

A Simple Analytical Theory for Interpreting Measured Total Pressure in Multiphase Flows

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This paper presents a simple, analytical theory for determining total pressure in multiphase flows, a subject of theoretical interest as well as of practical importance. It is shown here that the nonequilibrium processes occurring in the vicinity of a measuring device have a significant influence on the magnitude of flow velocity inferred from Pitot measurements. The present theory predicts that, depending on the size of the particles or droplets, the total pressure varies monotonically between the two limiting values: the frozen total pressure (when there is no interphase mass, momentum, and energy transfer in the decelerating flow toward the stagnation point) and the equilibrium total pressure (when the dispersed phase, either liquid droplets, or solid particles, is always at inertial and thermodynamic equilibrium with the continuous vapour phase). The presented analytical theory is a relation between nondimensional total pressure and Stokes number, representing particle size or inertia, and specifies the total pressure under different nonequilibrium conditions. One simple equation applies to diverse multiphase mixtures, solid particle laden gas as well as vapour-droplet mixtures, and at a wide range of flow conditions, both subsonic and supersonic. The associated issue of interpreting total temperature, and the relation between measured total pressure and entropy production in multiphase flows have been discussed at length by Guha (1998).

1 Introduction

Mixtures of gases laden with solid particles or mixtures of a vapor and liquid droplets occur frequently in many practical flow situations. Pitot measurements are often used for inferring velocity or loss (entropy generation) in multiphase mixtures. However, in single phase fluids, it is normally permissible to assume that the fluid is brought to rest at the mouth of the Pitot tube isentropically. Hence flow Mach number and entropy generation (in steady, adiabatic flow) are uniquely determined by the total pressure measured by a Pitot tube, together with an independent measurement of the static pressure. In supersonic flow in an ideal gas, application of Rankine-Hugoniot equations across the detached shock wave in front of a Pitot tube retains the utility of Pitot measurements for deducing flow Mach number and entropy generation. Pitot measurements in a multiphase mixture, however, require careful considerations (Guha, 1998).

The solid particles or the liquid droplets respond to changes in temperature, velocity, etc. of the gas phase through interphase exchanges of mass, momentum, and energy. These are essentially rate processes and hence significant departures from equilibrium can take place if the rate of change of external conditions, imposed by the deceleration in the stagnating flow, is comparable to the internal time scales. Thus, for example, if the size of the liquid droplets or the solid particles is very small, then inertial and thermodynamic equilibrium between the two phases are maintained always, and a Pitot tube would measure the equilibrium total pressure, p_{e0} . On the other hand, if the size of the droplets or the particles is very large, all interphase transfer processes remain essentially frozen. The Pitot tube records the pressure which it would have recorded if the vapour phase alone was brought to rest from the same velocity. The total pressure in this case is termed the frozen total pressure, p_{f0} .

As an example, consider low-pressure wet steam with a typical wetness fraction of 10 percent and at a Mach number 1.5. Calculations show that $p_{f0}/p_{e0} = 3.3$ and $p_{e0}/p_{s0} = 3.79$, where p_{s0} is the static pressure. Therefore, in this particular example, the equilibrium total pressure is about 15 percent higher than the frozen total pressure.

The subject of interpretation of measured total pressure in two-phase mixtures has been considered by Taylor (1945), Dussourd and Shapiro (1958), Soo (1964), Rudinger (1964), and Crane and Moore (1972). None of these previous authors attempted any generalization for all flow conditions and different two-phase mixtures. For example, Crane and Moore considered Pitot measurements in wet steam at low subsonic velocities and concluded that the total pressure measured would lie between the frozen and the equilibrium value, and would virtually be independent of the droplet size. Soo, on the other hand, asserted that, for low mass fraction of the dispersed phase, a total head probe in a gas-particle flow would always measure the frozen total pressure.

However, as explained earlier, it is expected that for intermediate sizes of the droplets or particles, the pressure recorded by the probe would neither be the equilibrium nor the frozen value. The imposed deceleration in front of the Pitot tube would render the two-phase mixture into nonequilibrium situations, both inertially and thermodynamically. The deceleration process consequently ceases to be isentropic, as nonequilibrium exchanges of mass, momentum, and energy between the two phases create entropy. The degree of the nonequilibrium effects will depend on the droplet size, mass concentration, etc. or more precisely on the relaxation times of the system. Hence, relaxation gas dynamics has to be called for a solution of the problem. (For a lucid overview of the relaxation gas dynamics in vapor-droplet flow, one may refer to Guha, 1995.)

