Thermal Choking Due to Nonequilibrium Condensation

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A theory of thermal choking due to nonequilibrium condensation in a nozzle is presented. An explicit equation for the critical quantity of heat in condensing flow has been derived. The equation is of general validity and applies to vapor-droplet flow with or without a carrier gas. It has been usually assumed in the literature that the classical gas dynamics result for the critical quantity of heat applies in condensing flow as well. The classical result is, however, obtained by considering external heat addition to an ideal gas in a constant area duct. In this paper it is shown that the area variation across the condensation zone (although small) and the depletion in the mass of vapor as a result of condensation have profound effects on the critical quantity of heat. The present equation (derived from an integral, control-volume approach) agrees very well with results from full time-marching solution of the nonequilibrium, differential gas dynamic equations. The classical gas dynamics result, on the other hand, seriously underpredicts the critical heat for condensing flow in nozzles (by a factor of three in the example calculation presented).

1 Introduction

It is well known that heat addition causes a reduction in Mach number in supersonic flow and an increase in Mach number in subsonic flow. In other words, heat addition to a flowing fluid drives the Mach number towards unity. Therefore, at a particular flow Mach number, the fluid can absorb a maximum quantity of heat before the local Mach number equals unity and the flow becomes thermally choked. It can be easily shown from classical gas dynamics that this critical quantity of heat, $\dot{q}_{\text{classical}}$, for simple heat addition (external heat addition without any change in flow cross-sectional area) to an ideal gas is given by (Shapiro, 1953, equation 7.14)

$$\frac{\dot{q}_{\text{classical}}}{c_p T_{01}} = \frac{[M_1^2 - 1]^2}{2(\gamma+1)M_1^2\left[1+\frac{\gamma-1}{2}M_1^2\right]} \qquad (1)$$

where c_p is the isobaric specific heat of the ideal gas, M_1 is the Mach number at which the heat addition begins, T_{01} is the stagnation temperature before heat addition, and γ is the isentropic index (ratio of the two specific heats).

In many practical situations, the energy is supplied internally. For example, when a dry, superheated vapor is expanded rapidly through a convergent-divergent nozzle, liquid droplets do not form as soon as the vapor reaches the saturation temperature. The vapor goes out of thermodynamic equilibrium and attains considerable subcooling, ΔT, (i.e., the vapor temperature, T, falls below the local saturation value, T_s) due to continued expansion. The rate of formation of liquid nuclei is very strongly dependent on the subcooling. Thus when the subcooling becomes appreciable (usually in the divergent part of the nozzle where the flow velocity is supersonic), a very large number of very small nuclei form over a relatively short time. These nuclei grow by exchanging heat and mass with the surrounding, subcooled vapor. The resulting release of latent heat is conducted back to the vapor and the vapor temperature quickly rises to the local saturation value (i.e., the subcooling decreases to almost zero). This rapid reversion to equilibrium is generally termed condensation shock and has been the topic of an extremely large number of studies. (The term condensation shock is, in general, a misnomer. Although heat addition in a supersonic flow results in an increase in pressure, the rise is gradual and the Mach number at the end of the condensation zone, in general, remains above unity.)

Similar to the case of external heat addition, the Mach number decreases in the condensation zone (the flow being supersonic). Therefore, for particular combinations of nozzle geometry, supply conditions and the working fluid, the liberation of latent heat could be such that the minimum Mach number becomes unity and the flow is thermally choked. (A numerical computation of this limiting case of thermal choking due to nonequilibrium condensation is later shown in Fig. 4.) If the inlet total temperature, T_{01}, is reduced any further, keeping the inlet total pressure, p_{01}, fixed, continuous variation of the flow variables is no longer possible and an aerodynamic shock wave appears inside the condensation zone (Barschdorff and Fillipov, 1970; Guha and Young, 1991).

