Effects of internal combustion and non-perfect gas properties on the optimum performance of gas turbines

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Abstract: With the help of a purpose-built computer program the separate and combined effects of the various aspects of internal combustion and non-perfect gas properties on the optimum performance of gas turbines are examined. The effects of variation of specific heat, addition of fuel mass, pressure losses and dissociation are elucidated. The numerical results have been extensively compared with closed-form analytical solutions and a linear perturbation analysis. The accuracy of a standard, approximate method for predicting gas turbine performance is assessed and its various limitations are identified. The newly established concept of optimum turbine entry temperature is further explored.

Keywords: power plant, gas turbines, optimization, optimum pressure, optimum temperature, thermal efficiency, cycles, real gas, pressure loss, fuel, internal combustion, dissociation

NOTATION

c_p	specific heat at constant pressure
$[CV]_0$	calorific value at temperature T_0
f	fuel–air mass ratio
h	specific enthalpy
n	$= (c_{pa})_{12}/(c_{pg})_{34}$
n'	$= (c_{pa})_{12}/(c_{pg})_{13}$
p	pressure
r	pressure ratio
R	characteristic gas constant
s	specific entropy
T	temperature
TET	turbine entry temperature, defined here as the temperature at entry to the turbine stator (and is the same as the burner exit temperature)
x	isentropic temperature ratio $= r^{(\gamma_A - 1)/\gamma_A}$
α	$= \eta_c \eta_t \theta$
β	$= 1 + \eta_c(\theta - 1)$
γ	ratio of specific heats
η_A	air standard efficiency
η_c	isentropic efficiency of the compressor
η_f	plant efficiency with the 'f effect' alone
η_n	plant efficiency with the 'n effect' alone
η_o	overall plant efficiency
η_p	plant efficiency with the pressure loss effect alone
η_t	isentropic efficiency of the turbine
θ	maximum–minimum temperature ratio $= T_3/T_1$
ϕ	$= (x_e - x_{eA})/x_{eA}$
φ	$= (\eta_e - \eta_{eA})/\eta_{eA}$

Subscripts

a	air
analyt	present analytical (non-linear)
av	average
A	air standard
e	maximum efficiency condition
f	addition of fuel effect
g	gas (combustion products)
isen	isentropic
linear	present linear analysis
n	specific heat effect
p	pressure loss effect
ref1	as in reference [**1**]
1	entry
2	exit of compressor
3	entry to turbine (stator)
4	exit of turbine

Two consecutive numerical subscripts indicate an average value over that range.

1 INTRODUCTION

The analysis of a real, open-circuit gas turbine differs

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from the closed-circuit air-standard analysis (incorporating turbine and compressor efficiencies) because of various effects: internal combustion of fuel instead of heat addition from an external source, variation with temperature of the specific heat of air and combustion products, dissociation becoming significant at high temperatures and pressure losses in the combustion chamber, turbine exhaust and other ducts. The theoretical determination of the optimum design parameters of a gas turbine power plant depends on the mathematical model used. The present paper provides qualitative understanding and quantitative determination of the dependence of optimum parameters on all of these four effects considered separately and when combined. (If turbine cooling is necessary, then calculations become further involved.)

The air-standard analysis is used as the datum (subscript A). The shifts in optimum conditions, when other mathematical models are used, are measured from this datum, where ϕ denotes the fractional change in optimum pressure ratio and φ denotes the fractional change in the corresponding maximum efficiency. In order to study the above effects separately four intermediate models are used:

1. The 'n effect'. This studies the deviation from air-standard analysis when the variation with temperature of the specific heat of air and combustion products is included. The subscript n is used to denote this model.
2. The 'f effect'. This studies the deviation from air-standard analysis when the effect of increased mass flow due to fuel addition is considered. The subscript f is used to denote this model.
3. The 'Δp effect'. This studies the deviation from air-standard analysis when the effect of pressure losses in the combustion chamber, turbine exhaust and other ducts are included. The subscript p is used to denote this model.
4. Dissociation. This studies the effects of dissociation of combustion products.