For a proper solution of the real flow field around the mouth of a Pitot tube, one therefore has to solve multidimensional (at least two dimensional axisymmetric) conservation equations with viscous, and thermal and inertial nonequilibrium effects. It is important to incorporate the inertial nonequilibrium effects, i.e., to allow a velocity slip between the two phases. It is so, not only

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Contributed by the Fluids Engineering Division for publication in the *JOURNAL OF FLUIDS ENGINEERING*. Manuscript received by the Fluids Engineering Division October 24, 1994; revised manuscript received February 9, 1998 (delayed due to editor's difficulties). Associate Technical Editor: D. E. Stock.

because the inertial effects are themselves significant but also because restraining the two phases to travel at the same velocity has serious implications for the thermal equilibration process. For example, a fluid particle moving along the stagnation streamline takes infinite time to reach the stagnation point. Therefore, in the absence of any velocity slip, the vapor-droplet mixture would have long enough time to come to thermal equilibrium, irrespective of the magnitude of the thermal relaxation time. In reality, however, large droplets may move without significant interphase mass and energy transfers and finally hit the back of the probe without influencing the pressure.

Most available computational schemes (explicit time-marching, e.g., Guha and Young, 1991; Young, 1992), however, neglect any velocity slip between the phases. In most situations with submicron size droplets, this is an acceptable assumption and saves computer time tremendously. (The relaxation time governing the velocity slip is very small and hence the computational time-step required for numerical simulation is small resulting in a large CPU time.) Even if all these effects are incorporated in the computer programs, it is quite likely that the numerical entropy generation would mask the entropy generation due to relaxation processes. Hence Guha (1998) presented a simple, quasi-one-dimensional model in which some plausible assumptions about the variation of the gas phase velocity were made. Step-by-step numerical integration of the conservation equations for a multiphase mixture, between a far-upstream point and the stagnation point, then determined the pressure that would have been recorded by a measuring device.

Guha (1998) considered a large number of two-phase mixtures, both gas-particle and vapour-droplet, at subsonic as well as supersonic velocities for many different sizes of the droplets (or particles). In the supersonic case a detached frozen shock wave stands in front of the Pitot tube. The relaxation mechanisms in a gas-particle mixture are different from those in a vapor-droplet flow. Despite all these complexities and differences, it was possible with proper nondimensionalization of flow parameters to adopt a *universal* plot, within acceptable tolerance, of nondimensional total pressure versus Stokes number (which is a nondimensional representation of particle inertia, Eq. (22)). The predicted total pressure correctly reduced to the frozen total pressure in the limit of large Stokes number (large particles) and to the equilibrium total pressure in the limit of small Stokes number (small particles). Maximum dependence of the total pressure on Stokes number was observed when the Stokes number was of the order unity.

Guha's (1998) calculations were, however, numerical in nature. In this paper, we develop an *analytical* theory for determining total pressure under nonequilibrium conditions, which is amenable to direct physical interpretation.

2 Conservation Equations

We consider a homogeneous, two-phase mixture consisting of a large number of small, spherical solid particles (or liquid droplets) dispersed in a continuous vapor phase. For simplicity, we assume that the particles (or droplets) are of the same size. Sufficient number density and uniform distribution of the particles (or droplets) make their interaction with the vapor describable by a continuous variation. We adopt the usual "two-fluid" model and view the particles (or droplets) as providing sources or sinks of mass, momentum, and energy for the vapor, each source term varying continuously in the x -direction.

