CHAPTER 6

CURVES USED IN ENGINEERING PRACTICE

6-0. Introduction:

The profile of number of objects consists of various types of curves. This chapter deals with various types of curves which are commonly used in engineering practice as shown below:

1. Conic sections
2. Cycloidal curves
3. Involute
4. Evolutes
5. Spirals
6. Helix.

We shall now discuss the above in details with reference to their construction and applications.

6-1. Conic sections:

The sections obtained by the intersection of a right circular cone by a plane in different positions relative to the axis of the cone are called conics. Refer to fig. 6-1.

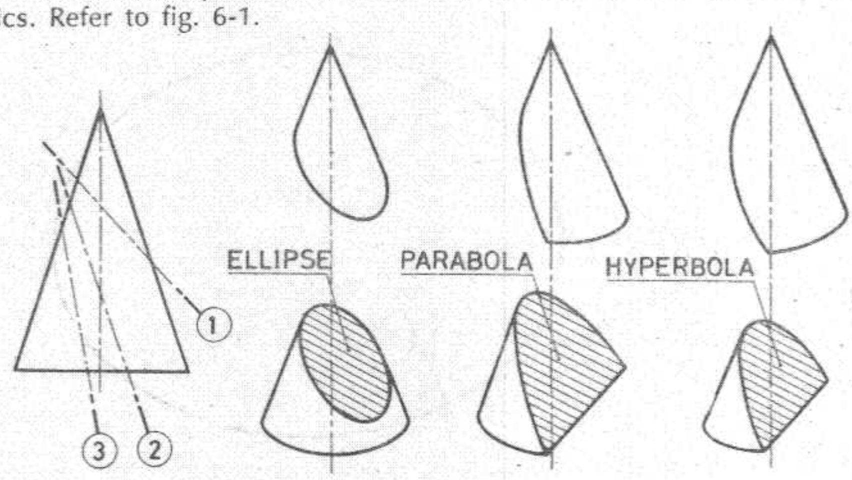


FIG. 6-1 FIG. 6-1(i) FIG. 6-1(ii) FIG. 6-1(iii)

(i) When the section plane is inclined to the axis and cuts all the generators on one side of the apex, the section is an *ellipse* [fig. 6-1(i)].

(ii) When the section plane is inclined to the axis and is parallel to one of the generators, the section is a *parabola* [fig. 6-1(ii)].

(iii) When the section plane cuts both the parts of the double cone on one side of the axis, the section is a *hyperbola* [fig. 6-1(iii)].

In fig. 6-2(i), AB is the major axis, CD the minor axis and F_1 and F_2 are the foci. The foci are equidistant from the centre O.

The points A, P, C etc. are on the curve and hence, according to the definition,

$$(AF_1 + AF_2) = (PF_1 + PF_2) = (CF_1 + CF_2) \text{ etc.}$$

But $(AF_1 + AF_2) = AB.$

$\therefore$ $(PF_1 + PF_2) = AB$, the major axis.

Therefore, *the sum of the distances of any point on the curve from the two foci is equal to the major axis.*

Again, $(CF_1 + CF_2) = AB.$

But $CF_1 = CF_2$

$\therefore$ $CF_1 = CF_2 = \frac{1}{2} AB.$

Hence, *the distance of the ends of the minor axis from the foci is equal to half the major axis.*

Problem 6-2. *To construct an ellipse, given the major and minor axes.*

The ellipse is drawn by, first determining a number of points through which it is known to pass and then, drawing a smooth curve through them, either freehand or with a french curve. Larger the number of points, more accurate the curve will be.

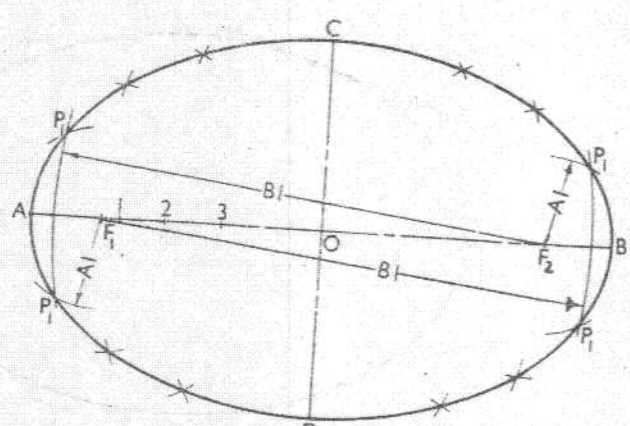
Method I: *Arcs of circles method* (fig. 6-3).

(i) Draw a line AB equal to the major axis and a line CD equal to the minor axis, bisecting each other at right angles at O.

