$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} f_1(x_1 \ldots x_n) \\ f_2(x_1 \ldots x_n) \\ \vdots \\ f_n(x_1 \ldots x_n) \end{bmatrix} = \begin{bmatrix} f_1(x^*) \\ f_2(x^*) \\ \vdots \\ f_n(x^*) \end{bmatrix} = 0$$

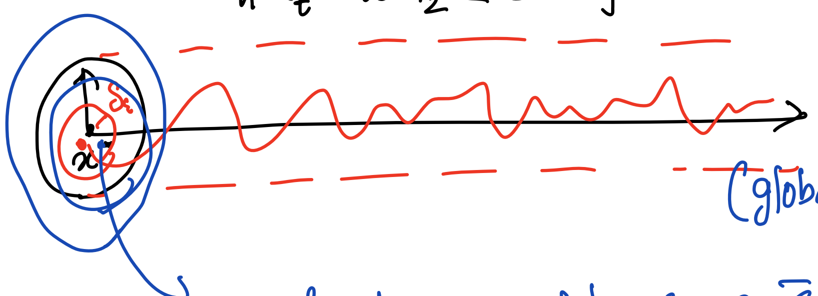
Lecture 24: 5th March

B. Stability

- Consider a continuous-time (autonomous) dynamical system: $\dot{x} = f(x)$ with initial state x_0.

- Equilibrium point: x^* is an eq^m point if $f(x^*) = 0$.

- Stability of an equilibrium point: x^* is said to be stable if for every $\varepsilon > 0$, there exists $\delta_\varepsilon > 0$ such that if $\|x_0 - x^*\|_2 \le \delta_\varepsilon$, then $\|x_t - x^*\|_2 \le \varepsilon$ for all $t \ge 0$.



If $\lim_{t \to \infty} x(t) = x^*$, we say that x^* is (globally) asymptotically stable.

x^* is unstable if there exists some $\bar{\varepsilon}$ s.t. we cannot find any $\delta > 0$ satisfying the above property.

- Lyapunov Stability Theorem: Let $V : \mathbb{R}^n \to \mathbb{R}_+$ such that following conditions are satisfied:

(i) $V(x^*) = 0$

(ii) $V(x) > 0$, for all $x \neq x^*$.

(iii) $\frac{d}{dt} V(x) < 0$ for all $x \neq x^*$.

(iv) When $\|x\|_2 \to \infty$, then $V(x) \to \infty$

Then, the equilibrium point x^* is globally asymptotically stable.

Applying chain rule,

$$\frac{d}{dt} V(x) = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} \cdot \frac{dx_i}{dt} = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} f_i(x) = \nabla_x V(x)^T f(x)$$

We often choose $V(x) = (x - x^*)^T P (x - x^*)$, $P = P^T$ is a positive definite matrix.

Stability of a Continuous-time LTI System

$x \in \mathbb{R}^n$

- An autonomous LTI System is stated as $\dot{x} = Ax$ with initial state x_0.

Let us derive conditions under which $x^* = 0$ is globally asymptotically stable (GAS).

Let us choose $V(x) = x^T P x$

$$\frac{d}{dt} V(x) = (\dot{x})^T P x + x^T P (\dot{x})$$
$$= (Ax)^T P x + x^T P (Ax)$$
$$= x^T \left[A^T P + P A \right] x$$

x^* is GAS if
$$\begin{cases} P = P^T \succ 0 & \text{(positive definite)} \\ A^T P + PA \prec 0 & \text{(negative definite)}. \end{cases}$$

Ex: $n = 2$, $P = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} = p_{11} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}_{E_1} + p_{12} \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{E_2} + p_{22} \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}_{E_3}$

E_1, E_2, E_3 are basis matrices in the space of symmetric 2×2 matrices.

For any n, we have $\frac{n(n+1)}{2}$ number of basis matrices.

$$\begin{aligned} P \succ 0 &\iff p_1 E_1 + p_2 E_2 + p_3 E_3 \succ 0 \\ A^T P + PA \prec 0 &\iff A^T \left(\sum_{i=1}^{3} p_i E_i \right) + \left(\sum_{i=1}^{3} p_i E_i \right) A \prec 0 \\ &\iff \sum_{i=1}^{3} p_i \left(A^T E_i + E_i A \right) \prec 0. \end{aligned}$$

$$\begin{aligned} \min \quad & 5 \\ \text{s.t} \quad & \sum_{i=1}^{3} p_i E_i \succ 0 \\ & \sum_{i=1}^{3} p_i \left(A^T E_i + E_i A \right) \prec 0 \end{aligned}$$

$F_0, F_1, \ldots F_n$: Known

Linear Matrix Inequalities

- Definition: $F_0 + x_1 F_1 + x_2 F_2 + \cdots + x_n F_n \preceq 0$

Consider the set $\left\{ x \in \mathbb{R}^n \;\middle|\; F_0 + \sum_{i=1}^{n} x_i F_i \preceq 0 \right\} = S$ $\left(\begin{matrix} \prec \\ \succeq 0 \\ \succ 0 \end{matrix}\right)$

$\to$ This set is a convex set.