The mass fraction of the dispersed phase, y , is given by

$$y = nm, \quad (1)$$

where there are n particles (or droplets), each of mass m , per unit mass of the mixture. The mass of an individual particle is connected to its radius, r , and the material density of the dispersed phase, ρ_d , via

$$m = 4/3\pi r^3 \rho_d. \quad (2)$$

If the vapor density is ρ_g , the mixture density ρ (neglecting the volume of the dispersed phase) is

$$\rho = \rho_g/(1 - y), \quad (3)$$

and the number of particles (or droplets) N per unit volume is given by:

$$N = \frac{n\rho_g}{(1 - y)}. \quad (4)$$

We assume that the vapor phase behaves as a perfect gas with constant isobaric specific heat capacity c_{pg} . Thus

$$p = \rho_g RT_g \quad (5)$$

and,

$$\frac{dh_g}{dx} = c_{pg} \frac{dT_g}{dx} \quad (6)$$

where R is the specific gas constant and T_g is the temperature of the vapor phase. For the dispersed phase,

$$\frac{dh_d}{dx} = c_d \frac{dT_d}{dx} \quad (7)$$

where h_d , T_d , and c_d are the specific enthalpy, temperature and the isobaric specific heat of the particles (or droplets).

Nomenclature

A = cross-sectional area of the stream-tube
 a_e = equilibrium speed of sound
 a_f = frozen speed of sound
 c_d = isobaric specific heat of the dispersed phase
 c_{pg} = isobaric specific heat of the gas phase
 D = diameter of the Pitot tube
 δ = c_d/c_{pg}
 γ = isentropic index of the gas phase
 $\bar{\gamma}$ = isentropic index of a gas-particle mixture
 h = enthalpy
 Kn = Knudsen number
 λ_g = thermal conductivity of the gas phase

m = mass of an individual particle (or droplet)
 M_e = equilibrium Mach number
 M_f = frozen Mach number
 μ_g = dynamic viscosity of the gas phase
 N = number of particles (or droplets) per unit volume
 n = number of particles (or droplets) per unit mass of the mixture
 p = pressure
 p_o = total pressure
 p_{oe} = equilibrium total pressure
 p_{of} = frozen total pressure
 Pr = Prandtl number of the gas phase
 R = specific gas constant
 ρ = density of the mixture
 r = radius of a particle (or droplet)

ρ_d = material density of the dispersed phase
 ρ_g = density of the gas phase
 R_p = dimensionless total pressure
 St = Stokes number
 T = temperature
 τ_i = inertial relaxation time
 τ_T = thermal relaxation time
 x = coordinate along the flow direction
 y = mass fraction of the dispersed phase

Subscripts

∞ = unperturbed flow variables far upstream of the Pitot tube
 g = gas phase
 d = dispersed phase (particles or droplets)

The basic one-dimensional gasdynamic equations for steady, non-nucleating flow for a two-phase system can be written in the usual way (Guha, 1992):

Droplet number conservation:

$$\frac{d}{dx} (ANV_d) = 0 \quad (8)$$

Continuity:

$$\frac{d}{dx} (A\rho_g V_g) + \frac{d}{dx} (ANmV_d) = 0 \quad (9)$$

Momentum:

$$A \frac{dp}{dx} + \frac{d}{dx} (A\rho_g V_g^2) + \frac{d}{dx} (ANmV_d^2) = 0 \quad (10)$$

Energy:

$$\frac{d}{dx} \left[\left(h_g + \frac{V_g^2}{2} \right) A\rho_g V_g \right] + \frac{d}{dx} \left[\left(h_d + \frac{V_d^2}{2} \right) ANmV_d \right] = 0 \quad (11)$$

where h is the specific enthalpy, V is the velocity, A is the flow area, and x is the distance along the flow direction. The subscript g denotes the vapor phase and subscript d refers to the dispersed phase. Equations (1)–(11) are valid for a solid particle laden gas as well as a vapor-droplet mixture.