It is often stated (Wegener and Mack, 1958; Pouring, 1965; Wegener, 1969; Wegener and Cagliostro, 1973; Skillings et al., 1987) that supercritical condensation with an inbuilt frozen shock wave occurs when $q > \dot{q}_{\text{classical}}$, where $\dot{q}_{\text{classical}}$ is given by Eq. (1). It has been argued by Guha (1994) that Eq. (1) is not appropriate for a condensing flow primarily for two reasons: (i) In case of a condensation shock, the energy is added

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as a result of condensation of a part of the fluid itself. Therefore, the mass flow rate of the condensable vapor changes as the vapor is continually transformed into the liquid phase. Equation (1), which is derived for external heat addition to an ideal gas, does not take into account this mass depletion. (ii) The droplets formed through homogeneous nucleation grow at a *finite rate* by exchanging mass and energy with the surrounding vapor. Therefore, the energy addition due to condensation is not instantaneous and takes place over a short but finite zone. Since condensation shock normally occurs in the diverging section (with dry vapor at inlet), this means the flow area increases between the upstream and downstream of the condensation zone. Equation (1), on the other hand, is derived by assumed heat addition in a constant area duct.

The purpose of the present paper is to derive an expression for the critical amount of heat taking proper account of the above two effects. This would then show the level of errors incurred as a result of the approximations in Eq. (1), which is referred extensively in the context of nonequilibrium condensation. Here we adopt an integral, control volume approach; conditions of choking from differential equations of motion may be found in Guha (1994) and Young (1984). All numerical results presented in this paper are obtained for pure steam, but the analysis is valid for other vapor-droplet mixtures with or without an inert carrier gas.

2 Thermal Choking Considering External Heat Addition With Area Variation

In order to appreciate the effects of area variation and mass depletion on the critical quantity of heat separately, the effects of area variation only are considered in this section. We, therefore, consider external heat addition to an *ideal gas* in a diverging flow section. We denote the pressure, density, velocity, temperature, sound speed, flow area and the rate of heat addition per unit mass by p, ρ, V, T, a, A, and q, respectively. The subscripts 1 and 2 respectively denote the upstream and downstream sections of the control volume within which the heat q is added. We can then write the continuity, momentum and the energy equation across the control volume as,

Continuity

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 \equiv \dot{m} \tag{2}$$

Momentum

$$p_1 A_1 - p_2 A_2 + 0.5(p_1 + p_2)(A_2 - A_1) = \dot{m}(V_2 - V_1) \tag{3}$$

Energy

$$a_1^2\left[\frac{1}{\gamma-1}+\frac{M_1^2}{2}\right]+q=a_2^2\left[\frac{1}{\gamma-1}+\frac{M_2^2}{2}\right] \tag{4}$$

Equation of state

$$p=\rho R T \tag{5}$$

where R is the specific gas constant. While writing the momentum conservation Eq. (3), the general term $\int A dp$ is expressed as $\int d(Ap) - \int p dA$, and the axial component of the pressure force on the nozzle wall ($\int p dA$) has been approximated by assuming a linear variation of pressure. It is advantageous to employ this approximation in an integral theory. The accuracy of this method has been compared shortly with an exact formulation for an isentropic flow, and later with an accurate time-marching solution of a condensation shock. Both comparisons show that this is a valid approximation for the present problem.

Combining (2) and (5), we obtain

$$p=\frac{\dot{m}}{A\gamma M^2}V \tag{6}$$

Substitute (6) in (3) to give

$$V_2\left[\gamma+\frac{\overline{A}}{A_2}\frac{1}{M_2^2}\right]=V_1\left[\gamma+\frac{\overline{A}}{A_1}\frac{1}{M_1^2}\right] \tag{7}$$

where,

$$\overline{A}=0.5(A_1+A_2) \tag{8}$$

From (4) and (7),

$$\frac{\frac{1}{M_2^2}\left[\frac{1}{\gamma-1}+\frac{M_2^2}{2}\right]}{\left[\gamma+\frac{\overline{A}}{A_2}\frac{1}{M_2^2}\right]^2}=\frac{\frac{1}{M_1^2}\left\{\left[\frac{1}{\gamma-1}+\frac{M_1^2}{2}\right]+\frac{q}{a_1^2}\right\}}{\left[\gamma+\frac{\overline{A}}{A_1}\frac{1}{M_1^2}\right]^2} \tag{9}$$

Now we are in a position to assess the accuracy of Eq. (3). Assuming the face of the control volume with subscript 1 to coincide with the geometric throat (with area A^*) where the Mach number is unity, we can write the variation of local isentropic Mach number as a function of local flow area by setting $q = 0$ in (9). The result is,

$$\left[2\gamma+1+\frac{A}{A^*}\right]^2 M^2\left[\frac{1}{\gamma-1}+\frac{M^2}{2}\right]=\left[\frac{1}{\gamma-1}+\frac{1}{2}\right]\left[2\gamma M^2+1+\frac{A^*}{A}\right]^2 \tag{10}$$