Let $y, z \in S$,

$$F_0 + \sum_{i=1}^{n} \left(\lambda y_i + (1-\lambda) z_i \right) F_i$$

$$= F_0 + \lambda \sum_{i=1}^{n} y_i F_i + (1-\lambda) \sum_{i=1}^{n} z_i F_i$$

$$= \lambda F_0 + (1-\lambda) F_0 + (\quad)$$

$\lambda, 1-\lambda \geq 0$

$$= \lambda \underbrace{\left[F_0 + \sum_{i=1}^{n} y_i F_i \right]}_{\preceq 0} + (1-\lambda) \underbrace{\left[F_0 + \sum_{i=1}^{n} z_i F_i \right]}_{\preceq 0}$$

$$\preceq 0.$$

Consequently, the problem:

$$\left[\begin{array}{ll} \min\limits_{x \in \mathbb{R}^n} & c^T x \\ \text{s.t.} & \boxed{F_0 + \sum_{i=1}^{n} x_i F_i \preceq 0}, \end{array} \right. \quad : Z \succeq 0$$

is a convex optimization problem.

(also for $\succ, \succeq 0, \prec$ type of constraints)

Now, let us try to derive the dual of the above problem.

For two matrices A, B, $\langle A, B \rangle = \text{trace}(A^T B)$.

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}, \quad A^T B = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 7 & 1 \end{bmatrix}$$

$\text{trace}(A^T B) = 2 + 4 = 6$

Primal and Dual forms of Optimization with LMI Constraints

We define the Lagrangian to be

$$L(x,Z) = c^T x + \left\langle Z, F_0 + \sum_{i=1}^{n} x_i F_i \right\rangle$$

$$= \underline{c^T x} + \langle Z, F_0 \rangle + \underbrace{\sum_{i=1}^{n} x_i \langle Z, F_i \rangle}$$

If we are able to find $Z \succeq 0$ such that $\underline{\langle Z, F_i \rangle = -c_i} \ \forall i$,

then $\sum_{i=1}^{n} x_i \langle Z, F_i \rangle = \sum_{i=1}^{n} x_i(-c_i) = -c^T x$

When $Z \succeq 0$, and x is feasible,

i.e., $Z \succeq 0$ and $F_0 + \sum_{i=1}^{n} x_i F_i \preceq 0$,

then $\left\langle Z, F_0 + \sum_{i=1}^{n} x_i F_i \right\rangle \leq 0$.

$$\Rightarrow \langle Z, F_0 \rangle \leq -\sum_{i=1}^{n} x_i \langle Z, F_i \rangle = \sum_{i=1}^{n} x_i c_i = c^T x$$

Therefore, the dual optimization problem can be written as

$$\begin{cases} \max\limits_{Z \in S} \ \langle Z, F_0 \rangle \\ \text{s.t.} \quad Z \succeq 0 \\ \qquad \langle Z, F_i \rangle = -c_i, \quad i = 1, 2 \ldots n. \end{cases}$$

This class of optimization problems are called semidefinite programs (SDPs).

Stability of a Discrete-time LTI System

- An discrete-time autonomous LTI System is stated as $x_{k+1} = Ax_k$, $k \in \mathbb{Z}$. with initial state x_0.

as before, define $V(x) = x^T P x$, where $P = P^T$

$$V(x_{k+1}) - V(x_k) = (x_{k+1})^T P x_{k+1} - x_k^T P x_k$$
$$= (Ax_k)^T P (Ax_k) - x_k^T P x_k$$
$$= x_k^T \left[A^T P A - P \right] x_k$$

For $x^* = 0$ to be GAS, we need to find $P \succ 0$, $A^T P A - P \prec 0$.

$$A^T \left(\sum_i p_i E_i \right) A - \sum_i p_i E_i$$

$$= \sum_i p_i \left(A^T E_i A - E_i \right) \prec 0$$

Both define linear matrix inequalities (LMIs).