Equations (1)–(11) have to be complemented with rate equations representing interphase transfer processes. For a solid-particle laden gas we need two equations representing the variation of particle velocity and temperature. These are given by (Guha 1998)

$$V_d \frac{dV_d}{dx} = \frac{V_g - V_d}{\tau_i} \quad (12)$$

$$V_d \frac{dT_d}{dx} = \frac{T_g - T_d}{\tau_T} \quad (13)$$

The inertial relaxation time τ_i and the thermal relaxation time τ_T are given by,

$$\tau_i = \frac{2r^2 \rho_d}{9\mu_g} [1 + 4.5 \text{Kn}], \quad (14)$$

$$\tau_T = \frac{r^2 \rho_d c_d}{3\lambda_g} \left(1 + \frac{4.5 \text{Kn}}{\text{Pr}} \right), \quad (15)$$

where μ_g , λ_g , and Pr are the dynamic viscosity, thermal conductivity and the Prandtl number of the gas phase and Kn is the particle Knudsen number ($\text{Kn} = l_p/2r$, where l_p is the molecular mean free path of the gas phase). Equation (13) needs to be replaced by a different equation in the case of vapour-droplet flow (Guha, 1998).

In a solid-particle-laden gas, there is no interphase mass transfer ($dm/dx = 0$). The mass conservation equation is, therefore, separately valid for each phase. Equations (1), (4), (8), and (9) can then be combined to give

$$\frac{Nm}{\rho_g} V_d = \frac{y_w}{1 - y_w} V_g, \quad (16)$$

where y_w is the mass fraction of the particles far upstream of the measuring device where the mixture is assumed to be at

equilibrium. After some algebraic manipulation of Eqs. (1)–(11), the above momentum Eq. (10) can be expressed as:

$$\frac{1}{\rho_g} \frac{dp}{dx} + V_g \frac{dV_g}{dx} + \frac{NmV_d}{\rho_g} \frac{dV_d}{dx} = 0. \quad (17)$$

Equations (16) and (17) can be combined to give

$$\frac{1}{\rho_g} \frac{dp}{dx} + V_g \frac{dV_g}{dx} + \frac{y_w}{1 - y_w} V_g \frac{dV_d}{dx} = 0. \quad (18)$$

Equation (18) is exact. Note that, in flows with velocity slip between the phases, the value of $y/(1 - y)$ does not remain constant along a streamline even in the absence of any interphase mass transfer. Equation (18) shows the correct form of the droplet momentum term (the third term in the LHS of Eq. 18) which contains the product of the gas phase velocity and dV_d/dx . The coefficient $y_w/(1 - y_w)$ remains constant for a specific upstream condition.

3 Analytical Solution for Total Pressure Under Non-equilibrium Conditions

For a complete solution of Eqs. (8)–(11), one has to prescribe the variation of the area of the stream tube in front of the Pitot tube (Fig. 1). Since the area variation is not known, the variation of the gas phase velocity is assumed. The vapor velocity is assumed to vary exponentially from the Pitot mouth,

$$V_g = V_\infty [1 - \exp(x/kD)], \quad (19)$$

where V_∞ is the unperturbed velocity far upstream, D is the pitot diameter, and k is an empirical constant which may be determined experimentally. Equation (19) shows that as $x \rightarrow 0$, $V_g \rightarrow 0$; as $x \rightarrow -\infty$, $V_g \rightarrow V_\infty$. (It is found that the nondimensional curve representing the variation in total pressure, as shown later in Fig. 3, does not depend on the value of k . The parameter k is introduced in the theory to increase its flexibility to match with experiments. If no other data is available then use $k \sim 1$.)

For the sake of a simple theory, we linearize Eq. (12) so that an analytical solution can be formulated,

$$V_g \frac{dV_d}{dx} = \frac{V_g - V_d}{\tau_i} \quad (20)$$

Variation in effective area

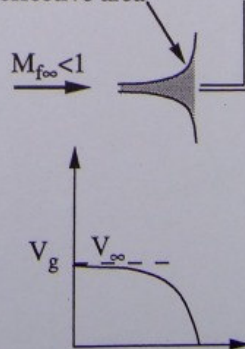


Fig. 1 Schematic flow structure in front of a Pitot tube and assumed variation in velocity of the gas phase

Equation (19) is substituted in Eq. (20) and the resulting equation is integrated subject to the boundary condition: at $x = -\infty$, $V_d = V_g = V_\infty$ to give

$$\frac{V_d}{V_\infty} = 1 - \frac{\exp(x/kD)}{1 + St} \quad (21)$$

where the dimensionless Stokes number, St , is defined by

$$St = \frac{\tau_p V_\infty}{kD} \quad (22)$$

The Stokes number, given by (22), is nothing but a Damkohler parameter signifying the importance of the relaxation processes.