When all these effects are considered together the general model for an open-circuit gas turbine power plant is obtained. The overall plant efficiency is then denoted by η_o.

Study of the above four effects, in isolation and when combined, has been undertaken by four different methods:

1. Full numerical solution developed in this paper.
2. Analytical (non-linear) solution developed in this paper. The subscript analyt has been used to denote this procedure.
3. Linear perturbation analysis developed in this paper. The subscript linear has been used to denote this procedure.
4. Linear perturbation analysis developed in a recent extensive paper by Horlock and Woods [**1**], who discussed the issues in a systematic way. The subscript ref1 has been used to denote this procedure.

The present numerical solutions have been compared with closed-form analytical solutions and the linear perturbation analyses. Two major conclusions of reference [**1**] were that the efficiency of CBT (compressor–burner–turbine) plant increases due to the 'n effect' and decreases due to the 'f effect'. The present study shows that the actual effects are the opposite of these; the reasons for the discrepancy are explained in sections 3.1 and 3.4. Accordingly, the linear perturbation analysis has been modified. The calculation of the 'n effect' and the 'f effect' contains a number of subtle and conceptual issues. These two effects are discussed first. The effects of pressure loss and dissociation are then discussed in sections 3.6 and 3.7 respectively. Section 3.7 and Appendix 3 describe a newly discovered concept of optimum turbine entry temperature. The accuracy of a standard, approximate method for predicting the performance of gas turbines is assessed in section 3.2 and Appendices 1 and 2.

2 MATHEMATICAL MODELS AND SOLUTION METHODS

2.1 Mathematical models

The performance of a gas turbine power plant can be modelled with various levels of approximations. The basic model for a closed-circuit plant and the general model for an open-circuit plant are discussed first. Modifications required for capturing the 'n effect' and the 'f effect' are then illustrated.

2.1.1 Air-standard cycle

The simplest description of the cycle is the air-standard model of a closed-circuit power plant. The working fluid, air, is assumed to be a perfect gas with constant specific heats. The heat addition and rejection are external (i.e. these take place through appropriate heat exchangers). There is no loss in pressure in the circuit and therefore the pressure ratio across the compressor is the same as that across the turbine. The irreversibilities in the turbine and compressor are accounted for by appropriate isentropic efficiencies. The efficiency of the air-standard cycle, with compressor and turbine losses, is given by [**2**]

$$\eta_A = \frac{\dfrac{T_3}{T_1}\left(1 - \dfrac{1}{x}\right)\eta_t - \dfrac{x-1}{\eta_c}}{\dfrac{T_3}{T_1} - \dfrac{x-1}{\eta_c} - 1} \tag{1}$$

Differentiation of equation (1) $(\partial\eta_A/\partial x = 0)$ gives the optimum value of the isentropic temperature ratio, x_{eA}, at which the maximum efficiency is achieved. It can be shown [2] that, for a given temperature ratio T_3/T_1,

$$x_{eA} = \frac{-B - \sqrt{B^2 - 4AC}}{2A} \tag{2}$$

where

$$A = \frac{T_1}{T_3}\frac{1}{\eta_c} + \frac{\eta_t}{\eta_c} - \frac{1}{\eta_c}$$
$$B = -2\frac{\eta_t}{\eta_c}$$
$$C = \frac{T_3}{T_1}\eta_t - \eta_t + \frac{\eta_t}{\eta_c}$$

Horlock and Woods [1] have considered the conditions $\theta = T_3/T_1 = 4, \eta_c = 0.8$ and $\eta_t = 0.9$ for their numerical illustrations. These values have been used here in all subsequent calculations unless otherwise stated. Equations (2) and (1) then give $x_{eA} = 2.0503, r_{eA} = 12.34$ and $\eta_{eA} = 0.3149$. The entry condition in the present work is taken as $p_1 = 1$ bar, $T_1 = 288$ K.

