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Optimum Fan Pressure Ratio for Bypass Engines with Separate or Mixed Exhaust Streams

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The optimum fan pressure ratio is determined both numerically and analytically for separate-stream as well as mixed-stream bypass engines. The results are applicable to civil and military engines. The optimum fan pressure ratio is shown to be predominantly a function of the specific thrust and a weak function of the bypass ratio. Two simple, explicit equations, one for each type of engine, have been derived that specify the optimum fan pressure ratio. The predictions compare very well against numerical optimization performed by a specialist computer package employing iterative and advanced search techniques and real gas properties. The analytical relations accelerate the optimization process and offer physical insight. It has been shown that the optimum fan pressure ratio achieves the condition $V_{jc}/V_{jh} = \eta_{KE}$ in a separate-stream engine and the condition $V_{163}/V_{63} \approx \eta_{KE}$ in a mixed-stream engine. The condition $V_{163}/V_{63} \approx \eta_{KE}$ applies even under situations when significant departures from the normally assumed condition $p_{016}/p_{06} \approx 1$ can occur.

Nomenclature

B = bypass ratio
c_p = specific heat of air at constant pressure
c_{pg} = specific heat of combustion products at constant pressure
E_k = kinetic energy added by the core engine
F_N = net thrust
$\hat{F}_N$ = specific thrust
M = Mach number of the aircraft
$\dot{m}_c$ = mass flow rate through the bypass duct (cold flow)
$\dot{m}_f$ = mass flow rate of fuel
$\dot{m}_h$ = mass flow rate through the core engine (hot flow)
p = static pressure
p_0 = total pressure
R = specific gas constant of air
T = static temperature
T_a = temperature of ambient air
T_0 = total temperature
V = velocity
V_a = forward speed of the aircraft
V_j = jet speed in a mixed stream engine
V_{jc} = fully expanded jet speed of the cold bypass stream
V_{jh} = fully expanded jet speed of the hot core stream
V_m = mean jet speed [given by Eq. (8)]
γ = isentropic index of air
γ_g = isentropic index of combustion products
η_f = isentropic efficiency of the fan or low pressure compressor
η_{KE} = efficiency of energy transfer between the core and bypass flow
η_{LPT} = isentropic efficiency of the low pressure turbine
η_{NB} = isentropic efficiency of the bypass nozzle

Subscript

op = optimum value

Introduction

IN this paper simple, explicit relations have been derived from fundamental principles for determining the optimum fan (compressor) pressure ratio in bypass engines. Separate analysis has been presented for the mixed-stream and separate-stream engines because the flow physics is different in the two cases. In both cases, the present theory gives results in good agreement with numerical optimization calculations.

Determination of the optimum fan pressure ratio (FPR_{op}) is important because, given all other variables such as the overall pressure ratio (OPR), bypass ratio B, and turbine entry temperature T_{04} the optimum value of FPR simultaneously ensures maximum specific thrust and minimum specific fuel consumption. Thus, the FPR_{op} achieves the two, usually contradictory, goals of jet engine design.

For civil engines, particularly for medium to long range aircrafts, the FPR_{op} can be calculated under cruise conditions. For military engines, optimization may be mission specific. However, Millhouse et al.[1] have shown that, although the most fuel-efficient values of such parameters as bypass ratio and OPR depend on the integrated effects of the various flight altitudes, Mach numbers, and thrust required throughout the mission, it is possible to locate heuristically the most efficient FPR to complement other engine parameters independent of the mission. Thus, the relations derived in this paper can be used in the design of both civil and military engines.

The design wisdom is that only a single-stage fan is used for a civil aircraft engine with large bypass ratios, and so the maximum FPR is about 1.8. Engines with low bypass ratios, for example, military engines, normally use a low-pressure (LP) compressor having 3–5 stages. In this case, the maximum possible FPR can be much higher. The mixing of the two streams in a mixed-stream turbofan engine offers performance gain (lower specific fuel consumption and higher specific thrust), even though mixing incurs a loss in total pressure because the enthalpies of the two streams are redistributed. Moreover, only one reheat system and only one variable nozzle are needed. A long nacelle can contain the fan/compressor noise more effectively. For the same specific thrust, the mixed-stream engine has lower optimum FPR than a separate-stream engine; thus, the LP turbine may have one fewer stage if both engines have the same bypass ratio. The outer cold stream also protects the jet pipe from severe temperatures resulting from afterburning. Low-specific-thrust, high-bypass-ratio engines, on the other hand, do not employ afterburning, and mixing of the two streams would necessitate a long nacelle increasing weight, cost, nacelle drag, and interference effects. These engines, therefore, use separate-stream configurations. When a common nozzle is, however, used for high-bypass-ratio engines, the small axial extent of the mixing zone is usually insufficient to achieve complete mixing of the two streams.