It can be seen that Eq. (21) prescribes qualitatively correct variation of V_d in different limits. For example in the frozen limit, $St \rightarrow \infty$, $V_d = V_\infty$ always. In the equilibrium limit, $St \rightarrow 0$, $V_d = V_g$ always. Figure 2 shows the variation of V_d predicted by Eq. (21) for three different values of St .

Equations (19) and (21) can be substituted in Eq. (18), and the resulting equation can then be integrated between $x = -\infty$ and $x = 0$ in order to find the change in pressure as the gas is brought to rest by the Pitot tube. As a first approximation, we assume that the gas phase is effectively incompressible, so that we can use some average value for its density, ρ_g . (Later the effects of such approximation is studied.) In the frozen limit, $dV_d/dx \rightarrow 0$. Therefore, integrating Eq. (18) between $x = -\infty$ and $x = 0$ in the frozen limit, we obtain,

$$p_{of} - p_\infty = 1/2 \rho_g V_\infty^2 \quad (23)$$

where, p_{of} is the frozen total pressure and p_∞ is the pressure far upstream.

In the equilibrium limit, $V_g = V_d$. Again, integrating Eq. (18) between $x = -\infty$ and $x = 0$ in the equilibrium limit, we obtain,

$$p_{oe} - p_\infty = 1/2 \frac{\rho_g}{1 - y_\infty} V_\infty^2 \quad (24)$$

where p_{oe} is the equilibrium total pressure. Now we integrate Eq. (18) between $x = -\infty$ and $x = 0$ in the general nonequilibrium case with V_g and V_d given by Eqs. (19) and (21), respectively. The result is,

$$p_o - p_\infty = 1/2 \rho_g V_\infty^2 \left(1 + \frac{y_\infty}{1 - y_\infty} \frac{1}{1 + St} \right) \quad (25)$$

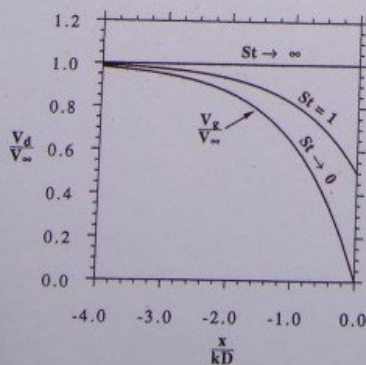


Fig. 2 Predicted variation in velocity of the particles (or droplets) in front of a Pitot tube as a function of Stokes number

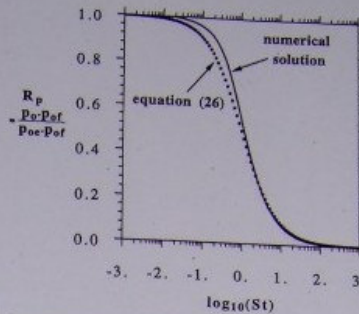


Fig. 3 Comparison of the analytical theory with numerical solutions for nondimensional total pressure. (For numerical solutions: $M_{f\infty} = 0.8$, $y_\infty = 0.1$, $\delta = 2.0$.)

Substituting Eqs. (23) and (24) in Eq. (25), one obtains,

$$R_p = \frac{p_o - p_{of}}{p_{oe} - p_{of}} = \frac{1}{1 + St} \quad (26)$$

where R_p is the nondimensional total pressure. Although the analysis in this section assumed the gas phase density to remain constant at some average value, Eq. (26), because of the particular definition of R_p , compares very well with full numerical solution of Eqs. (8)–(13) even at very high subsonic velocity. Figure 3, for example, shows the comparison of Eq. (26) with an accurate numerical solution of Eqs. (8)–(13) at $M_{f\infty} = 0.8$, where $M_{f\infty}$ is the frozen Mach number far upstream. The numerical solution is obtained for air with solid particles ($\delta = c_d/c_{pg} = 2.0$).