In practice, we write $P \succeq \varepsilon_1 I$, $A^T P A - P \preceq -\varepsilon_2 I$, for $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ being a small constants

to avoid situations where optimal solution may not be defined.

Lecture 25: 6th March

Properties of LMIs.

$F : \mathbb{R}^m \to \mathcal{S}^n$

$$F(x) = F_0 + x_1 F_1 + x_2 F_2 + \cdots + x_m F_m$$
$$= F_0 + \sum_{i=1}^{m} x_i F_i$$

find x s.t. $F(x) \succeq 0$ (or $\prec, \preceq, \succ$)

(A) Combining multiple LMIs

Suppose we are given $F_1 : \mathbb{R}^m \to \mathcal{S}^{n_1}$, $F_2 : \mathbb{R}^m \to \mathcal{S}^{n_2}$

$$F_1(x) \preceq 0, \quad F_2(x) \preceq 0 \iff \begin{bmatrix} F_1(x) & 0 \\ 0 & F_2(x) \end{bmatrix} \preceq 0.$$

The above holds because the eigenvalues of a block diagonal matrix are union of eigenvalues of the constituent diagonal blocks.

(B) Schur complement Lemma

Consider a matrix $M = \left[\begin{array}{c|c} A & C^T \\ \hline C & B \end{array}\right]$, A and B are symmetric

$\Rightarrow$ M is symmetric.

(i) $M \succeq 0 \iff A \succ 0$ and $B - CA^{-1}C^T \succeq 0$

(ii) $M \succeq 0 \iff B \succ 0$, and $A - C^T B^{-1} C \succeq 0$

(remember the clockwise movement along the block matrix)

(iii) Suppose $A = 0$. Then, $M \succeq 0$ would require $B \succeq 0$ and $C = 0$.

$$t^2 I - A(x)^T A(x) \succeq 0.$$

(C) Congruence Transformation

Suppose M that is symmetric. Let R be any other matrix.

Then $M \succeq 0 \Rightarrow \underbrace{R^T M R}_{} \succeq 0$.

$$w^T R^T M (Rw) = v^T M v \geq 0$$

For positive definiteness to be preserved, we require R to have full column rank.

$M \succ 0 \Rightarrow R^T M R \succ 0$ when R has full column rank.

If R is a square matrix, and is non-singular, then $M \succeq 0 \Leftrightarrow R^T M R \succeq 0$.

Example: Suppose $A: \mathbb{R}^m \to S^n$,

$$\begin{cases} \min\limits_{x \in \mathbb{R}^m} & \underbrace{\|A(x)\|_2}_{} \end{cases}$$

largest singular value of the matrix $A(x)$.

Recall that $\sigma_{max}(A(x))^2 = \lambda_{max}(A(x)^T A(x))$

converting the above problem to the epigraph form, we obtain

$$\min_{\substack{x \in \mathbb{R}^m \\ t \in \mathbb{R}}} t$$

$$\text{s.t.} \quad \|A(x)\|_2 \leq t$$

If we impose $t \geq 0$, then $\|A(x)\|_2^2 \leq t^2$

$$\Rightarrow \lambda_{max}(A(x)^T A(x)) \leq t^2$$

$$\Rightarrow \lambda_{max}(A(x)^T A(x)) - t^2 \leq 0.$$

$$\Rightarrow \lambda_i \underbrace{(A(x)^T A(x))}_{} \leq t^2 \quad \forall i = 1, 2 \dots k$$

$$(A(x)^T A(x)) \preceq (t^2 I)$$

$$\Rightarrow t^2 I - A(x)^T A(x) \succeq 0.$$

If $t > 0$, then $tI - A(x)^T \left(\frac{1}{t}\right) A(x) \succeq 0$

applying Schur complement lemma, we can write

$$\begin{bmatrix} t^2 I & A(x)^T \\ A(x) & I \end{bmatrix} \succeq 0 .$$

applying schur complement Lemma, we obtain

$$\begin{bmatrix} tI & A(x)^T \\ A(x) & tI \end{bmatrix} \succeq 0$$: linear in both x as well as t

we can now write
$$\min_{\substack{x \in \mathbb{R}^m \\ t \geq 0}} \quad t$$

which is a convex optimization problem.
$$\text{s.t.} \quad \begin{bmatrix} tI & A(x)^T \\ A(x) & tI \end{bmatrix} \succeq 0 .$$

C. State Feedback for a Continuous-time LTI System

$\Rightarrow \dot{x} = Ax + BKx$

$= \underbrace{(A+BK)}_{A_{cl}} x .$

- Consider the continuous-time system with input: $\dot{x} = Ax + Bu$.
- Suppose A is not stable. Can we use feedback to stabilize the system?
- Consider static full state feedback $u = Kx$ where K is a matrix of suitable dimension.
- Dynamics of the closed-loop system: $\dot{x} = (A + BK)x = A_{cl} x$
- Problem: under what condition is $(A + BK)$ stable? we need to find $P = P^T$ s.t. $P \succ 0$

$(A+BK)^T P + P(A+BK) \prec 0$

- Is this a LMI? If not, how to tackle this issue?