Calculation and analysis of the performance of turbofan engines are discussed in details in several excellent texts.[2–4] In comparison,

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the issue of optimized design has received less attention, and analytical solutions did not exist. The real store house of expertise and knowledge obviously exists with the aeroengine manufacturers who have actively pursued the optimization of gas turbine performance for the past 60 years. Ruffles[5] shows that there has been a 50% improvement in the specific fuel consumption (SFC) of the bare engine, as the engines evolved from the turbojet of 1958 to the high-bypass Trent engines of modern time. Improvements in propulsive efficiency, component efficiency and cycle efficiency contribute approximately one-third each to this gain in performance. The publications made by experts from industry (e.g., Jackson,[6] Bennett,[7] Wilde,[8] and Birch[9]) contain a wealth of experience, information, and calculation results. However, the details of calculation method are not known to the readers. The industry would typically have its own, sophisticated computer packages to which the public does not have access. Moreover, the published results often would involve an adopted family of engine designs. It is then not always obvious to the general reader how to generalize the results or how to proceed on a clean-sheet analysis.

In this paper analytical relations have been derived for calculating FPR_{op}. The analytical results not only accelerate the optimization process but also offer valuable physical insight. It is shown that the FPR_{op} is predominantly a function of the specific thrust and only weakly depends on the bypass ratio. The FPR_{op} monotonically increases with increasing specific thrust at two different levels corresponding to the separate-stream and mixed-stream turbofan engines.

The theory has been verified by comparing its predictions with calculations performed by the computer program GasTurb,[10] which is a general purpose commercial package for the calculation of design and off-design performance of various types of gas turbine engines, turbojet, turboshaft, separate- or mixed-stream turbofan, and geared turbofan, all with single- or twin-spool configurations. It uses real gas properties with dissociation and has several features including optimization, iteration, transient, and Monte Carlo simulation. Each numerical point subsequently illustrated has been calculated by GasTurb by a lengthy optimization process that uses advanced search techniques to find the optimum FPR that minimizes the SFC while, at each trial, the primary variables are iterated to give the prescribed specific thrust. This numerical optimization is used not only to verify the theoretical prediction, but also to discover what exactly happens to the many variables when the FPR_{op} is achieved.

Based on the principles discussed in the present paper, a new methodology for the thermodynamic optimization of bypass engines (turbofan or advanced propulsors) has been developed by Guha[11] in which the optimum combination of all variables is determined concurrently. The process starts with establishing an optimum specific thrust for the engine based on an economic analysis (installation constraints, noise regulations, etc., also need to be considered). The task of the optimization process is then to find the combination of optimum values of OPR, B, FPR, and T_{04} concurrently that minimizes SFC at the fixed specific thrust. This procedure is quite different from the usual parametric studies of engine performance,[2,12] in which a single parameter is varied each time, while keeping all other parameters fixed, therefore at their nonoptimum values. Moreover the usual single-variable parametric studies may involve a large excursion in the value of speficic thrust unacceptable for a specific mission. Guha[11] has also discussed at length the determination of optimal jet velocity, with the derivation of a new analytical expression that performs well against numerical optimization results.

Following industrial practice, the SFC and the specific thrust, are expressed here respectively in pounds mass per hour per pounds force and pounds force per pounds mass per second where 1 lbf/lbm/s = 9.81 m/s and 1 lbm/hr/lbf = 28.316 × 10^{-6} kg/s/N.

Separate-Stream Bypass Engines

The schematic arrangement and flow structure in a separate-stream bypass engine is shown in Fig. 1. The description of an equivalent turbojet engine is also shown in Fig. 1 as a reference point in the analysis. The core gas generator produces, for a particular value of fuel consumption rate, a jet with speed $V_{j,\mathrm{ref}}$ in the absence of any bypass flow. The kinetic energy added by the gas generator is then, neglecting the fuel mass flow rate which is usually a very small proportion of the air mass flow rate in a jet engine, equal to