2.1.2 *Open-circuit power plant*

The definition of rational overall efficiency for the open-circuit CBT plant is given by [1]

$$\eta_o = \frac{(1+f)(h_{g3} - h_{g4}) - (h_{a2} - h_{a1})}{f[\mathrm{CV}]_0} \tag{3}$$

where the fuel–air ratio can be calculated from

$$f[\mathrm{CV}]_0 = (1+f)(h_{g3} - h_{g1}) - (h_{a2} - h_{a1}) \tag{4}$$

This is the general model of a gas turbine power plant. In this, fuel is directly added in the burner (combustion chamber) and energy is released as a result of combustion. The mass flowrate and the composition of the gas that flows through the turbine (which is situated downstream of the combustion chamber) are different from those going through the compressor. The variation of specific heats with temperature for both air and combustion products are included (section 2.2 shows how this is actually calculated). Any pressure losses can easily be included in the analysis; this would mean that the pressure ratio across the compressor would be different from the pressure ratio across the turbine.

2.1.3 *Model to capture the 'n effect' only*

In this, the only deviation from the air-standard analysis is that the variation of gas and air specfic heats with temperature are included. Simple equations for calculating thermodynamic properties of air and combustion products have been given by Guha [3]. The study shows that the specific heat increases with temperature. For example, c_p of air increases by 25 per cent as the temperature changes from 300 to 2100 K. In addition to the temperature, the specific heat of combustion products depends on the fuel and fuel–air ratio. The specific heat ratio (γ) of air and combustion products decreases with temperature. For example, for air, $\gamma = 1.4$ at 300 K but decreases monotonically to $\gamma \approx 1.3$ at 2100 K. At any temperature, c_p of combustion products is higher than that of air and γ of combustion products is lower than that of air, the effects increasing with an increasing fuel–air ratio [3].

The efficiency considering the 'n effect' only, η_n, is given by

$$\eta_n = \frac{(h_{g3} - h_{g4}) - (h_{a2} - h_{a1})}{(h_{g3} - h_{g1}) - (h_{a2} - h_{a1})} \tag{5}$$

While evaluating equation (5) various enthalpies are determined by neglecting pressure losses and dissociation. Comparison with the air-standard cycle might suggest the term $(h_{g3} - h_{a2})$ as the heat input. However, the particular denominator in the right-hand side (RHS) of equation (5) is used in order to be compatible with the definition of rational overall efficiency of an open-circuit plant defined by equations (3) and (4). The difference is not great and does not make any qualitative change in the result.

2.1.4 *Model to capture the 'f effect' only*

In this, it is assumed that energy is added as a result of combustion, which increases the mass flow through the turbine but does not alter the specific heat of the working fluid from its constant value as in the air-standard analysis. The efficiency of this cycle can be calculated by the equation

$$\eta_f = \frac{(1+f)(h_{A3} - h_{A4}) - (h_{A2} - h_{A1})}{(1+f)(h_{A3} - h_{A1}) - (h_{A2} - h_{A1})} \tag{6}$$

where the subscript A is used to indicate that the enthalpy values are the same as in the air-standard analysis considering constant specific heat (as the 'f effect' is being studied in isolation). Substitution of $f = 0$ in equation (6) recovers the air-standard result. [The particular form of denominator is used in equation (6) to be compatible with equations (3) and (4). Since the aim is to find the difference in efficiency from an air-standard cycle, it could perhaps be proposed that $(1+f)h_{A3} - h_{A2}$ be used as the heat input, which also reduces to the appropriate air-standard form when $f = 0$. The effect of this choice is shown later in Table 2 and section 3.5.]

2.1.5 Model to capture pressure losses

It is a straightforward matter to calculate the effects of pressure losses both in isolation as well as in conjunction with all other effects. The turbine pressure ratio is lower than the compressor pressure ratio by a factor $(1 - \sum \Delta p/p)$, where $\sum \Delta p/p$ is the total fractional pressure loss in the combustion chamber, turbine exhaust and other ducts. The turbine work output therefore decreases, reducing the cycle efficiency. This topic is discussed further with numerical illustrations in section 3.6.