It should be noted that if the droplet momentum term in Eq. (17) were approximated by $[y_\infty/(1 - y_\infty)] V_d dV_d/dx$, then a similar analysis would have given

$$R_p = \frac{p_o - p_{of}}{p_{oe} - p_{of}} = 1 - \left(\frac{St}{1 + St} \right)^2 \quad (27)$$

4 Analytical Theory Incorporating Density Changes of the Gas Phase and Thermal Nonequilibrium Effects

In this section, we introduce the effects of density change of the gas phase. For this, we rewrite Eq. (18) as

$$\frac{1}{p} \frac{dp}{dx} + \frac{V_g}{RT_g} \frac{dV_g}{dx} + \frac{y_\infty}{1 - y_\infty} \frac{V_g}{RT_g} \frac{dV_d}{dx} = 0 \quad (28)$$

Equation (28) can be integrated in the same manner as we integrated Eq. (18) in the previous section, if the additional variation in T_g is substituted in Eq. (28). In the frozen limit, the energy equation (11) shows that the temperature of the gas phase, T_{gf} , is given by

$$T_{gf} = (c_{pg} T_\infty + \frac{1}{2} V_\infty^2 - \frac{1}{2} V_g^2) / c_{pg} \quad (29)$$

where, T_∞ is temperature at the far upstream. In the equilibrium limit, when the gas and the particles are always at thermal equilibrium, the temperature T_{ge} is again given by the energy equation,

$$T_{ge} = (c T_\infty + \frac{1}{2} V_\infty^2 - \frac{1}{2} V_g^2) / c \quad (30)$$

where the mixture specific heat, \bar{c} , is given by

$$\bar{c} = (1 - y)c_{pg} + yc_d \quad (31)$$

Since a complete solution of all the equations to determine T_x is not feasible in the present context of an analytical theory, we assume that, in analogy with Eq. (26), the temperature under nonequilibrium situations varies as

$$T_x = T_{gf} + \frac{T_{ge} - T_{gf}}{1 + St} \quad (32)$$

where T_{gf} and T_{ge} are given by Eqs. (29) and (30), respectively. Since V_x varies with x , Eq. (32) prescribes the variation of T_x with x . Equations (19), (21), and (32) can be substituted in Eq. (28) and the resulting equation integrated between $x = -\infty$ to $x = 0$. The result is

$$\frac{p_o}{p_\infty} = \left(1 + C \frac{V_\infty^2}{T_\infty}\right)^{(1+K)/(2RC)} \quad (33)$$

where,

$$C = \frac{1}{2} \left[\frac{1}{c_{pg}} + \frac{1}{1 + St} \left(\frac{1}{\bar{c}} - \frac{1}{c_{pg}} \right) \right] \quad (34)$$

and,

$$K = \frac{y_\infty}{1 - y_\infty} \frac{1}{1 + St} \quad (35)$$

Equation (33) reduces to exact solutions (Guha, 1998) in the limits of the frozen and equilibrium flow. In frozen flow, Eq. (33) reduces to

$$\frac{p_{of}}{p_\infty} = (1 + 0.5(\gamma - 1)M_{f\infty}^2)^{\gamma/(\gamma-1)} \quad (36)$$

where, p_{of} is the frozen total pressure, γ is the isentropic exponent of the gas phase alone, and, the far upstream frozen Mach number, $M_{f\infty}$, is calculated from,

$$M_{f\infty} = \frac{V_\infty}{a_{f\infty}} = \frac{V_\infty}{\sqrt{\gamma RT_\infty}} \quad (37)$$

In the equilibrium limit, Eq. (33) reduces to,

$$\frac{p_{oe}}{p_\infty} = (1 + 0.5(\bar{\gamma} - 1)M_{e\infty}^2)^{\bar{\gamma}/(\bar{\gamma}-1)} \quad (38)$$

where, p_{oe} is the equilibrium total pressure, $\bar{\gamma}$ is the mixture isentropic exponent, and $M_{e\infty}$ is the equilibrium Mach number at far upstream. $\bar{\gamma}$ and $M_{e\infty}$ are given by

$$\bar{\gamma} = \frac{(1 - y)c_{pg} + yc_d}{(1 - y)(c_{pg} - R) + yc_d} \quad (39)$$

$$M_{e\infty} = \frac{V_\infty}{a_{e\infty}} = \frac{V_\infty}{\sqrt{\bar{\gamma}(1 - y)RT_\infty}} \quad (40)$$