$\Rightarrow A^T P + K^T B^T P + PA + PBK \prec 0$

not a Linear matrix inequality.

$I = P^{-1}P = PP^{-1}$

Let us start with

$$I\left[A^T P + K^T B^T P + PA + PBK\right] I \prec 0$$

$$\Leftrightarrow P P^{-1}\left[A^T P + K^T B^T P + PA + PBK\right] P^{-1} P \prec 0$$

$$\Leftrightarrow P\left[P^{-1} A^T P + P^{-1} K^T B^T P + A + BK\right] P^{-1} P \prec 0$$

$$\Leftrightarrow P\left[P^{-1} A^T + P^{-1} K^T B^T + AP^{-1} + BKP^{-1}\right] P \prec 0$$

$$\Leftrightarrow P^{-1} A^T + \underbrace{P^{-1} K^T}_{(KP^{-1})^T} B^T + AP^{-1} + BKP^{-1} \prec 0 \quad \text{since } P \succ 0$$

Let us introduce new variables: $Q = P^{-1}$, $X = KP^{-1}$, $Q = Q^T \succ 0$

then

$$Q^T A^T + X^T B^T + AQ + BX \prec 0 \rightarrow \text{linear matrix inequality.}$$

Thus, we need to find (Q, X) such that

$$\Rightarrow \begin{cases} Q \succ 0 \\ Q^T A^T + X^T B^T + AQ + BX \prec 0 \end{cases}$$

Suppose we obtain (Q^*, X^*) satisfying the above LMIs.

then $X^* = KQ^* \Rightarrow \underline{K^* = X^* (Q^*)^{-1}}$

When we apply input $u = K^* x$, then origin is stable equilibrium of the closed loop system.

State Feedback for a Discrete-time LTI System

- An discrete-time LTI System is stated as $x_{k+1} = Ax_k + Bu_k$ with initial state x_0.

$u_k = Kx_k$

- What is the condition for the closed-loop system $x_{k+1} = (A + BK)x_k$ to be stable? Is this a LMI?

that is need to find $P = P^T \succ 0$ s.t.

$$(A+BK)^T P (A+BK) - P \prec 0$$

$X = P^{-1}$, $Z = P$

$Y = A+BK$

- Schur Complement Lemma: Consider three matrices $X \in \mathbb{S}^n, Y \in \mathbb{R}^{n\times m}, Z \in \mathbb{S}^m$. Then, the following are equivalent:

  1. $\begin{bmatrix} X & Y \\ Y^\top & Z \end{bmatrix} \succ 0.$
  2. $X - YZ^{-1}Y^\top \succ 0$ and $Z \succ 0$.
  3. $Z - Y^\top X^{-1} Y \succ 0$ and $X \succ 0$.

$$P - (A+BK)^T P (A+BK) \succ 0$$

$$\underbrace{\begin{bmatrix} P^{-1} & A+BK \\ (A+BK)^T & P \end{bmatrix} \succ 0}_{\text{not a LMI due to presence of } P^{-1}.}$$

let us examine

$$(A+BK)^T P (A+BK) - P \prec 0$$

$$\Leftrightarrow P^{-1}\left[(A+BK)^T P (A+BK) - P\right] P^{-1} \prec 0$$

$$\Leftrightarrow P^{-1}(A+BK)^T P (A+BK) P^{-1} - P^{-1} \prec 0$$

$$\Leftrightarrow \begin{bmatrix} P^{-1} & (A+BK)P^{-1} \\ ((A+BK)P^{-1})^T & P^{-1} \end{bmatrix} \succ 0.$$

now introduce $Q = P^{-1}$, $KP^{-1} = X$,

$$\begin{bmatrix} Q & AQ+BX \\ (AQ+BX)^T & Q \end{bmatrix} \succ 0$$ LMI in Q and X

once we solve it and obtain (Q^*, X^*), then

$$KQ^* = X^* \Rightarrow K = X^*(Q^*)^{-1}$$

Lecture 26 : 7th March

D. State Estimation of a Discrete-time LTI System

- An discrete-time autonomous LTI System is stated as $x_{k+1} = Ax_k + Bu_k$ with initial state x_0.