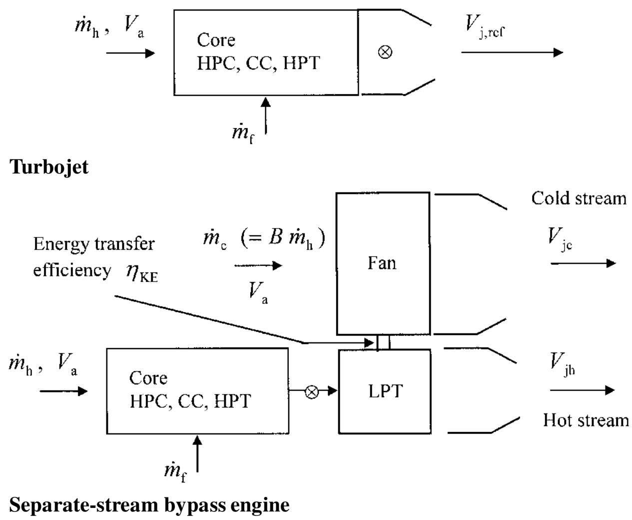

Fig. 1 Schematic description of a separate-stream turbofan engine and an equivalent turbojet engine used for analysis; energy is the same at point ⊗ in both engines.

$$E_k = \tfrac{1}{2}\dot{m}_h\left(V_{j,\mathrm{ref}}^2 - V_a^2\right) \tag{1}$$

In the bypass engine, the whole of this kinetic energy is, of course, not present in the core jet. The LP turbine extracts a portion of this energy and turns the fan, which, in turn, adds kinetic energy to the bypass flow. The efficiency η_{KE} of this energy transfer between the core and bypass flow depends on the component efficiencies of the LP turbine, the fan, and the bypass nozzle and also on the flow losses in the ducts and mechanical efficiency of the shafts. Assuming the mechanical efficiency is close to 1,

$$\eta_{KE} \approx \eta_{LPT}\eta_f\eta_{NB} \tag{2}$$

At the current level of technologies,[4,9] $\eta_{LPT} \approx 0.9$, $\eta_f \approx 0.9$, and $\eta_{NB} \approx 1$. Equation (2), therefore, suggests that $\eta_{KE} \approx 0.8$.

In a bypass engine, the kinetic energy given by Eq. (1) is related to the sum of the kinetic energies of the two streams by

$$E_k = \tfrac{1}{2}\dot{m}_h\left(V_{j,\mathrm{ref}}^2 - V_a^2\right) = \tfrac{1}{2}\dot{m}_h\left(V_{jh}^2 - V_a^2\right) + \tfrac{1}{2}\dot{m}_h(B/\eta_{KE})\left(V_{jc}^2 - V_a^2\right) \tag{3}$$

In Eq. (3) we have expressed the mass flow rate of the bypass stream, $\dot{m}_c$, in terms of $\dot{m}_h$ and B. Similarly the net thrust F_N produced by the bypass engine is the sum of the thrusts of the two streams and is given by

$$F_N = \dot{m}_h(V_{jh} - V_a) + B\dot{m}_h(V_{jc} - V_a) \tag{4}$$

The pressure thrust has been taken into account because V_{jh} and V_{jc} are the fully expanded jet velocities of the two streams.

The objective of the optimization process is to determine the ratio V_{jc}/V_{jh}, which would maximize the net thrust F_N while keeping the fuel consumption fixed. The condition for maximum net thrust is given by

$$\frac{\partial F_N}{\partial V_{jh}} = 0 \tag{5}$$

At constant thermal efficiency, a fixed fuel consumption means E_k in Eq. (3) is constant. Therefore,

$$\frac{\partial (E_k)}{\partial V_{jh}} = 0 \tag{6}$$

For a constant bypass ratio B, Eqs. (3–6) can be combined to give

$$(V_{jc}/V_{jh})_{\text{op}} = \eta_{\text{KE}} \tag{7}$$

While evaluating the partial derivatives in Eqs. (5) and (6), B and $\dot{m}_h$ are assumed constant. Together they specify a constant inlet airflow. Because the specific thrust is the ratio of net thrust and mass flow rate of air at the inlet, Eq. (5) is also the condition for maximum specific thrust. SFC is the ratio of mass flow rate of fuel and net thrust. Equation (6) implies a fixed mass flow rate of fuel; Eq. (5) is, thus, also the condition for minimum SFC. Because Eq. (7) gives both maximum specific thrust and minimum SFC, this condition would be used in the analytical formulation of the optimization process that follows.