(ii) With centre C and radius equal to half AB (i.e. AO) draw arcs cutting AB at F_1 and F_2, the foci of the ellipse.

(iii) Mark a number of points 1, 2, 3 etc. on AB.

(iv) With centres F_1 and F_2 and radius equal to $A1$, draw arcs on both sides of AB.



Arc of circle method
FIG. 6-3

(b) Construction of parabola by other methods:

Method I: *Rectangle method* [fig. 6-14].

Problem 6-10. *To construct a parabola given the base and the axis.*

(i) Draw the base *AB*.

(ii) At its mid-point *E*, draw the axis *EF* at right angles to *AB*.

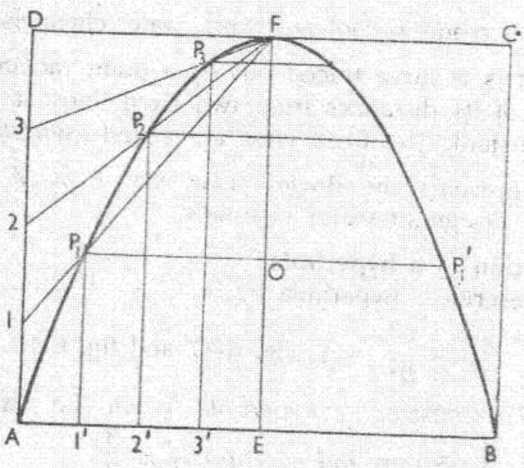
(iii) Construct a rectangle *ABCD*, making side *BC* equal to *EF*.

(iv) Divide *AE* and *AD* into the same number of equal parts and name them as shown (starting from *A*).

(v) Draw lines joining *F* with points 1, 2 and 3. Through 1′, 2′ and 3′, draw perpendiculars to *AB* intersecting *F*1, *F*2 and *F*3 at points P_1, P_2 and P_3 respectively.

(vi) Draw a curve through *A*, P_1, P_2 etc. It will be a half parabola.

Repeat the same construction in the other half of the rectangle to complete the parabola. Or, locate the points by drawing lines through the points P_1, P_2 etc. parallel to the base and making each of them of equal length on both the sides of *EF*, e.g. $P_1O = OP_1'$. *AB* and *EF* are called the base and the axis respectively of the parabola.



Rectangle method
FIG. 6-14

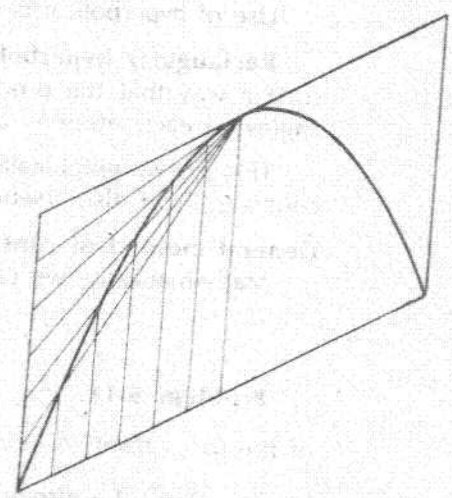


FIG. 6-15

Fig. 6-15 shows a parabola drawn in a parallelogram by this method.

Method II: *Tangent method* (fig. 6-16).

(i) Draw the base *AB* and the axis *EF*. (These are taken different from those in method I.)

(ii) Produce *EF* to *O* so that *EF* = *FO*.

(iii) Join *O* with *A* and *B*. Divide lines *OA* and *OB* into the same number of equal parts, say 8.

(iv) Mark the division-points as shown in the figure.

Problem 6-18. (fig. 6-18): *To draw a tangent to a hyperbola at a point P on it when the axis and the foci are given.*

Draw lines joining P with foci F and F_1. Draw the bisector RS of $\angle FPF_1$. RS is the required tangent to the hyperbola.

Note: In fig. 6-19, the line AB is the tangent to the hyperbola at the point A.

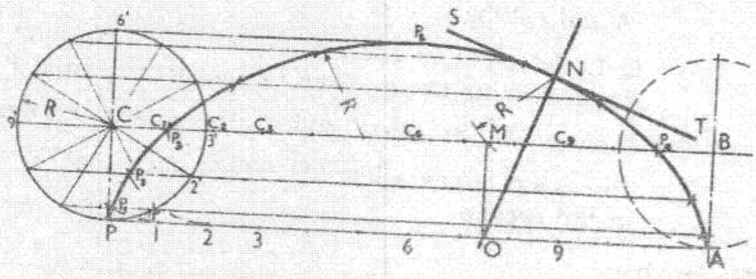
6-2. Cycloidal curves:

These curves are generated by a fixed point on the circumference of a circle, which rolls without slipping along a fixed straight line or a circle. The rolling circle is called *generating circle* and the fixed straight line or circle is termed *directing line* or *directing circle*. Cycloidal curves are used in tooth profile of gears of a dial gauge.