Suppose that the state vector is not known, rather we observe output $y_k = Cx_k$, C is not an identity matrix.

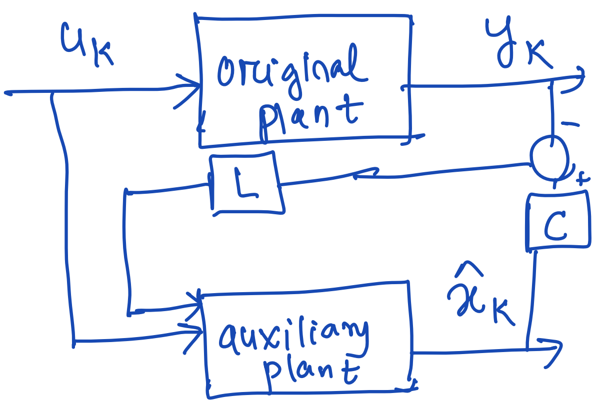
The state estimation problem is to estimate the state x_k as a function of $(u_k, y_k)_{k \geq 0}$.

A simple way to estimate the state is to create an auxiliary system

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + L\underbrace{(C\hat{x}_k - y_k)}_{\text{correction term}}$$

$\hat{x}_0$: picked in an arbitrary manner.

L → observer gain that we need to determine.



We define the estimation error $e_k = x_k - \hat{x}_k$

$$e_{k+1} = x_{k+1} - \hat{x}_{k+1} = (Ax_k + Bu_k) - (A\hat{x}_k + Bu_k + L(C\hat{x}_k - y_k))$$

$$= Ax_k - A\hat{x}_k - L(C\hat{x}_k - Cx_k)$$

$$= Ax_k - A\hat{x}_k - LC\hat{x}_k + LCx_k$$

$$= (A+LC)x_k - (A+LC)\hat{x}_k$$

$$\boxed{e_{k+1} = (A+LC)e_k}$$

$$\dot{e} = (A+LC)e$$

$\Longrightarrow$ discrete-time linear system, and we would like to find L s.t. origin is GAS.

we have already seen that for a DT system $x_{k+1} = \bar{A} x_k$, origin is GAS if $P = P^T \succ 0$ s.t. $\bar{A}^T P \bar{A} - P \prec 0$.

For the e_k dynamics: $P = P^T \succ 0$ s.t.

$$(A+LC)^T \underbrace{P}_{\to P P^{-1} P} (A+LC) - P \prec 0$$

$$\Leftrightarrow \quad (A+LC)^T P \, P^{-1} \, P(A+LC) - P \prec 0$$

$$\Leftrightarrow \quad P - \big(P(A+LC)\big)^T P^{-1} (PA + PLC) \succ 0$$

$$\Leftrightarrow \quad \begin{bmatrix} P & (PA + \overset{X}{PL}C)^T \\ PA+PLC & P \end{bmatrix} \succ 0$$

$$\Leftrightarrow \quad \begin{cases} \begin{bmatrix} P & (PA+XC)^T \\ PA+XC & P \end{bmatrix} \succ 0 \\ P \succ 0. \end{cases}$$

which is a LMI in (P, X).

once we obtain solution (P^*, X^*), we can find L as

$$P^* L = X^* \quad \Rightarrow \quad \boxed{L = (P^*)^{-1} X^*}$$

E. Robust Control

- So far, we have assumed that the system matrices (A, B, C) are known. However, in practice, we don't have completely accurate knowledge of the system dynamics.

- Here, we assume $A = \underline{A_{nom}} + \underline{\underline{\Delta(t)}}$, where

  A_{nom}: nominal dynamics that is known.

  $\Delta(t)$: perturbation that changes with time.