Nomenclature

- A = flow cross-sectional area
- a = speed of sound in gas, or vapor phase
- A^* = area at geometric throat of nozzle
- c_p = isobaric specific heat of gas, or vapor phase
- $\overline{e}$ = specific internal energy of vapor-droplet mixture
- γ = isentropic exponent of gas or vapor phase
- $\overline{h}$ = specific enthalpy of vapor-droplet mixture
- h_{fg} = specific enthalpy of evaporation
- h_l = specific enthalpy of liquid phase
- h_v = specific enthalpy of vapor phase
- M = Mach number (frozen value)
- $\dot{m}$ = mass flow rate of gas, or vapor phase
- $\dot{m}_{total}$ = combined mass flow rate of vapor plus liquid phase
- p = pressure
- p_0 = stagnation pressure
- q = rate of heat addition per unit mass
- q^* = critical heat ($q^* \equiv y^* h_{fg}$ in condensing flow)
- ρ = density of gas, or vapor phase
- $\overline{\rho}$ = density of vapor-droplet mixture
- T = temperature of gas, or vapor phase
- T_s = saturation temperature
- ΔT = subcooling ($\Delta T = T_s - T$)
- T_0 = stagnation temperature
- V = velocity
- y = wetness fraction
- y^* = critical amount of condensation

Subscripts

- 1 = upstream of control volume (beginning of condensation zone)
- 2 = downstream of control volume (point of thermal choking)

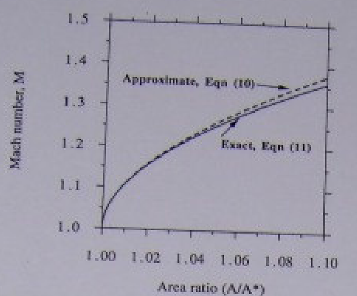

Fig. 1 Comparison of approximate and exact solutions for isentropic flow

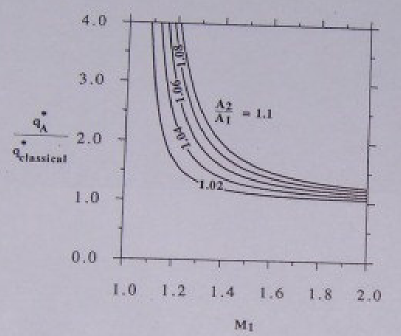

Fig. 2 Effects of area ratio across the zone of heat addition on the critical amount of heat

The exact variation for isentropic flow is given in classical gas dynamics (Shapiro, 1953)

$$\left[\frac{A}{A^*}\right]^2=\frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{\gamma-1}} \tag{11}$$

Figure 1 shows the comparison of the approximate result from the present theory, Eq. (10), with the exact solution, Eq. (11). It may be seen that for area ratios of the order of 1.1, the present theory gives acceptable solution. [The area ratio across the condensation zone of the practical example shown later (Section 4) is 1.028. The prediction of Eq. (10) is then little different from the exact solution (11).]

The critical quantity of heat needed to choke the flow thermally may be determined by substituting $M_2 = 1$ in (9). We denote this critical amount of heat by $\dot{q}_A$, to distinguish this from the classical prediction $\dot{q}_{\text{classical}}$ of Eq. (1).

$$\frac{\dot{q}_A}{c_pT_{01}}=\frac{\gamma+1}{2\left(\gamma+\dfrac{\overline{A}}{A_2}\right)^2}\frac{\left[\gamma M_1^2+\dfrac{\overline{A}}{A_1}\right]^2}{M_1^2\left[1+\dfrac{\gamma-1}{2}M_1^2\right]}-1 \tag{12}$$

where $\overline{A}$ is given by (8). *Equation (12) reduces to equation (1) if* $A_1 = A_2$. Figure 2 plots the ratio $\dot{q}_A/\dot{q}_{\text{classical}}$ for different values of the area ratio A_2/A_1. It may be seen that, at low supersonic Mach numbers, even a small increase in area may increase the critical heat many times that predicted by the classical Eq. (1).

