EE61012 – Spring Semester 2024-25

Homework 4: Convex Optimization in Control and Signal Processing

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Q 4.1: Worker - Job Assignment

Consider the matrix `worker_assignment_cost_matrix_W.mat` whose (i, j)-th entry indicates the wage demanded by worker i if it is assigned job j. Find the optimal assignment of workers to jobs such that each worker is assigned exactly one job, and each job is allocated to exactly one worker.

Q 4.2: Estimating Filter Coefficients

Consider the matrix `FIR_Data.mat` whose first row indicates the inputs applied to a FIR filter and the second row indicates the outout. For a FIR filter, the input and output are related as:

$$y[n] = \sum_{k=0}^{p} b_k x[n-k] + \omega[n]$$

where p is the order of the filter, b_k are the coefficients and $\omega[n]$ is a small noise term. For the given data set, p is known to be between 1 and 8. Determine what is the most likely choice of p and the find the coefficients that best explain the input-output behavior.

Q 4.3: ARMA Model Identification

Consider the matrix `ARMA_Data.mat` whose first row indicates the inputs applied to a ARMA model and the second row indicates the outout. The input and output are related as:

$$y[n] = \sum_{k=0}^{p} b_k x[n-k] + \sum_{k=1}^{q} a_k y[n-k] + \omega[n]$$

where $p = 3$ and $q = 3$. Determine the coefficients that best explain the input-output behavior.

Q 4.4: Minimum Energy and Minimum Fuel Control

Consider the following discrete-time LTI system given by

$$x(k+1) = Ax(k) + Bu(k), \qquad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad x(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad x_T = \begin{bmatrix} 20 \\ 20 \\ 20 \end{bmatrix}.$$

1. Does there exist a sequence of control inputs to drive the states from $x(0)$ to x_T for $T = 1$?

2. What is the smallest value of T for which you can find a control input that drive the states from $x(0)$ to x_T stated above?
3. For $T = 10$, find a sequence of control inputs that minimizes the total energy $\sum_{k=0}^{T-1} u(k)^2$.
4. For $T = 10$, find a sequence of control inputs that minimizes the "fuel" $\sum_{k=0}^{T-1} |u(k)|$.
5. Compare both the inputs on the same plot.
6. Find the state trajectory under both inputs and compare each component of the state in a separate plot.

Q 4.5: Portfolio Optimization

Suppose there are seven stocks whose return per unit investment is denoted by a vector $r \in \mathbb{R}^7$ which is assumed to be a Gaussian random variable with mean and covariance given by $\hat{r}$ and Σ_r, respectively. The numerical values are uploaded as `Portfolio_problem.mat`. Suppose the initial amount you have is 1 unit. Answer the following problems by formulating and solving suitable optimization problems.

1. Determine how much to invest in each stock to maximize the expected return.
2. Determine how much to invest in each stock to minimize the variance subject to the expected return being at least $\mu \in \{0.0001, 0.0002, \ldots, 0.0007, 0.0008\}$. Plot the variance of the return under the respective optimal allocation, and plot them with respect to μ.
3. Suppose the initial allocation has assigned equal amount to each stock. If you purchase or sell stock worth amount u, the the platform charges a fee $c = 0.00005$. Determine the optimal redistribution of your portfolio including the transaction fee. Is this problem convex? If not, find a suitable convex relaxation and solve it.
4. Determine how much to invest in each stock to maximize expected return subject to constraint that the return exceeds 0.0005 with probability at least 0.8. How does the allocation change if the probability is set to 0.95?