2.2 Solution methods

2.2.1 Approximate linear perturbation analysis

Various cycle efficiencies can be calculated from equations (3) to (6) when appropriate enthalpy values for air and combustion products are used. A linear perturbation analysis seeks to find a closed-form analytical estimate of the changes from the air-standard analysis when 'real' effects are incorporated. Two levels of approximations are used for this in reference [**1**]:

(a) *Approximations in property values.* Changes in enthalpies are approximated by using *average* specific heats for air and combustion products. This is done by introducing two parameters n and n' [$n = (c_{pa})_{12}/(c_{pg})_{34}$; $n' = (c_{pa})_{12}/(c_{pg})_{13}$]. As an example, equation (5) then specializes into equation (11) given later in section 3.1.1. Further discussion on this approximation can be found in section 3.2 and Appendices 1 and 2.

Moreover, reference [**1**] assumes a constant fuel–air ratio ($f = 0.014$). The impact of this apparently innocuous assumption is explored in section 3.3.

(b) *Approximations due to linearization.* In the linear perturbation analysis, approximations to changes in plant efficiency from air-standard values are made by assuming these are small quantities. As an example, φ_n can be calculated directly from its definition $\varphi_n = (\eta_n - \eta_A)/\eta_A$, where η_A is calculated from equation (1) and η_n is calculated from equation (5) or equation (11). In the linear perturbation analysis [**1**], this is not done directly but φ_n is expressed as a linear combination of small quantities: $\varphi_n = E_n(1 - n) + F_n\phi_n$ and $\phi_n = D_n(1 - n)$, where D_n, E_n and F_n may be determined from α, β and x_{eA}. The advantage is that D_n, E_n and F_n can all be calculated from the air-standard analysis itself.

Similarly, in the linear perturbation analysis, φ_f and ϕ_f (changes in maximum efficiency and optimum pressure ratio due to the 'f effect') are determined from a linear combination of small quantities: $\varphi_f = E_f f + F_f\phi_f$ and $\phi_f = D_f f$. In the linear perturbation analysis, higher-order terms of small quantities are neglected.

2.2.2 Full numerical solution

It is intended here to determine the 'n', 'f', pressure loss and dissociation effects directly by full numerical computations. It is not possible to study the effects of various elements in isolation by employing a commercially available computer program, e.g. GasTurb [**4**], which only gives the combined effects as a single output. A purpose-built computer program has been developed by the author for understanding the physics involved.

For this, accurate relations for properties of air and combustion products are needed. In particular, specific enthalpy and specific entropy of air and combustion products have to be determined as a function of temperature to be used in equations (3) to (6). Although the simple equations developed by Guha [**3**] would have made the computer program usable for any hydrocarbon fuel, the following equations, (7) to (10), comprehensive but specific only for kerosene [**5**], have been used here. The coefficients are given in Table 1. These equations do not model any dissociation of combustion products:

$$h_a\,(\mathrm{MJ/kg}) = A_0\left(\frac{T}{1000}\right) + \sum_{i=2}^{9}\frac{1}{i}A_{i-1}\left(\frac{T}{1000}\right)^i + A_9 \tag{7}$$

$$h_g\,(\mathrm{MJ/kg}) = A_0\left(\frac{T}{1000}\right) + \sum_{i=2}^{9}\frac{1}{i}A_{i-1}\left(\frac{T}{1000}\right)^i + A_9 + \frac{f}{1+f} \times \left[B_0\left(\frac{T}{1000}\right) + \sum_{i=2}^{7}\frac{1}{i}B_{i-1}\left(\frac{T}{1000}\right)^i + B_8\right] \tag{8}$$

$$s_{a2} - s_{a1}\,(\mathrm{kJ/kg\,K}) = A_0 \ln\left(\frac{T_2}{T_1}\right) + \sum_{i=1}^{8}\frac{1}{i}A_i \times \left[\left(\frac{T_2}{1000}\right)^i - \left(\frac{T_1}{1000}\right)^i\right] - R_a \ln\left(\frac{p_2}{p_1}\right) \tag{9}$$