- We will look at the following two types of perturbations:

  (a) $\|\Delta(t)\|_2 \le \gamma$ (norm-bounded)

  (b) $\Delta(t) \in \underline{conv(\Delta_1, \Delta_2 \cdots \Delta_K)}$. (convex hull)

- The notion of stability we will look into is called $\underline{\text{quadratic}}$ stability, which requires us to find $P = P^T \succ 0$ s.t.

$$\frac{d}{dt} V(x) = \frac{d}{dt}(x^T P x) = (\dot{x})^T P x + x^T P \dot{x} < 0$$

$$\Rightarrow x^T (A_{nom} + \Delta(t))^T P x + x^T P (A_{nom} + \Delta(t)) x < 0$$

$$\Rightarrow x^T \left[\underbrace{A_{nom}^T P + P A_{nom} + \Delta(t)^T P + P \Delta(t)}_{} \right] x < 0$$

i.e.,

$$(*)\quad P^{-1}\left[A_{nom}^T P + P A_{nom} + \Delta(t)^T P + P \Delta(t)\right]P^{-1} \prec 0 \quad \text{for all } \|\Delta(t)\|_2 \le 1$$

$G = A_{nom}^T P + P A_{nom}$, $G = G^T$

$M = \underline{P}$, $N = I$

Petersen's Lemma

Let $G = G^T$, and M, N be two other matrices. Then

$$\boxed{G + M\Delta N + N^T\Delta^T M^T \preceq 0 \quad \forall \ \|\Delta\|_2 \leq 1} \quad \text{--(1)}$$

if and only if

there exists $\varepsilon \in \mathbb{R}$ s.t. $\begin{bmatrix} G + \varepsilon M M^T & N^T \\ N & -\varepsilon I \end{bmatrix} \preceq 0.$

LMI in ε

Therefore, $(*)$ is equivalent to

$$\begin{bmatrix} A_{nom}^T P + P A_{nom} + \varepsilon P P^T & I \\ I & -\varepsilon I \end{bmatrix} \preceq 0$$

which is not LMI in ε and P.

In order to tackle this,

$$P^{-1}\left[A_{nom}^T P + P A_{nom} + \Delta^T P + P\Delta\right]P^{-1} \prec 0$$

$$\Leftrightarrow \quad P^{-1}A_{nom}^T + A_{nom}P^{-1} + P^{-1}\Delta^T + \Delta P^{-1} \prec 0 \quad \text{----(2)}$$

Comparing (2) with (1), we obtain $G = P^{-1}A_{nom}^T + A_{nom}P^{-1}$
$M = I$, $N = P^{-1}$.

then (2) is equivalent to

$$\begin{bmatrix} P^{-1}A_{nom}^T + A_{nom}P^{-1} + \varepsilon I & P^{-1} \\ P^{-1} & -\varepsilon I \end{bmatrix} \preceq 0.$$

define $Q = P^{-1} \succ 0$, and the above is LMI in ε and Q.

Robust State Feedback Controller: $u = Kx$,

Closed-loop System is $\dot{x} = (A_{nom} + BK + \Delta(t))x$.

the problem of finding K such that origin is quadratically stable can be written as:

$$Q = Q^T \succ 0 \quad \& \quad \begin{bmatrix} Q(A_{nom}+BK)^T + (A_{nom}+BK)Q + \epsilon I & Q \\ Q & -\epsilon I \end{bmatrix} \preceq 0$$

- we will proceed as before to convert the above condition to a LMI.

Case 2: $\Delta(t) \in \text{conv}(\Delta_1, \ldots \Delta_k)$

Proposition: For any matrices H, $(L_i)_{i=1..n}$, $(R_i)_{i=1..n}$, the condition $H + \sum_{i=1}^{n} L_i \Delta R_i \succ 0 \quad \forall \Delta \in \text{conv}(\Delta_1 \ldots \Delta_k)$

$$\Leftrightarrow \quad H + \sum_{i=1}^{n} L_i \Delta_j R_i \succ 0 \quad \text{for } j = 1, 2 \ldots K$$

In our case, the goal is to find $P = P^T \succ 0$ s.t.

$$(A_{nom} + \Delta)^T P + P(A_{nom} + \Delta) \prec 0 \quad \forall \Delta \in \text{conv}(\cdot)$$

$$\Leftrightarrow \quad A_{nom}^T P + P A_{nom} + \Delta^T P + P\Delta \prec 0 \quad \forall \Delta \in \text{conv}(\cdot)$$

$$\Leftrightarrow \quad \left.\begin{aligned} &A_{nom}^T P + P A_{nom} + \Delta_1^T P + P\Delta_1 \prec 0 \\ &A_{nom}^T P + P A_{nom} + \Delta_2^T P + P\Delta_2 \prec 0 \\ &\quad \vdots \\ &A_{nom}^T P + P A_{nom} + \Delta_k^T P + P\Delta_k \prec 0 \end{aligned}\right\} \text{K number of LMIs needs to be satisfied.}$$