A designer can satisfy Eq. (7) in a real engine by choosing the FPR_{op}, as will be shown. The mean jet speed V_m can be expressed, using Eq. (7), as

$$V_m = [1/(1+B)](B + 1/\eta_{\text{KE}})V_{jc} \tag{8}$$

and it follows that

$$\hat{F}_N = F_N/(\dot{m}_h + \dot{m}_c) = V_m - V_a \tag{9}$$

Suppose the rise in total temperature in the bypass flow for a hypothetical, isentropic compression by the fan, for the same pressure ratio that exists across a real fan, is $\Delta T_{0,\text{isen}}$. If the fan and the bypass nozzle were isentropic, then the static temperature of the fully expanded jet would be equal to the ambient temperature. Application of the steady flow energy equation then gives

$$\Delta T_{0,\text{isen}} = (1/2c_p)\left(V_{jc}^2 - V_a^2\right) \tag{10}$$

Equations (8–10) can be combined with the isentropic pressure–temperature relation. One then obtains, after some algebraic manipulation, the final expression for the FPR_{op}:

$$(\text{FPR})_{\text{op}}^{(\gamma-1)/\gamma} = 1 + \frac{(\gamma-1)}{2+(\gamma-1)M^2} \times \left[\frac{(1+B)^2}{(B+1/\eta_{\text{KE}})^2}\left\{\frac{\hat{F}_N}{\sqrt{\gamma R T_a}} + M\right\}^2 - M^2\right] \tag{11}$$

All quantities in Eq. (11) are nondimensional numbers. Figure 2 shows graphically the prediction of Eq. (11).

One interesting feature of Eq. (11) is that if $\eta_{\text{KE}} = 1$, that is, in the case of ideal energy transfer from the core stream to the bypass stream, the bypass ratio B drops out of the equation. FPR_{op} would then have depended only on $\hat{F}_N$. At any rate, Eq. (11) predicts that, for a fixed $\hat{F}_N$, the dependence of FPR_{op} on B is weak, especially at higher values of B ($B > 4$). Equation (11) also produces the expected limiting result:

$$\lim_{\substack{B\to\infty \\ \hat{F}_N\to 0}} (\text{FPR})_{\text{op}} = 1$$

For a chosen specific thrust $\hat{F}_N$ and bypass ratio B, Eq. (11) immediately specifies the FPR_{op}. The allowable range of FPR is rather restricted. The design wisdom is that only a single-stage fan is used for a civil aircraft with large-bypass ratios, the maximum

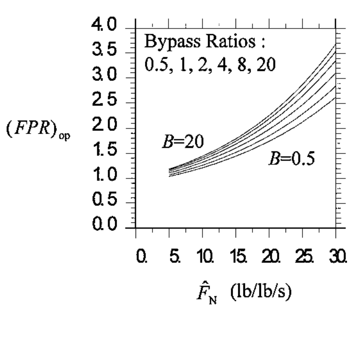

Fig. 2 Variation of FPR_{op} with specific thrust in separate-stream bypass engines: prediction of Eq. (11); $M = 0.82$, $T_a = 216.65$ K, $\eta_{\text{KE}} = 0.81$, and $\gamma = 1.4$.

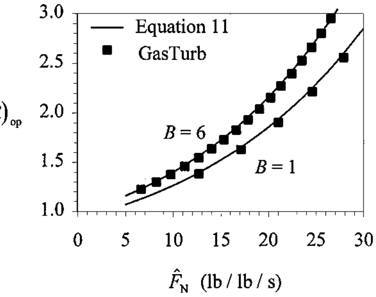

Fig. 3 Comparison of present theory with numerical optimization results for FPR_{op} in separate-stream bypass engines. (Altitude = 11 km, $M = 0.82$, isentropic efficiency of compressors and turbines = 0.9).

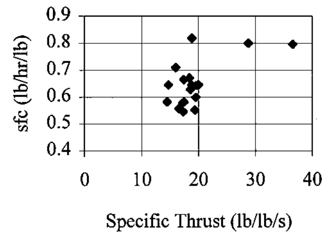

Fig. 4 Data for various current civil turbofan engines under cruise conditions (typically 35,000 ft, $M = 0.8$–0.85).

FPR is, therefore, restricted to 1.8. A limit of minimum FPR may also arise due to fan instabilities at partload, off-design conditions. According to Rüd and Lichtfuss,[13] below an FPR of 1.4, the engine will require variable geometry either via a variable pitch fan or a variable area bypass nozzle to provide aerodynamic fan stability under partload conditions. Therefore, knowing FPR_{op} at an early stage of the design is an advantage. If its value does not lie within the desired limit, the choice of $\hat{F}_N$ and B can be altered (of course, another option would be not to adhere strictly to the optimum value of FPR).