3 Thermal Choking Considering Condensation With Area Variation and Mass Depletion of the Vapor

In this section, we discuss the case of condensing flow. First, consider a mixture of vapor and liquid phase of the same chemical species. The mass of the vapor is continually depleted as a result of condensation. The vapor phase is assumed to obey the perfect gas law. Assuming no velocity slip between the vapor and the liquid phase, the momentum equation may be written from (3) as,

$$(p_1-p_2)\overline{A}=\dot{m}_{\text{total}}(V_2-V_1) \tag{13}$$

where $\dot{m}_{\text{total}}$ denotes the *combined* mass flow rate of the vapor and the liquid phase, and is a constant. Assuming that the vapor is dry at Section 1, the energy equation across the condensation zone may be written as (Guha, 1992a),

$$a_1^2\left[\frac{1}{\gamma-1}+\frac{M_1^2}{2}\right]+y_2h_{fg}=a_2^2\left[\frac{1}{\gamma-1}+\frac{M_2^2}{2}\right] \tag{14}$$

where y_2 is the wetness fraction at Section 2 and h_{fg} is the specific enthalpy of evaporation. a and M in Eq. (14) are the frozen speed of sound and the frozen Mach number respectively. $a^2 = \gamma RT$, where γ is the isentropic exponent of the vapor phase alone and R is the specific gas constant of the vapor phase. (For a discussion on sound speeds in vapor-droplet mixtures see Guha 1992b, Guha 1992a.) The mass flow rate of the vapor alone, $\dot{m}$, at Sections 1 and 2 are related by,

$$\dot{m}_1=\frac{\dot{m}_2}{1-y_2}=\dot{m}_{\text{total}} \tag{15}$$

Substitution of (6) and (15) in (13) results in

$$V_2\left[\gamma+\frac{\overline{A}}{A_2}(1-y_2)\frac{1}{M_2^2}\right]=V_1\left[\gamma+\frac{\overline{A}}{A_1}\frac{1}{M_1^2}\right] \tag{16}$$

Combining (14) and (16),

$$\frac{\dfrac{1}{M_2^2}\left[\dfrac{1}{\gamma-1}+\dfrac{M_2^2}{2}\right]}{\left[\gamma+\dfrac{\overline{A}}{A_2}(1-y_2)\dfrac{1}{M_2^2}\right]^2}=\frac{\dfrac{1}{M_1^2}\left\{\left[\dfrac{1}{\gamma-1}+\dfrac{M_1^2}{2}\right]+\dfrac{y_2h_{fg}}{a_1^2}\right\}}{\left[\gamma+\dfrac{\overline{A}}{A_1}\dfrac{1}{M_1^2}\right]^2} \tag{17}$$

Compare Eq. (7) with (16) and (9) with (17) to see the effects of mass depletion. The condition of thermal choking is obtained when the local frozen Mach number is unity (Guha, 1994). Therefore, substituting $M_2 = 1$ in (17),

$$\left(1+\frac{y_2^*h_{fg}}{c_pT_{01}}\right)\left[\gamma+\frac{\overline{A}}{A_2}(1-y_2^*)\right]^2=\frac{\gamma+1}{2}\frac{\left[\gamma M_1^2+\dfrac{\overline{A}}{A_1}\right]^2}{M_1^2\left[1+\dfrac{\gamma-1}{2}M_1^2\right]} \tag{18}$$

where y_2^* is the critical amount of condensation and $\overline{A}$ is given by (8). Although the flow is overall adiabatic, we define the critical quantity of heat (for comparisons with previous cases) as

$$\dot{q}_{\text{integral}}\equiv y_2^*h_{fg} \tag{19}$$

Equation (18) can be solved as a cubic equation in y_2^* (or $\dot{q}_{\text{integral}}$). However, the solution is greatly simplified if the squared term in the LHS of (18) is approximated considering y_2^* is a small quantity.

$$\left[\gamma+\frac{\overline{A}}{A_2}(1-y_2^*)\right]^2\approx\left[\gamma+\frac{\overline{A}}{A_2}\right]^2\left[1-\frac{2y_2^*}{\gamma A_2/\overline{A}+1}\right]$$

When this approximation is substituted in (18), (18) becomes a simple quadratic equation.

