CHAPTER 6

CURVES USED IN ENGINEERING PRACTICE

6-0. Introduction:

The profile of number of objects consists of various types of curves. This chapter deals with various types of curves which are commonly used in engineering practice as shown below:

1. Conic sections
2. Cycloidal curves
3. Involute
4. Evolutes
5. Spirals
6. Helix.

We shall now discuss the above in details with reference to their construction and applications.

6-1. Conic sections:

The sections obtained by the intersection of a right circular cone by a plane in different positions relative to the axis of the cone are called conics. Refer to fig. 6-1.

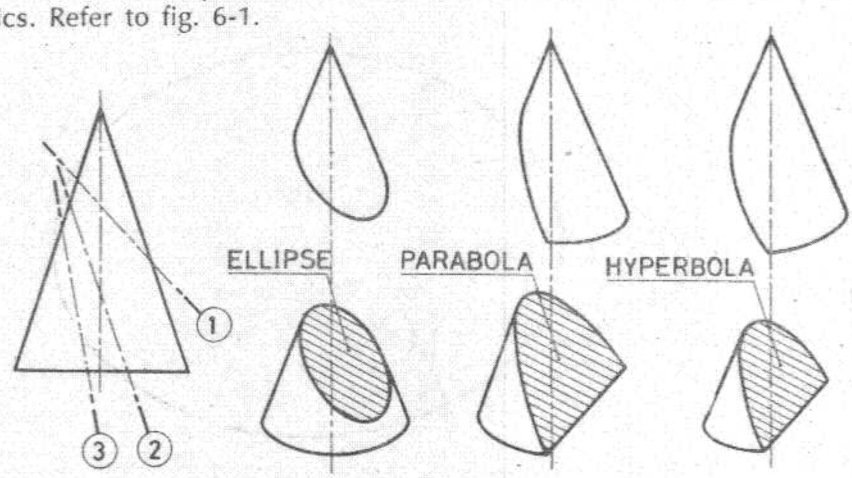


FIG. 6-1 FIG. 6-1(i) FIG. 6-1(ii) FIG. 6-1(iii)

(i) When the section plane is inclined to the axis and cuts all the generators on one side of the apex, the section is an *ellipse* [fig. 6-1(i)].

(ii) When the section plane is inclined to the axis and is parallel to one of the generators, the section is a *parabola* [fig. 6-1(ii)].

(iii) When the section plane cuts both the parts of the double cone on one side of the axis, the section is a *hyperbola* [fig. 6-1(iii)].

In fig. 6-2(i), AB is the major axis, CD the minor axis and F_1 and F_2 are the foci. The foci are equidistant from the centre O.

The points A, P, C etc. are on the curve and hence, according to the definition,

$$(AF_1 + AF_2) = (PF_1 + PF_2) = (CF_1 + CF_2) \text{ etc.}$$

But $(AF_1 + AF_2) = AB$.

$\therefore$ $(PF_1 + PF_2) = AB$, the major axis.

Therefore, *the sum of the distances of any point on the curve from the two foci is equal to the major axis.*

Again, $(CF_1 + CF_2) = AB$.

But $CF_1 = CF_2$

$\therefore$ $CF_1 = CF_2 = \frac{1}{2} AB$.

Hence, *the distance of the ends of the minor axis from the foci is equal to half the major axis.*

Problem 6-2. *To construct an ellipse, given the major and minor axes.*

The ellipse is drawn by, first determining a number of points through which it is known to pass and then, drawing a smooth curve through them, either freehand or with a french curve. Larger the number of points, more accurate the curve will be.

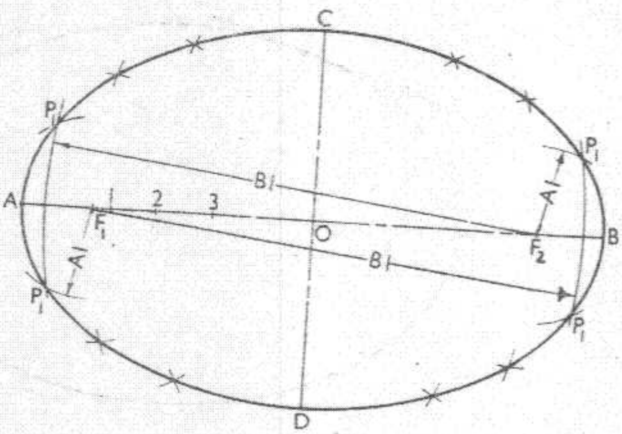
Method I: *Arcs of circles method* (fig. 6-3).