Equation (11) has been tested against optimization performed by the computer program GasTurb.[10] Figure 3 shows the comparison. Each numerical point has been calculated by GasTurb by a lengthy optimization process that uses advanced search techniques to find the optimum FPR that minimizes the SFC while, at each trial, the primary variables are iterated to give the prescribed specific thrust. Figure 3 shows that Eq. (11) performs well against sophisticated numerical optimization and can, therefore, be used for the design of real engines.

To appreciate the current design standard, Fig. 4 plots the SFC and specific thrust of some current civil turbofan engines of various manufacturers.[14] Specific thrust data in Fig. 4 are approximate because although net thrust is known at cruise, the mass flow rate is estimated from given values at sea level static conditions by dynamic scaling, that is, assuming the same nondimensional operating point. The specific thrust data indicates over which range any theory needs to be applicable in order to be relevant for current and future designs.

Figure 4 shows that for most existing engines, the cruise specific thrust lies in the band 15–20 lbf/lbm/s. Over the past 40 years of civil engine design, the specific thrust has reduced significantly (producing appreciable improvement in propulsive efficiency) while the bypass ratio has increased from 1–2 in the 1960s to 7–9 in the 1990s.[9] The forecast,[9,13] is that the design driver for future engines would be toward even lower specific thrust. A surge in fuel price and/or the introduction of more stringent noise regulation may necessitate such designs. Future geared turbofan or advanced ducted propulsors would reduce the specific thrust significantly. Figure 3 thus shows that Eq. (11) is applicable to both current and future bypass engines.

Mixed-Stream Turbofan Engines

Before developing the theory for FPR_{op} for mixed-stream turbofan engines, the computer package GasTurb was used to determine the same numerically. Figure 5 shows the results of this numerical optimization. Calculations are shown at two bypass ratios. Mixed-stream engines used for military purposes employ much higher specific thrust than that used for separate-stream civil applications. These engines usually have a smaller bypass ratio. Therefore, maximum tolerable turbine entry temperatures can produce a large specific thrust. Of course, such high values of specific thrust would mean low propulsive efficiency giving higher fuel consumption. The reduction in engine size and weight is, however, more crucial for such applications.

Figure 6 shows the standard station numbering for a mixed-flow turbofan engine. In the following analysis these numbers are used as subscripts to denote flow variables at various locations. The following representative values give reasonable approximation to the properties of air and combustion products: $R = 287$ J/kg/K, $\gamma = 1.4$, $c_p = \gamma R/(\gamma - 1)$, $\gamma_g = 1.33$, $c_{pg} = \gamma_g R/(\gamma_g - 1)$. A simple, analytical procedure for calculating the properties of combustion products as a function of temperature, fuel–air ratio, and fuel composition is given by Guha.[15]

The analysis can be formulated more easily if the optimum FPR and the specific thrust are expressed in terms of the primary variables OPR, B, and T_{04}.

An energy balance gives (with $T_{05} = T_{06}$ and $T_{013} = T_{016}$)

$$(c_{pg}/c_p)(T_{04} - T_{06}) = (T_{03} - T_{02}) + B(T_{016} - T_{02}) \tag{12}$$

where the various total temperatures can be calculated from

$$T_{02}/T_a = 1 + 0.5(\gamma - 1)M^2 \tag{13}$$

$$T_{03} - T_{02} = (1/\eta_c)T_{02}\left[\mathrm{OPR}^{(\gamma-1)/\gamma} - 1\right] \tag{14}$$

$$T_{016} - T_{02} = (1/\eta_f)T_{02}\left[\mathrm{FPR}^{(\gamma-1)/\gamma} - 1\right] \tag{15}$$

$$T_{04} - T_{06} = \eta_t T_{04}\left[1 - (\mathrm{OPR/FPR})^{(1-\gamma_g)/\gamma_g}\right] \tag{16}$$

where η_c is the isentropic efficiency of the compression between points 2 and 3 and η_t is the isentropic efficiency of the expansion between points 4 and 6.

While writing Eq. (16) it is assumed that $p_{016}/p_{06} = 1$ when optimum fan pressure ratio is achieved. Strictly speaking, this condition is not always fully satisfied. However, the value of the ratio p_{016}/p_{06} rises under off-design conditions: If the value of this ratio used at the design point is greater than one, serious mixing losses may result at off-design operations. Therefore, it is prudent to choose the value of p_{016}/p_{06} between 0.95 and 1 at the design point even if it is slightly suboptimal at that particular operating point. Numerical calculations of Millhouse et al.[1] show that if the condition $p_{016}/p_{06} \approx 1$ is used at the design point, the computed FPR then gives the optimum value independent of a specific mission of the aircraft.