Figure 3 plots the results of three models: $\dot{q}_{\text{classical}}$ given by (1) which deals with external heat addition to an ideal gas in

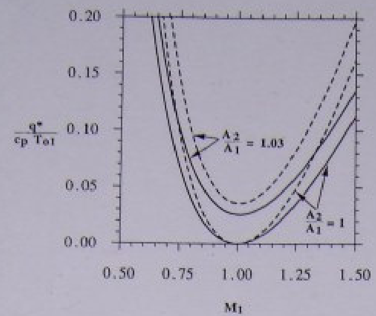

Fig. 3 Effects of area ratio across the condensation zone and the depletion in vapor mass flow rate on the critical amount of condensation [— Eq. (12), - - - - Eq. (19)]

a constant area duct, q_A^* given by (12) which deals with external heat addition with area variation, and q^*_{integral} given by (19) which takes into account of the area variation as well as of depletion in the mass of vapor due to condensation. The dotted lines in Fig. 3 represent Eq. (19) while the solid lines are the plots of Eq. (12). Note that the solid line corresponding to unity area ratio ($A_2/A_1 = 1$) is nothing but the classical solution, Eq. (1). Figure 3 shows clearly that the effects of mass depletion and (even a small) area variation are quite dramatic, especially when the Mach number is close to unity. Equation (1) is a very poor approximation then, and Eq. (19) must be used for determining the critical quantity of heat in non-equilibrium, condensing flow.

Figure 3 shows that Eq. (18) is valid in supersonic as well as in subsonic flow. The same equation can be used for $A_2/A_1 < 1$, i.e., in a converging flow section. In this respect, (18) is quite a versatile equation.

Equation (18) is as well valid if an inert, carrier gas is present. γ and a then represent the *frozen* values of the isentropic exponent and the speed of sound respectively in the mixture of the inert gas and the condensable vapor. They are calculated as weighted-averages of the respective values of the components (Guha, 1994). Assume that ω_g represents the mass fraction of the inert gas, and ω_{l2} is the mass fraction of the liquid phase at the downstream section. (As before, the vapor at the upstream section is assumed dry.) The corresponding quantities for c_p, γ, y, and M in Eq. (18) may then be calculated by starting from the conservation equations for a gas-vapor-droplet flow (Eqs. (28)–(32) in Guha, 1994) and then following exactly the same procedures presented in this section. The results (with some minor approximations) are:

$$c_p = \omega_g c_{pg} + (1 - \omega_g) c_{pv}$$

$$R = \omega_g R_g + (1 - \omega_g) R_v$$

$$\gamma = \frac{c_p}{c_p - R}$$

$$\mathrm{M} = V/a = V/\sqrt{\gamma R T}$$

$$y_2 = \omega_{l2}$$

The subscripts g and v refer to the respective properties of the inert gas and condensable vapor, respectively.

It should be realized that with finite transition thickness, pressure rise and area change, the boundary layer displacement thickness will change. Therefore, any reference to area such as in Eq. (18) should be interpreted as the effective flow area of the nozzle.

4 Time-Marching Solution of Condensation Zone

The equations presented in the previous section rest on an integral, control-volume analysis. We assess its accuracy by comparing the prediction of (19) with a full-blown time-marching solution of the governing differential equations for non-equilibrium condensation. Details of the time-marching calculation procedure may be found in Guha and Young (1991). Here we mention the salient points only. Assuming that there is *no velocity slip* between the vapor and the liquid droplets (which is usually an acceptable approximation as homogeneous nucleation normally produces very small, sub-micron size droplets), the gas dynamic equations for inviscid adiabatic unsteady two-phase vapor-droplet flow becomes:

Continuity

$$\frac{\partial \bar{\rho}}{\partial t} + \nabla \cdot (\bar{\rho} \mathbf{V}) = 0 \tag{20}$$

Momentum

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{V} + \frac{\nabla p}{\bar{\rho}} = 0 \tag{21}$$

Energy

$$\frac{\partial}{\partial t}\left[\bar{\rho}\left(\bar{e} + \frac{V^2}{2}\right)\right] + \nabla \cdot \left[\bar{\rho}\mathbf{V}\left(\bar{h} + \frac{V^2}{2}\right)\right] = 0 \tag{22}$$

where the vector quantity $\mathbf{V}$ is the common velocity of the two phases, and $\bar{\rho}$, $\bar{h}$, and $\bar{e}$ are, respectively, the density, the specific enthalpy and the specific internal energy of the *mixture*.

The mixture density, $\bar{\rho}$, is connected to the vapor density, ρ, (neglecting the volume of the liquid phase and assuming no carrier gas is present) via

$$\bar{\rho} = \rho/(1 - y) \tag{23}$$

and the mixture specific enthalpy is:

$$\bar{h} = (1 - y) h_v + y h_l \tag{24}$$

where h_v and h_l are the specific enthalpies of the vapor and the liquid phase, respectively.

