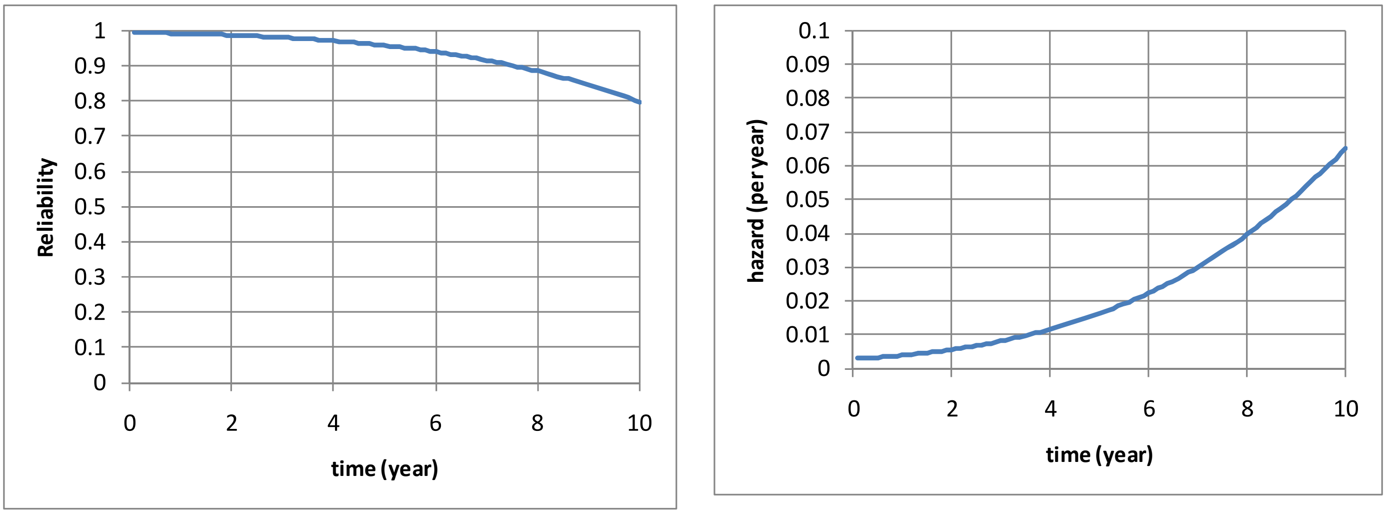


Figure 20-2: Reliability and hazard functions of corroding cable

20.3 Perfect vs. imperfect maintenance

Now let us consider repair. Say, only one maintenance operation is performed, which occurs at time t_R. It is convenient to start with the hazard function. It is altered due to the maintenance operation:

$$h(t) = \begin{cases} h_0(t), \; t < t_R \\ h_1(t), \; t \geq t_R \end{cases} \tag{20.8}$$

The reliability function (Eq. (17.11)) becomes:

$$R(t) = \begin{cases} R(t), & t < t_R \\ R(t_R) \exp\left[-\int_{t_R}^{t} h_1(\tau) d\tau \right], & t \geq t_R \end{cases} \tag{20.9}$$

The idea behind preventive repair is to maintain the reliability above the acceptable limit throughout the design life.

20.3.1 Perfect repair

If perfect repair is undertaken at t_R, then the hazard function undergoes a time shift:

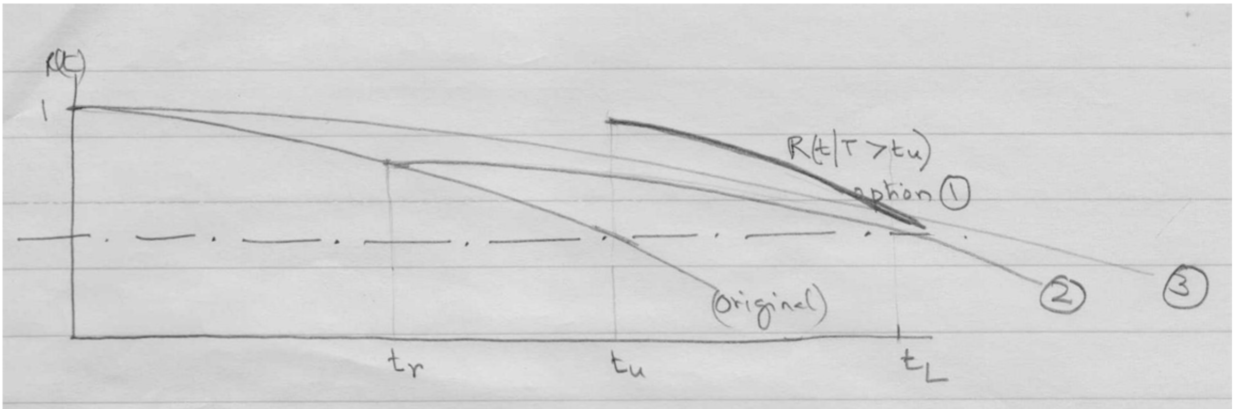


Figure 20-3: Various repair options

$$\text{Perfect repair at } t_R: \quad h_1(t) = h_0(t - t_R), \quad t \geq t_R \tag{20.10}$$

and the reliability function is repeated as a scaled version of itself:

$$\text{Perfect repair at } t_R: \quad R(t \mid M_{t_R}^{100\%}) = \begin{cases} R(t), & t < t_R \\ R(t_R) \cdot R(t - t_R), & t \geq t_R \end{cases} \tag{20.11}$$

The event $M_{t_R}^{100\%}$ signifies 100% repair at time t_R. We repeat example 3 above with 100% repair performed at 5 years (the red lines in Figure 20-7). It is clear that due to the repair, the reliability function stays above 0.9 at the end of the 10 year life as required.

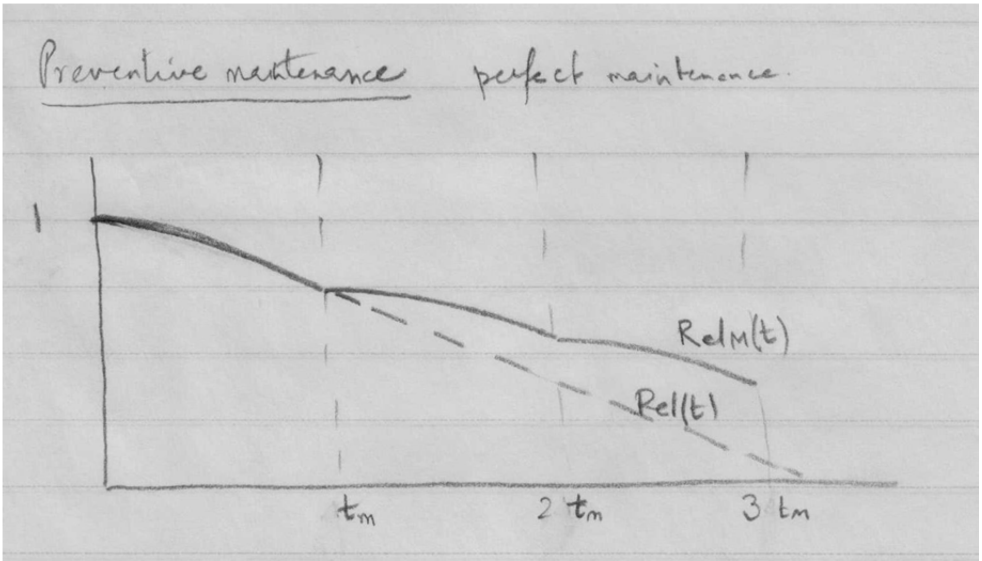


Figure 20-4: Reliability under repeated perfect maintenance

$$R_m(t) = R(t_m)R(t-t_m), \qquad t_m < t < 2t_m$$
$$= \{R(t_m)\}^2 R(t-2t_m), \quad 2t_m < t < 3t_m$$
...
$$= \{R(t_m)\}^n R(t-nt_m), \quad nt_m < t < (n+1)t_m$$

Recall, MTTF = $\int_0^\infty \mathrm{Re}l(t)dt$

$= \int_0^\infty R_m(t)dt$ in this case

$$\therefore MTTF = \sum_{n=0}^{\infty} \int_{nt_m}^{(n+1)t_m} \mathrm{Re}l_m(t)dt$$
$$= \sum_{n=0}^{\infty} \int_{nt_m}^{(n+1)t_m} \mathrm{Re}l(t_m)^n \,\mathrm{Re}l(t-nt_m)dt$$
$$= \sum_{n=0}^{\infty} \mathrm{Re}l(t_m)^n \int_0^{t_m} \mathrm{Re}l(t')dt'$$