Equations (12–16) can be solved to determine the optimum FPR. This will, however, require an iterative solution. For small-bypass ratios, as is usual for mixed-stream military engines, the second term in the right-hand side of Eq. (12) is small compared to the first term. The index for FPR in Eq. (15) can then be approximated by substituting γ_g for γ. With this approximation, the equations can be combined to give an explicit relation for the $\mathrm{FPR_{op}}$:

$$\mathrm{FPR}_{\mathrm{op}}^{(\gamma_g-1)/\gamma_g}\left[\frac{BT_{02}}{\eta_f} + \frac{(c_{pg}/c_p)\eta_t T_{04}}{\mathrm{OPR}^{(\gamma_g-1)/\gamma_g}}\right] = \left(\frac{c_{pg}}{c_p}\right)\eta_t T_{04} - \frac{1}{\eta_c}T_{02}\left[\mathrm{OPR}^{(\gamma-1)/\gamma} - 1\right] + \frac{BT_{02}}{\eta_f} \tag{17}$$

If V_j is the jet speed and V_a is the aircraft speed, the specific thrust is calculated from

$$\hat{F}_N = V_j - V_a \tag{18}$$

where

$$V_j = \sqrt{2c_{pg}(T_{064} - T_j)} \tag{19}$$

$$T_{064} = \frac{T_{06} + BT_{016}}{1 + B} \tag{20}$$

$$T_j = T_{064}(\mathrm{FPR_{op}} f_M)^{(1-\gamma_g)/\gamma_g} \tag{21}$$

$$f_M = [1 + 0.5(\gamma - 1)M^2]^{\gamma/(\gamma-1)} \tag{22}$$

Equation (20) gives the total temperature of the mixed stream and Eq. (21) gives the static temperature of the jet. While writing the preceding equations, the losses in the fan, compressors and turbines have been accounted for, but other losses, for example, that in the various ducts, have been neglected.

Figure 7 shows the comparison of the present theory and the numerical optimization results using GasTurb. Each numerical point has been calculated by GasTurb to find the optimum FPR that minimizes the SFC, while, at each trial, the primary variables are iterated to give the prescribed specific thrust. The nozzle area was also optimized at each point so that it produced a fully expanded jet. The convergence of the numerical optimization may be slow and sometimes, depending on the starting point, the numerical search technique may not find the optimum or may find a local rather than the global optimum. The analytical equations, on the other hand, readily provide the answer. Figure 7 shows that the present theory gives accurate results: The iterative analytical solution [Eqs. (12–16)] agrees almost

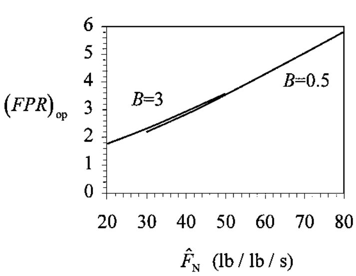

Fig. 5 $\mathrm{FPR_{op}}$ for mixed-stream turbofan engines determined by numerical optimization (GasTurb); for all calculations, OPR = 17.5, $M = 0.82$, isentropic component efficiencies = 0.9 and $\eta_{\mathrm{mix}} = 1$.

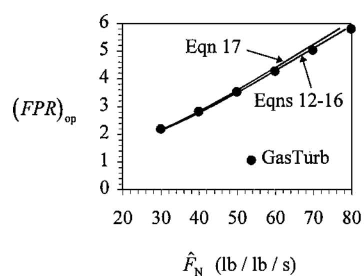

Fig. 7 Comparison of the present theory with numerical optimization results for $\mathrm{FPR_{op}}$ in mixed-stream turbofan engines; for all calculations, OPR = 17.5, $B = 0.5$, $M = 0.82$, isentropic component efficiencies = 0.9 and $\eta_{\mathrm{mix}} = 1$.

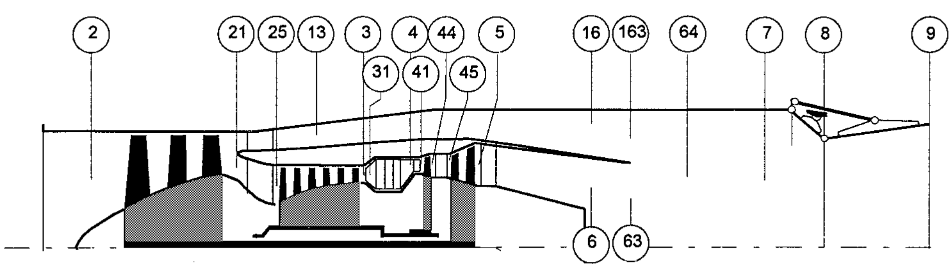

Fig. 6 Standard nomenclature for various locations in a mixed-stream turbofan engine (only half of the engine is shown).