Equations (20)–(22) are identical to those describing the adiabatic flow of an inviscid single phase fluid and are valid for unsteady, three-dimensional flow. The differences from an equilibrium calculation are apparent, however, when it is recalled that the wetness fraction y in Eqs. (23) and (24) is not necessarily the equilibrium value and that h_v and h_l in Eq. (24) are evaluated at the respective phase temperatures which are not necessarily equal to the local saturation value T_s. In order to close the set of equations for nonequilibrium condensation, one needs a nucleation rate equation specifying the rate of production of new droplets and a droplet growth law specifying the rate of condensation on existing droplets (providing the nonequilibrium value of y to be used in (23) and (24)). Both the rates of nucleation and droplet growth depend on the local subcooling, and have been described by Guha and Young (1991).

One of the most effective methods of calculation is to write a computational "black-box" which contains the nucleation and droplet growth equations, and the energy equation in its thermodynamic form. (The equation $d\bar{h} - dp/\bar{\rho}_e = 0$, derivable from (20)–(22), does not necessarily imply zero entropy increase in multi-phase flow.) Together they furnish the full set of equations that describe completely the formation and growth of liquid droplets in a fluid particle (from a Lagrangian viewpoint) if the pressure-time variation is specified. The pressure-time variation is obtained by time marching solutions of the conservation equations such as Denton's method (Denton, 1983), extensively used for single-phase calculations in turbomachinery blade rows. In this respect, the thermodynamic aspects of phase-change can be completely divorced from fluid dynamical considerations so that the use of the black-box is

Table 1 Comparison of Integral Predictions with Exact Solution

Equation (1) External heat addition to ideal gas in constant area duct	Equation (12) External heat addition to ideal gas in duct of varying area	Equation (19) Condensational heat release with area variation and mass depletion of vapor	Exact solution Time-marching solution of nonequilibrium gas dynamic equations
$\dot{q}_{classical}/c_pT_{01}$	$\dot{q}_A/c_pT_{01}$	$\dot{q}_{integral}/c_pT_{01}$	$\dot{q}_{actual}/c_pT_{01}$
0.01978	0.0423	0.0574	0.0587

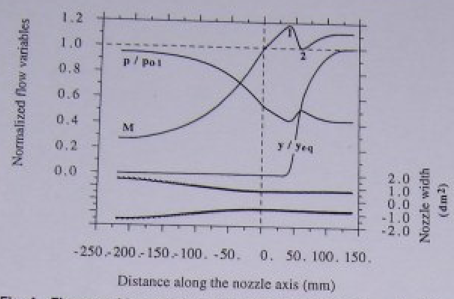

Fig. 4 Time-marching solution of a limiting condensation shock in a quasi-one-dimensional convergent-divergent nozzle. (Continuous variation of flow variables leading to thermal choking at point 2.)

effectively independent of any particular CFD application. Thus established single-phase CFD codes can, rather easily, be modified to deal with non-equilibrium two-phase flow with the above-mentioned modular approach. (The flexibility of this scheme may be appreciated from Guha and Young, 1994 where the same black-box has been grafted into a streamline curvature calculation procedure.)

The development of the computational routines within the black-box represents a comparatively major undertaking and has been fully described by Guha and Young (1991). The routines are sufficiently general and robust to deal with any type of nucleating or wet steam flow and (in contrast to many procedures reported in the literature) full details of the droplet size spectrum following nucleation are retained in the calculations. The last aspect is essential for accurate modelling of the nucleation zone. Successive nucleations after the primary are dealt with as a matter of course should the expansion be sufficiently rapid to generate the high levels of subcooling required. The computational scheme has been validated against measurements of steady (both sub and supercritical) and unsteady condensation shock waves (Guha and Young, 1991). Here, we present one calculation corresponding to thermal choking.

Figure 4 shows the nozzle employed for calculation. The working fluid is steam and the upstream total pressure, p_{01}, is 35140 N/sq m. Keeping the total pressure fixed, the total temperature, T_{01}, was varied until the condition of thermal choking is obtained. This is found to occur at T_{01} = 356.3 K. The frozen Mach number profile, M, shows that initially steam expands like a dry gas and attains M = 1 at the throat of the nozzle. Continued expansion to supersonic velocity ultimately generates sufficient subcooling for appreciable nucleation rate. The large number of small nuclei grow very fast, and the resulting release of latent heat erodes the subcooling and reduces the frozen Mach number. The point of maximum frozen Mach number constitutes the point 1 for the integral analysis described in Sections 2 and 3. Numerical calculations show that the Mach number at point 1, M_1, is 1.182.