Table 1 Values of coefficients in equations (7) to (10)

i	A_i	B_i
0	0.992313	−0.718874
1	0.236688	8.747481
2	−1.852148	−15.863157
3	6.083152	17.254096
4	−8.893933	−10.233795
5	7.097112	3.081778
6	−3.234725	−0.361112
7	0.794571	−0.003919
8	−0.081873	0.0555930
9	0.422178	−0.0016079
10	0.001053	

$$s_{g4} - s_{g3}\,(\mathrm{kJ/kg\,K}) = A_0 \ln\left(\frac{T_4}{T_3}\right) + \sum_{i=1}^{8} \frac{1}{i} A_i \left[\left(\frac{T_4}{1000}\right)^i - \left(\frac{T_3}{1000}\right)^i\right] + \frac{f}{1+f}\left\{B_0 \ln\left(\frac{T_4}{T_3}\right) + \sum_{i=1}^{7} \frac{1}{i} B_i \left[\left(\frac{T_4}{1000}\right)^i - \left(\frac{T_3}{1000}\right)^i\right]\right\} - R_g \ln\left(\frac{p_4}{p_3}\right) \quad (10)$$

where $R_g = 287.05 - 0.0099f + 10^{-7}f^2$.

The procedure for calculating the overall rational efficiency η_o in the open-circuit plant is as follows. Calculate h_{a1} from equation (7) since T_1 is known. Now set $s_{a2} - s_{a1} = 0$ in equation (9) and solve iteratively for the temperature $T_{a2,\,isen}$ at the exit of a hypothetical isentropic compressor. Use equation (7) to determine $h_{a2,\,isen}$. With the help of the isentropic efficiency of the compressor, h_{a2} can now be determined. Equations (8) and (4) are to be solved simultaneously and iteratively to determine f and h_{g3}. Set $s_{g4} - s_{g3} = 0$ in equation (10) to calculate iteratively the temperature at the exit of a hypothetical isentropic turbine, $T_{g4,\,isen}$. Use equation (8) to calculate $h_{g4,\,isen}$. With the help of the turbine isentropic efficiency, h_{g4} can then be determined. Application of equation (3) will now give the rational overall efficiency of the power plant, η_o. Similarly, η_A, η_n and η_f can be calculated respectively by equations (1), (5) and (6). The whole procedure is then repeated at different pressure ratios. Figure 1 shows various computed efficiencies as a function of the pressure ratio. Table 2 shows some relevant parameters of the simulation. Table 3 shows the comparison of present full numerical calculations with the linear perturbation prediction of reference [**1**].

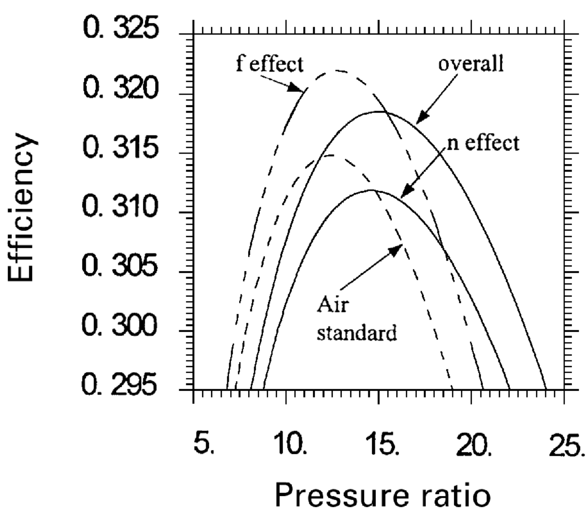


Fig. 1 Variation of gas turbine thermal efficiency as a function of pressure ratio for various mathematical models. [All curves are numerically simulated as detailed in section 2. For all calculations, $\theta = T_3/T_1 = 4$, $\eta_c = 0.8$, $\eta_t = 0.9$, no pressure loss. ———, η_o given by equation (3); -----, η_A given by equation (1); ——–——, η_n given by equation (5); ———–––, η_f given by equation (6)]