(Geometric series)

$t' = t - nt_m$

$$= \int_0^{t_m} \mathrm{Re}l(t)dt \sum_{n=0}^{\infty} \mathrm{Re}l(t_m)^n$$
$$= \frac{\int_0^{t_m} \mathrm{Re}l(t)dt}{1-\mathrm{Re}l(t_m)}$$

Case (1) $\lambda(t) \equiv \lambda.$ $\Rightarrow$ No increase in reliability

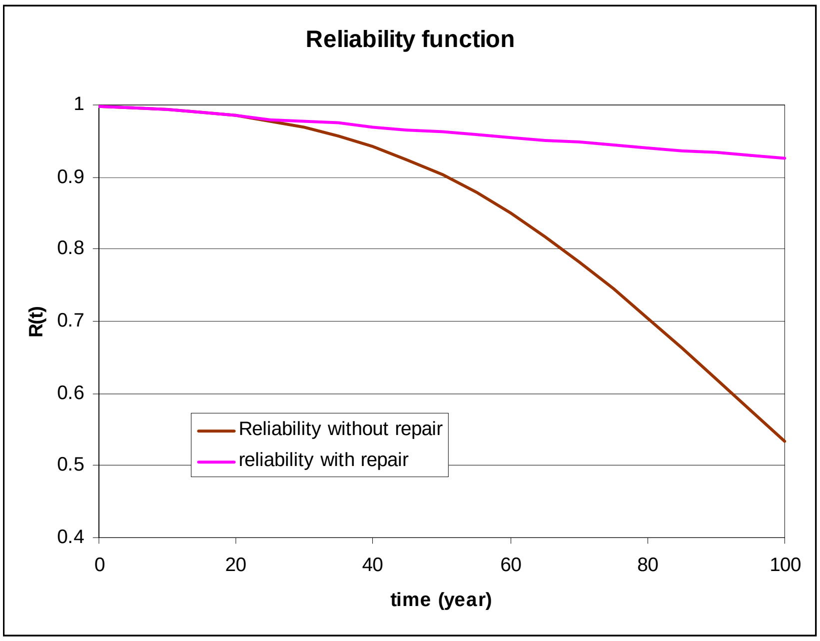
$$\mathrm{Re}l_m(t) = \left(e^{-\lambda t_m}\right)^n e^{-\lambda(t-nt_m)}$$
$$= e^{-n\lambda t_m} e^{-\lambda t + n\lambda t_m}$$
$$= e^{-\lambda t}$$
$$= \mathrm{Re}l(t)$$

Can show: $\mathrm{Re}l_m(t) > \mathrm{Re}l(t)$ if aging occurs

$< \mathrm{Re}l(t)$ if strengthening occur

20.3.1.1 Example: perfect repair and the reliability function

- Capacity degrades with time
  - C = C0 exp(-t/150) t in years
  - C0 is Normal, mean=20, s.d. =2
- Load is time invariant
  - Q is Normal, mean =10, s.d.=3
- C0 and Q are mutually independent
- tL= 100 years
- Safety limit: R(t)>0.95, h(t)<0.002/yr
- Repair Options
  - Option 1: no repair
  - Option 2: repair to full strength every 20 years