(i) Draw a line AB equal to the major axis and a line CD equal to the minor axis, bisecting each other at right angles at O.

(ii) With centre C and radius equal to half AB (i.e. AO) draw arcs cutting AB at F_1 and F_2, the foci of the ellipse.

(iii) Mark a number of points 1, 2, 3 etc. on AB.

(iv) With centres F_1 and F_2 and radius equal to $A1$, draw arcs on both sides of AB.



Arc of circle method
FIG. 6-3

(b) Construction of parabola by other methods:

Method I: *Rectangle method* [fig. 6-14].

Problem 6-10. *To construct a parabola given the base and the axis.*

(i) Draw the base *AB*.

(ii) At its mid-point *E*, draw the axis *EF* at right angles to *AB*.

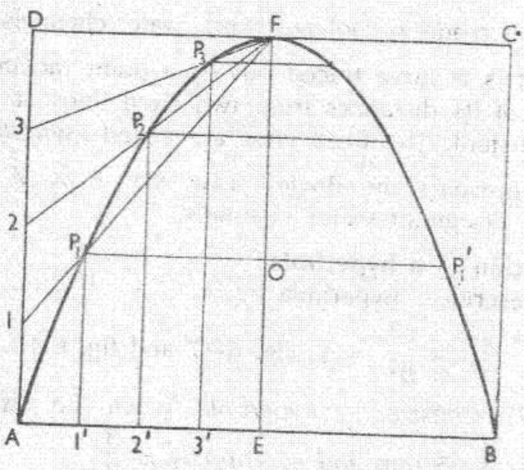
(iii) Construct a rectangle *ABCD*, making side *BC* equal to *EF*.

(iv) Divide *AE* and *AD* into the same number of equal parts and name them as shown (starting from *A*).

(v) Draw lines joining *F* with points 1, 2 and 3. Through 1′, 2′ and 3′, draw perpendiculars to *AB* intersecting *F*1, *F*2 and *F*3 at points P_1, P_2 and P_3 respectively.

(vi) Draw a curve through *A*, P_1, P_2 etc. It will be a half parabola.

Repeat the same construction in the other half of the rectangle to complete the parabola. Or, locate the points by drawing lines through the points P_1, P_2 etc. parallel to the base and making each of them of equal length on both the sides of *EF*, e.g. $P_1O = OP_1'$. *AB* and *EF* are called the base and the axis respectively of the parabola.



Rectangle method
FIG. 6–14

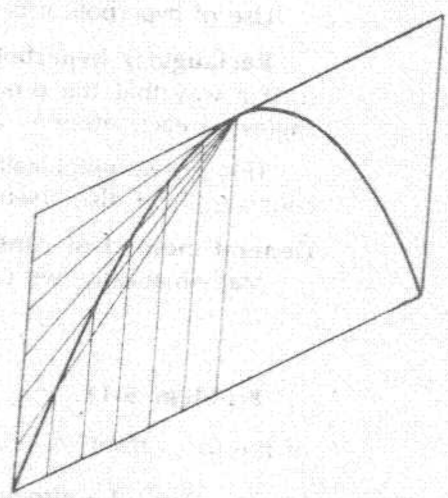


FIG. 6–15

Fig. 6-15 shows a parabola drawn in a parallelogram by this method.

Method II: *Tangent method* (fig. 6-16).

(i) Draw the base *AB* and the axis *EF*. (These are taken different from those in method I.)

(ii) Produce *EF* to *O* so that *EF* = *FO*.

(iii) Join *O* with *A* and *B*. Divide lines *OA* and *OB* into the same number of equal parts, say 8.

(iv) Mark the division-points as shown in the figure.