6-2-1. Cycloid:

Cycloid is a curve generated by a point on the circumference of a circle which rolls along a straight line. It can be described by an equation, $y = a\ (1 - \cos\theta)$ or $x = a\ (\theta - \sin\theta)$.

Problem 6-19. (fig. 6-24): *To construct a cycloid, given the diameter of the generating circle.*



Cycloid

FIG. 6-24

(i) With centre C and given radius R, draw a circle. Let P be the generating point.

(ii) Draw a line PA tangential to and equal to the circumference of the circle.

(iii) Divide the circle and the line PA into the same number of equal parts, say 12, and mark the division-points as shown.

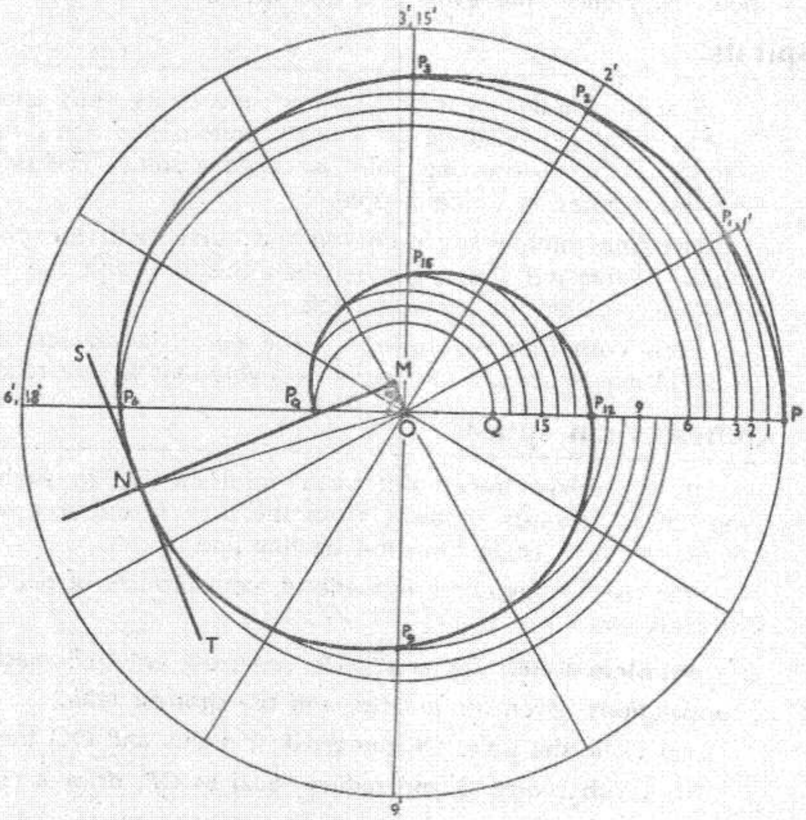
(iv) Through C, draw a line CB parallel and equal to PA.

(v) Draw perpendiculars at points 1, 2 etc. cutting CB at points C_1, C_2 etc.

Assume that the circle starts rolling to the right. When point 1′ coincides with 1, centre C will move to C_1. In this position of the circle, the generating point P will have moved to position P_1 on the circle, at a distance equal to P_1' from point 1. It is evident that P_1 lies on the horizontal line through 1′ and at a distance R from C_1.

Normal and tangent to an Archemedian spiral: The normal to an Archemedian spiral at any point is the hypotenuse of the right-angled triangle having the other two sides equal in length to the *radius vector* at that point and the *constant of the curve* respectively.

The constant of the curve is equal to the difference between the lengths of any two radii divided by the circular measure of the angle between them.



Archemedian spiral
FIG. 6-47

OP and OP_3 (fig. 6-47) are two radii making 90° angle between them. In circular measure, $90° = \frac{\pi}{2} = 1.57$. Therefore, the constant of the curve, $C = \frac{OP - OP_3}{1.57}$.

Problem 6-39. (fig. 6-47): *To draw a normal to the Archemedian spiral at a point N on it.*

(i) Draw the radius vector *NO*.

(ii) Draw a line *OM* equal in length to the constant of the curve *C* and perpendicular to *NO*.

(iii) Draw the line *NM* which is the normal to the spiral.

(iv) Through *N*, draw a line *ST* perpendicular to *NM*. *ST* is the tangent to the spiral.