3 DISCUSSION

The numerical calculations of Fig. 1 and Table 2 show that both the 'n effect' and 'f effect' increase the optimum pressure ratio, the influence of the 'n effect' in this regard being the dominant one. The 'n effect' decreases the maximum efficiency while the 'f effect' increases it, the overall effect being a slight increase in maximum efficiency. These results were calculated for $\theta = T_3/T_1 = 4$, $\eta_c = 0.8$ and $\eta_t = 0.9$. Numerical calculations were also conducted for other values of (T_3/T_1), η_c and η_t. For all calculations undertaken the above conclusions remained valid.

It is easy to see why the 'f effect' increases maximum efficiency. Consider equations (1) and (6) together. These equations could be written in contracted form as $\eta_A = (A - B)/(C - B)$ and $\eta_f = (A - B')/(C - B')$, where $C > A$ and $B' = B/(1+f) < B$. This means that, at *all* pressure ratios, $\eta_f > \eta_A$. This is indeed borne out by the numerical calculations shown in Fig. 1 and means that η_f is positive.

The 'n effect' is complex. As a result of variation of c_p and γ with temperature, the compressor work input (w_c), turbine work output (w_t) and energy added (q_{in}) are all affected, though the changes in the latter two are much more prominent. Suppose that Δw_t and Δq_{in} are the differences between the n-effect analysis and the corresponding air-standard analysis at the same pressure ratio. Both Δw_t and Δq_{in} are positive quantities. The main reason why Δq_{in} is positive is the increased value of c_p. Δw_t, although positive, is however affected by the opposing effects of c_p and γ, the increase in c_p tending to increase w_t and the decrease in γ (which reduces the temperature drop across the turbine) tending to decrease w_t. At lower pressure ratios, the effect of positive Δq_{in} therefore dominates, and it is found that $\eta_n < \eta_A$. It should be remembered that the study, such as in Fig. 1, is conducted with a fixed T_3. At high pressure ratios, T_2 comes closer to T_3 and the overall magnitude of q_{in} diminishes. At high pressure ratios, therefore, the effect of Δw_t dominates over that of Δq_{in} and it is found that $\eta_n > \eta_A$. Figure 1 shows that the η_n curve stays below η_A at low pressure ratios but the η_n curve stays above η_A at high pressure ratios, the crossing-over taking place at about $r = r_{en}$. Thus η_f is negative.

Table 2 Numerically determined maximum efficiency conditions for various levels of modelling (for $\theta = 4,\ \eta_c = 0.8,\ \eta_t = 0.9$)

	Maximum efficiency	$\varphi = (\eta_e - \eta_{eA})/\eta_{eA}$	Optimum pressure ratio r_e	$x_e = r_e^{(\gamma-1)/\gamma}$	$\phi = (x_e - x_{eA})/x_{eA}$
Air standard	$\eta_{eA} = 0.3149$		12.34	2.0503	
'n effect'	$\eta_{en} = 0.3120$	−0.0092	14.56	2.1495	0.048
'f effect'*	$\eta_{ef} = 0.3220$	0.0225	12.65	2.0648	0.0071
'(n + f) effect'	$\eta_{eo} = 0.3186$	0.012	14.9	2.1637	0.0553

* If $[(1+f)h_{A3} - h_{A2}]$ is used as the heat input in equation (6) then $\varphi = 0.0038, \phi = 0.006$.