Table 1 Dependence of FPR_{op} on OPR for mixed-stream engines

$\hat{F}_N$, lbf/lbm/s	OPR	T_{04}, K	B_{op}	FPR_{op}
60	17.5	1454	0.822	4.288
60	30	1548	1	4.896

identically with GasTurb optimization; the explicit equation (17) also gives quite acceptable answer until the specific thrust becomes very high.

The present theory can be extended to include the effects of incomplete mixing by introducing a parameter η_{mix}. It is then assumed that three separate streams having total temperatures T_{016}, T_{06}, and T_{064} expand through the same pressure ratio. The total thrust is the sum of the thrust produced by these individual streams:

$$V_{j16} = \sqrt{2c_p(T_{016} - T_{j16})}, \qquad T_{j16} = T_{016}(\mathrm{FPR_{op}} f_M)^{(1-\gamma)/\gamma}$$

$$V_{j6} = \sqrt{2c_{pg}(T_{06} - T_{j6})}, \qquad T_{j6} = T_{06}(\mathrm{FPR_{op}} f_M)^{(1-\gamma_g)/\gamma_g}$$

$$\hat{F}_N = \eta_{mix} V_j + (1 - \eta_{mix})(V_{j6} + BV_{j16})/(1 + B) - V_a \quad (23)$$

Separate Versus Mixed Stream Engines

Figure 5 can be compared with Figs. 2 and 3. Three observations can be made.

1) The FPR_{op} for mixed-stream engines rises monotonically with the specific thrust, as it does for separate stream engines.

2) The FPR_{op} is predominantly a function of the specific thrust and only weakly depends on the bypass ratio. The dependence on bypass ratio is weaker in mixed-stream engines than that in separate-streams engines.

3) At a particular value of specific thrust, the FPR_{op} for mixed-stream engines is lower than that in the separate-streams engines.

In a separate-stream engine, altering the values of OPR does not alter the value of FPR_{op}, so long as the specific thrust and bypass ratio are kept fixed. This is predicted by Eq. (11) and borne out by numerical optimization. In a mixed-stream engine, even at a fixed specific thrust, various values of OPR and M change the value of FPR_{op}. Either the analytical relations (12–16) or the numerical optimizations show this. Table 1 shows the results of optimization performed by GasTurb (both B and FPR are optimized at fixed specific thrust at two levels of OPR, minimising SFC).

Effects of FPR_{op} on the Conditions of Other Variables

The behavior of other flow variables are now examined when the FPR_{op} is achieved. Separate discussion is provided for the mixed-stream and separate-stream engines because different physical principles are involved.

Separate-Stream Bypass Engines

The conditions to which the FPR_{op} correspond have been determined analytically in this case. Equation (7) shows that there is a particular ratio of the fully expanded jet velocities of the cold and hot stream $[(V_{jc}/V_{jh})_{op} = \eta_{KE}]$ that simultaneously achieves minimum SFC and maximum specific thrust. The designer can make the two jet velocities take this value by choosing the FPR_{op}.

The computational program GasTurb was used to find the ratio V_{jc}/V_{jh} that occurs in an engine whose FPR has been numerically optimized for minimizing SFC. Table 2 shows these values (for OPR = 30, $T_{04} = 1200$ K, $\eta_{LPT} = 0.9$, $\eta_f = 0.9$, and $\eta_{NB} = 1$), against the prediction of Eq. (7).

Numerical optimizations were performed at several other OPR and T_{04}: V_{jc}/V_{jh} lay between 0.77 and 0.82 (for the assumed component efficiencies). The numerical calculations show that the derived analytical relation [Eq. (7)] is approximately valid.

Mixed-Stream Bypass Engines

The ratio of the total pressure of the two streams at the beginning of the mixer zone, p_{016}/p_{06}, is an important variable and the

Table 2 Comparison of theory and numerical results for optimum V_{jc}/V_{jh}

Bypass ratio	$(V_{jc}/V_{jh})_{op}$ Numerical optimization with GasTurb	$(V_{jc}/V_{jh})_{op} = \eta_{KE}$ $[\eta_{KE} \approx \eta_{LPT}\,\eta_f\,\eta_{NB}]$
1	0.808	0.81
3	0.791	0.81
6	0.794	0.81

Table 3 Validity of Eq. (24) for various component efficiencies

η_f	η_{LPT}	$(p_{016}/p_{06})_{op}$[a]	$(V_{163}/V_{63})_{op}$[a]	$\eta_{KE} = \eta_f\eta_{LPT}$
0.8	0.8	1.016	0.658	0.640
0.8	0.9	1.031	0.742	0.720
0.9	0.9	1.044	0.818	0.810
0.95	0.95	1.061	0.908	0.903

[a]Calculated by numerical optimization with GasTurb.