The released heat is conducted back to the vapor and the frozen Mach number decreases. The inlet stagnation conditions are chosen such that, as the Mach number just reaches unity, the effect of area variation assumes dominance over that of heat addition. The flow, therefore, expands subsequently to supersonic velocity. This is the limiting condition for obtaining a continuous variation in all flow properties. Note that two sonic points exist in the flow field: one at the geometric throat and the second at the point of thermal choking. The point of thermal choking (M = 1) constitutes the point 2 in the integral analysis of the previous sections 2 and 3. Numerical calculations show that y_2 = 0.0172, and the area ratio between Sections 1 and 2 is A_2/A_1 = 1.0284. We refer the critical amount of heat from this direct numerical solution of the nozzle flow by $\dot{q}_{actual}$. We may now construct Table 1 to compare the results of different integral analyses with the exact solution $\dot{q}_{actual}$.

5 Conclusions

A theory of thermal choking due to nonequilibrium condensation in a nozzle is presented. The theory is based on a simple control volume approach. (A differential theory of thermal choking is discussed by Guha 1994.) It applies to vapor-droplet flow with or without a carrier gas. The expression for critical heat (or condensation) derived is valid for either supersonic or subsonic flow, and for heat release either in the diverging or in the converging part of a nozzle.

Table 1 shows that the present theory, Eq. (19), is in very close agreement with the full numerical solution of the differential equations of motion (giving the detailed structure of a condensation shock wave leading to thermal choking). The usually quoted Eq. (1) underestimates the critical heat by a factor of three in the example calculation presented. The variation of area across the condensation zone (although small) and the depletion in vapor mass as a result of condensation cannot be neglected in determining the critical heat in condensing nozzle flow.

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7th International Symposium on Flow Visualization

Seattle, Washington USA
September 11–14, 1995

Preliminary Announcement and First Call for Papers

The 7th International Symposium on Flow Visualization will be held September 11–14, 1995 in Seattle, Washington, USA. The Symposium is co-sponsored by the International Flow Visualization Society, the University of Washington, the American Institute of Aeronautics and Astronautics, and The Boeing Company.

This Symposium is the latest in the series of International Flow Visualization Symposia, first held in Tokyo in 1977, and held every three years since then. The objective of the Symposium is to provide a forum for communication and information exchange in the broad field of Flow Visualization applied over the entire range of disciplines that have come to depend upon these techniques. These include: experimental and computational fluid dynamics, aerodynamics, mechanical and chemical engineering, bio-medical technology, metallurgy, meteorology, oceanography, food and agricultural technology.

Original papers are solicited covering all aspects of Flow Visualization. Topics for sessions include, but are not limited to, the following:

Flow Field Visualization
- Direct injection (smoke)
- Field pressure mapping
- Laser sheet imaging
- Laser induced fluorescence
- Speckle photography
- Shadowgraph
- Schlieren
- Interferometer
- Particle image velocimetry

Surface Flow Visualization
- Mechanical interactions
  (tufts, liquid indicators, liquid crystals)
- Thermal and mass transfer interactions
  (IR radiometry, sublimation)
- Chemical interactions
  (reacting indicators, luminescent paints)

Graphical Display of Data Sets

Numerical Flow Visualization

Instructions for Authors

Papers will be accepted on the basis of extended abstracts of no more than 4 pages including figures and references. They should include the title, authors' names and affiliations, a concise statement of the problem, method of approach, results, conclusions, and sample figures. Please identify the corresponding author with an asterisk and provide the complete address, telephone number (and, if available, fax number and e-mail address) of the corresponding author. Four copies of the abstract must be submitted by December 10, 1994. Notice of acceptance of papers will be provided by February 10, 1995. Final manuscripts will be due by May 10, 1995 and will be published in the bound Proceedings to be distributed at the Symposium.

Authors' Schedule

Deadline for submission of abstracts: December 10, 1994
Notification of acceptance: February 10, 1995
Camera-ready manuscripts due: May 10, 1995

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