40

Figure 20-5: Reliability and hazard under repeated perfect maintenance

20.3.1.2 Example: perfect repair and the reliability function

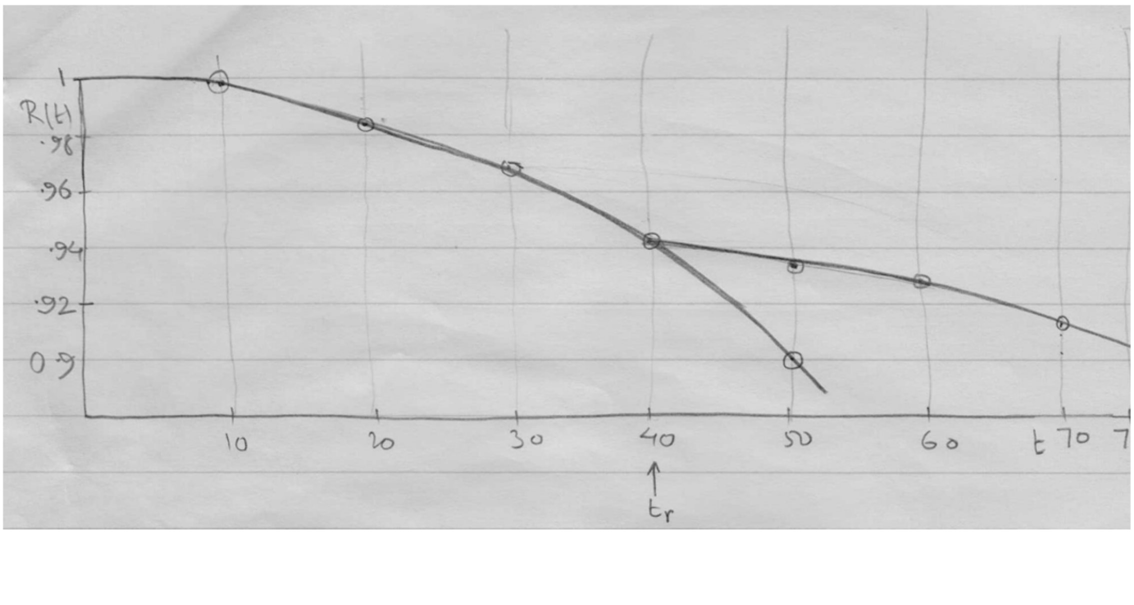


Figure 20-6: Reliability under repeated perfect maintenance

Repair at 40 yrs $\equiv t_r$

$$(t > t_r)$$
$$R'(t) = P\left[T > t\right] = P\left[C(t_r) > D \cap C_{new}(t) > D\right]$$

$$= P\left[C_{new}(t) > D \mid C(t_r) > D\right] P\left[C(t_r) > D\right]$$
$$= R\left[t \mid T > t_r\right] R(t_r)$$
$$= R_{rep,tr}(t - t_r) R(t_r)$$

$$F_T(t) = P\left[T \le t\right] = P\left[T \le t_m\right] + P\left[t_m < T \le t\right]$$
$$= P\left[T \le t_m\right] + P\left[T \le t \mid T > t_m\right] P\left[T > t_m\right]$$
$$\wedge - R(t) = X - R(t_m) + P\left[T' \le t - t_r \mid T' > 0\right] R(t_r)$$
$$-R(t) = -R(t_m)\left[1 - P\left(T' \le t - t_m\right)\right] \qquad ; T' = T - t_r$$
$$R(t) = R(t_m)\left[R'(t - t_r)\right] = R(t_m) R(t - t_m), \qquad F_{T'} \equiv F_T$$

Say $C_{new}(0) \equiv C_0$ (repaired to original strength)

$$C_{new}(t - t_r) \equiv C(t)$$

Table 20-2: Reliability improvement by perfect repair at 40 years of age

t	$R_{rep.40}(t - t_r)$	$R(t_r)$	$R'(t)$
40	1	.943	.943
50	.992	.943	.935
60	.984	.943	.928
70	.969	.943	.914
80	.943	.943	.889

20.3.2 Imperfect repair

Generalizing, if the repair is imperfect, we start with the second factor in Eq. (20.9) for $t \ge t_R$ and rewrite it as:

$$\begin{aligned}\exp\left[-\int_{t_R}^{t} h_1(\tau)d\tau\right] &= \exp\left[-\int_{0}^{t-t_R} h_1(\tau+t_R)d\tau\right] \\ &= \exp\left[-\int_{0}^{t-t_R} h_1^{'}(\tau)d\tau\right] \\ &= \mathrm{Rel}^{'}(t-t_R)\end{aligned} \tag{20.12}$$

where $h_1^{'}$ is a legitimate hazard function (generally different from h_0 due to the imperfect nature of the repair) and Rel'(t) is the corresponding reliability function which is generally different from (and less benign than) Rel(t). The reliability function due to imperfect repair can then be written as:

$$\text{Imperfect repair at } t_R: \quad \mathrm{Rel}(t \mid M_{t_R}^{\alpha\%}) = \begin{cases} \mathrm{Rel}(t), & t < t_R \\ \mathrm{Rel}(t_R)\cdot \mathrm{Rel}^{'}(t-t_R)\ , & t \geq t_R \end{cases} \tag{20.13}$$