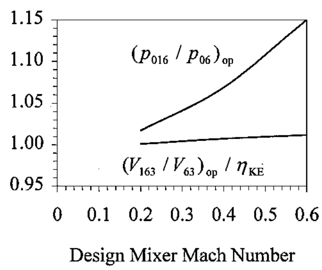

Fig. 8 Effects of design mixer Mach number on the two characteristic ratios when FPR_{op} is achieved; for all calculations, $M = 0.82$, $B = 0.5$, $\hat{F}_N = 60$ lb/lb/s, and component isentropic efficiencies = 0.9.

FPR controls this. It is generally recognized,[1,10] that when FPR_{op} is achieved, the ratio p_{016}/p_{06} is close to one. The physical reason is that, when the total pressures of the two streams are nearly equal, the loss (entropy generation) due to mixing would be small.

The computer program GasTurb was used to determine the value of FPR (by random adaptive search) that minimizes the SFC at several specific thrust levels that were fixed by iterating OPR and T_{04}. It was found that optimum p_{016}/p_{06} did not remain fixed but varied. Optimum p_{016}/p_{06} increased with increasing specific thrust and bypass ratios. The optimum value of p_{016}/p_{06} was found to vary particularly strongly with design mixer Mach number M_{64}, as shown in Fig. 8. However, the present study discovered a new relation between the velocities of the two streams at the mixer inlet under optimum conditions:

$$(V_{163}/V_{63})_{op} \approx \eta_{KE} \quad (24)$$

where $\eta_{KE} = \eta_f\eta_{LPT}$. At the FPR_{op}, the relation $V_{163}/V_{63} \approx \eta_{KE}$ remained nearly valid at all values of specific thrust, bypass ratio, and design mixer Mach number. The similarity of this condition with Eq. (7) derived for separate-stream bypass engines is noticeable. The variation in the velocity ratio is shown in Fig. 8 (with $\eta_{KE} = 0.81$).

To assess the accuracy of Eq. (24), a large number of numerical experiments were conducted with GasTurb in which the isentropic efficiencies of various components were changed systematically. It was found that indeed the efficiencies of the high-pressure (HP) compressor, η_{HPC}, and the HP turbine, η_{HPT}, did not influence the value of $(V_{163}/V_{63})_{op}$, as Eq. (24) suggests. Equation (24) was also satisfied when various values of the LP compressor and LP turbine efficiencies were tested. A few of these numerical optimization results are given in Table 3 to demonstrate this point. (For all calculations shown in Table 3, $\hat{F}_N = 60$ 1b/1b/s, $B = 1$, OPR = 17.5, $M_{64} = 0.3$, $M = 0.82$, and $\eta_{mix} = 1$. Similar results are obtained at other values of these variables.)

Conclusions

The optimum fan (compressor) pressure ratio is determined both numerically and analytically for separate-stream as well as mixed-stream bypass engines. The FPR_{op} is shown to be predominantly a function of the specific thrust and a weak function of the bypass ratio. For mixed-stream engines, the dependence of FPR_{op} on bypass ratio is very weak, and FPR_{op} also depends on OPR. At the same specific thrust and bypass ratio, the FPR_{op} for a mixed-stream engine is lower than that of a separate-stream engine.

Two simple, explicit equations have been derived from fundamental principles. Equation (11) gives the FPR_{op} for separate-stream engines, whereas Eq. (17) [or the equation set (12–16)] gives the FPR_{op} for mixed-stream engines. The accuracy of the analytical formulas has been established through extensive verification by numerical optimization results of the commercial computer package GasTurb (Figs. 3 and 7). The analytical results accelerate the optimization process and offer physical insight.

It has been shown that the FPR_{op} achieves the condition $V_{jc}/V_{jh} = \eta_{KE}$ in a separate-stream engine and the condition $V_{163}/V_{63} \approx \eta_{KE}$ in a mixed-stream engine. The condition $V_{163}/V_{63} \approx \eta_{KE}$ applies even under situations when significant departures from the normally assumed condition $p_{016}/p_{06} \approx 1$ occur.

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