Table 3 Comparison of present numerical calculations with linear analysis [1]

	φ_n	φ_f	φ_{n+f}	$\varphi_n + \varphi_f$	ϕ_n	ϕ_f	ϕ_{n+f}	$\phi_n + \phi_f$
Reference [1]	0.024*	−0.017†	Assumed $\varphi_{n+f} = \varphi_n + \varphi_f$		0.05*	0.012	Assumed $\phi_{n+f} = \phi_n + \phi_f$	
Present numerical calculations	−0.0092	0.0225	0.012	0.013	0.048	0.0071	0.0553	Near equality demonstrated

* See section 3.1, Table 4.
† 0.0237 in Erratum [6] (see section 3.4).

3.1 Comments on ϕ_n and φ_n

3.1.1 *Linear perturbation analysis of reference* [1]

A linear perturbation method determines analytically the small departure from the air-standard results as each effect (e.g. 'n', 'f', 'Δp', etc.) is introduced in the analysis. First consider the 'n effect'. The variations in specific heat and specific heat ratio with temperature are captured only *approximately* by using their average values in the appropriate temperature ranges. Thus, the various enthalpy differences in equation (5) are evaluated as $h_{g3} - h_{g4} = (c_{pg})_{34}(T_3 - T_4)$, $h_{a2} - h_{a1} = (c_{pa})_{12}(T_2 - T_1)$, etc. With these approximations, equation (5) then specializes into

$$\eta_{n,\,ref1} = \frac{(\alpha/n)(1 - 1/x^n) - (x - 1)}{(\beta - 1)/n' - (x - 1)} \qquad (11)$$

where $\alpha = \eta_c\eta_t\theta$, $\beta = 1 + \eta_c(\theta - 1)$, $n = (c_{pa})_{12}/(c_{pg})_{34}$ and $n' = (c_{pa})_{12}/(c_{pg})_{13}$. With the chosen values of various parameters, $\alpha = 2.88$ and $\beta = 3.4$. When there is no variation in specific heat, $n = n' = 1$ and equation (11) reduces to the air-standard analysis given by equation (1). The linear perturbation analysis [1] gives

$$\phi_{n,\,ref1} = 0.4(1 - n) \qquad (11a)$$

$$\varphi_{n,\,ref1} = 0.19(1 - n) \qquad (11b)$$

Equations (11a) and (11b) are derived under the assumption $n = 7/8 = 0.875$ and $n' = 0.5(1 + n) = 0.9375$. With $n = 0.875$, the two equations give $\phi_{n,\,ref1} = 0.05$ and $\varphi_{n,\,ref1} = 0.024$, as shown in Table 3. Table 3 shows that the conclusion made in reference [1], that maximum efficiency increases due to the 'n effect', is contradicted by the present numerical calculations. The reasons for this discrepancy is explained next.

One advantage of the computer program used in this study is that it can determine the exact values of n and n' at each operating point. As the enthalpies are accurately calculated in the program using equations (7) to (10), it can numerically determine the average specific heat over various temperature ranges. Present numerical calculations show that actual values of n and n' vary with the pressure ratio and various other parameters, such as T_1, T_3, η_c, η_t, etc. (Appendix 1). When the pressure ratio is in the range 12–15, with other parameters fixed at their chosen levels as in reference [1], the data in Appendix 1 show that representative values may be taken as $n \approx 0.9$ and $n' \approx 0.935$. When these values are used directly in equation (11), then $\varphi_{n,\,eqn11} = -0.0067$ [$\varphi_{n,\,eqn11}$ is calculated from $(\eta_{n,\,eqn11} - \eta_A)/\eta_A$]. Thus, it is found that φ_n is extremely sensitive to the values of n and n' and apparently innocuous changes in n and n' bring qualitative changes in the value of φ_n. A mere 2.8 per cent decrease in n (from 0.9 to 0.875) in the example causes a 530 per cent increase in $\varphi_{n,\,eqn11}$ (from −0.0067 to +0.0288). It should be remembered that $\varphi_{n,\,eqn11} = -0.0067$ is calculated using the actual value of x_{en} (= 2.1495) calculated by the computer program. In addition, equation (11) has been used directly to evaluate the efficiency. With $n = 0.9$, equation (11b) would have given $\varphi_{n,\,ref1} = 0.019$ and equation (11a) would have given $\phi_{n,\,ref1} = 0.04$.