$M_{t_R}^{\alpha\%}$ represents imperfect repair at time t_R in which the strength is restored to α% of the initial value. The green lines in Figure 20-7 correspond to α = 90. The effect is not as good as perfect repair, as can be expected.

$$p = \mathrm{p(failure\ immediately\ after\ repair)}$$

$$\mathrm{Re}l_m(t) = \mathrm{Re}l(t_m)^n(1-p)^n\,\mathrm{Re}l(t-nt_m)$$
$$nt_m < t < (n+1)t_m$$

<u>Ex.</u> T~weibull (u=7.5y, k=2.5)

$$b = k-1 = 1.5$$

$$\lambda(t) = \qquad c = \frac{k}{uk}$$

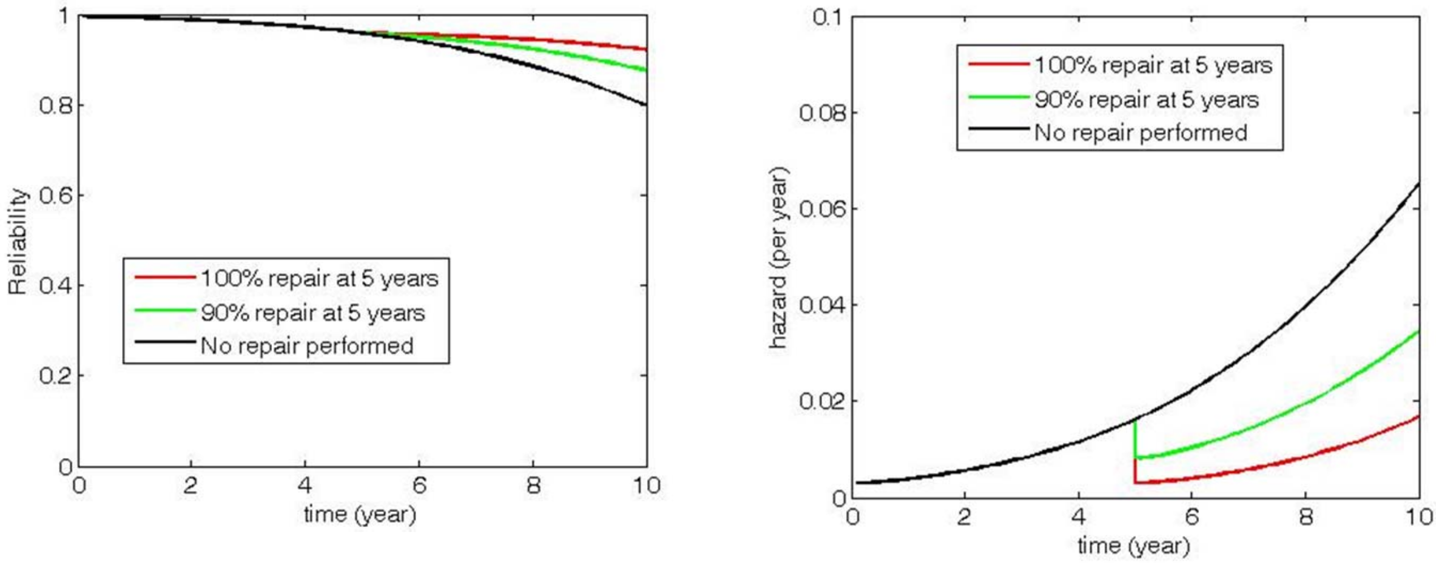


Figure 20-7: Effect of perfect and partial repair on reliability and hazard functions

20.4 The point of view issue

If in addition, the condition is imposed that the structure is found to survive at t_R, then the conditional reliability starts from 1 at t_R as stated before, and all past information is erased:

$$\begin{aligned}
\mathrm{Rel}(t \mid T > t_R, M_{t_R}^{\alpha\%}) &= \frac{P[T > t \mid M_{t_R}^{\alpha\%}]}{P[T > t_R \mid M_{t_R}^{\alpha\%}]}, \qquad t \geq t_R \\
&= \frac{\mathrm{Rel}(t \mid M_{t_R}^{\alpha\%})}{\mathrm{Rel}(t_R)}, \qquad t \geq t_R \\
&= \mathrm{Rel}^{'}(t - t_R), \qquad t \geq t_R
\end{aligned} \tag{20.14}$$