The nondimensional total pressure R_p [$R_p = (p_o - p_{of})/(p_{oe} - p_{of})$] can be calculated from Eqs. (33), (36), and (38) and plotted against Stokes number, St . The nature of the variation of R_p with St is, however, not immediately obvious from the complicated formulae. When the plot of R_p versus St obtained in this more

rigorous way was compared with the prediction of Eq. (26), the results were almost identical at all Mach numbers and for different two-phase mixtures. Equation (26), although derived less rigorously, is simple, amenable to direct physical interpretation and gives almost exact answer. Equation (26) is, therefore, recommended for practical calculation methods. (The slight difference between Eq. (26) and the numerical solutions in Fig. 3 arises from the use of the linearized Eq. (20) in the derivation of Eq. 26.)

5 Conclusions

A simple, analytical theory is presented for calculation of the total pressure in two-phase flows. Equation (26) presented here is a very simple relation between the nondimensionalized total pressure, R_p , and Stokes number, St . The equation correctly reduces to the frozen total pressure in the limit of large St (large particles) and to the equilibrium total pressure in the limit of small St (small particles). Under nonequilibrium conditions for intermediate St , the prediction of this equation compares well with results from full numerical solution of the gas dynamic equations for multiphase mixtures.

Although Eq. (26) has been derived here by considering subsonic two-phase flow without any interphase mass transfer, it has been shown (Guha, 1998) that the same calibration curve of R_p versus St could be adopted with acceptable accuracy for diverse multiphase mixtures, vapor-droplet as well as solid particle laden gases, at a wide range of subsonic and supersonic Mach numbers. Equation (26) can, therefore, be used for inferring flow velocity from measured Pitot pressures in any multiphase mixture at a wide range of flow conditions. The associated problem of interpreting total temperature, analytical relations for determining p_{of} and p_{oe} in vapor-droplet mixtures, and, the relation between measured total pressure and entropy production in multiphase flow are discussed at length by Guha (1998).

Acknowledgment

The author is grateful to Gonville & Caius college, Cambridge, for electing him as a Research Fellow.

References

- Crane, R. I., and Moore, M. J., 1972, "Interpretation of Pitot Pressure in Compressible Two-Phase Flow," *Journal of Mechanical Engineering Science*, Vol. 14, No. 2, pp. 128-133.
- Dussourd, J. L., and Shapiro, A. H., 1958, "A Deceleration Probe for Measuring Stagnation Pressure and Velocity of a Particle-Laden Gas Stream," *Jet Propulsion*, Vol. 28, pp. 24-34.
- Guha, A., and Young, J. B., 1991, "Time-Marching Prediction of Unsteady Condensation Phenomena Due to Supercritical Heat Addition," *Proc. Conf. Turbomachinery: Latest Developments in a Changing Scene*, London, IMechE, pp. 167-177.
- Guha, A., 1992, "Structure of Partly Dispersed Normal Shock Waves in Vapor-Droplet Flows," *Physics Fluids A*, Vol. 4, No. 7, pp. 1566-1578.
- Guha, A., 1995, "Two-Phase Flows with Phase Transition," *VKI Lecture Series*, 1995-06 (ISSN0377-8312), pp. 1-110, von Karman Institute for Fluid Dynamics.
- Guha, A., 1998, "A Unified Theory for the Interpretation of Total Pressure and Temperature in Two-Phase Flows at Subsonic and Supersonic Speeds," *Proceedings of Royal Society, Series A*, Vol. 454, pp. 671-695.
- Rudinger, G., 1964, Reply to comments by S. L. Soo, *Physics Fluids*, Vol. 7, pp. 1884-1885.
- Soo, S. L., 1964, Comments on "Shock Relaxation in Flows carrying Particles," *Physics Fluids*, Vol. 7, pp. 1883-1884.
- Taylor, G. I., 1945, Pitot pressures in moist air, reports and memoranda of the Aeronautical Research Council no. 2248.
- Young, J. B., 1992, "Two-Dimensional, Nonequilibrium Wet Steam Calculations for Nozzles and Turbine Cascades," *ASME Journal of Turbomachinery*, Vol. 114, pp. 569-579.