3.1.2 *An improved linear perturbation analysis*

Equations (11a) and (11b) are based on the assumption $n' = (n + 1)/2$. Data presented in Appendix 1 show that this relation is not exactly valid at the relevant operating points. Additionally, section 3.1.1 and Appendix 2 show that φ_n depends very strongly on the values of n and n'. Hence, the linearized analysis of reference [1] needs to be reworked by keeping both n and n' as separate entities.

When this is done, the following results are obtained:

$$\phi_{\mathrm{n,\,linear}} = \frac{(\alpha - \beta + 1)x_{\mathrm{eA}}^2 \ln x_{\mathrm{eA}}(1-n) + \alpha\left[\left(n' + \dfrac{n'}{n} - 2\right)x_{\mathrm{eA}} - \left(\dfrac{n'}{n} - 1\right)x_{\mathrm{eA}}^2 + (1-n')\right]}{2\alpha(x_{\mathrm{eA}} - \beta)} \tag{12a}$$

$$\varphi_{\mathrm{n,\,linear}} = \frac{(x_{\mathrm{eA}} - 1 - \ln x_{\mathrm{eA}})(1-n)}{(x_{\mathrm{eA}} - 1)(1 - x_{\mathrm{eA}}/\alpha)} - \frac{(\beta - 1)(1/n' - 1)}{\beta - x_{\mathrm{eA}}} \tag{12b}$$

[While writing equation (12b) a simplification is made. The RHS of equation (12b) contained another term, $\phi_{\mathrm{n}}[(\alpha/x_{\mathrm{eA}} - x_{\mathrm{eA}})/(\alpha - \alpha/x_{\mathrm{eA}} - x_{\mathrm{eA}} + 1) + x_{\mathrm{eA}}/(\beta - x_{\mathrm{eA}})]$; the coefficient within the brackets is found to be almost identically zero under all combinations of θ, η_{c} and η_{t} that were tested. Thus equation (12b) does not explicitly depend on ϕ_{n}.] With $n = 0.9$ and $n' = 0.935$, equation (12a) gives $\phi_{\mathrm{n,\,linear}} = 0.0377$ and equation (12b) gives $\varphi_{\mathrm{n,\,linear}} = -0.0138$. Equation (12b) produces the correct sign for φ_{n} and brings its magnitude closer to the numerical result than equation (11b).

A small change in n induces a much larger relative change in $(1-n)$ since n is close to 1. Both ϕ_{n} and φ_{n} are very sensitive to small variations in n and n' (Appendix 2). Accurate values of n and n' depend on r, $T_3/T_1, \eta_{\mathrm{c}}, \eta_{\mathrm{t}}, \sum \Delta p/p, T_1, p_1$ and the fuel (Appendix 1), and they can only be determined by detailed numerical computation such as the present one. Moreover, even the use of exact values of n and n' in the linear analysis does not give exact values of ϕ_{n} and φ_{n}. The discrepancy in the linear analysis would increase at a higher value of turbine entry temperature (giving a higher optimum pressure ratio) than the one used for the example calculations.

3.1.3 Non-linear analytical solution

The error due to linearization itself (section 2.2.1) may be eliminated by differentiating equation (11) directly. However, before this is done, it is worth pointing out that while writing equation (11) it was tacitly assumed in reference [**1**] that $\gamma_{12} = \gamma_A$, where γ_{A} is the *constant* isentropic exponent (specific heat ratio) used in the air-standard analysis. In this work (as well as in reference [**1**]) γ_{A} is taken as 1.4. The definition of the isentropic temperature ratio, x, is based on $\gamma_{\mathrm{A}}(x = r^{(\gamma_{\mathrm{A}}-1)/\gamma_{\mathrm{A}}})$. All calculations of x, ϕ and φ use this value of γ_{A}. On the other hand, γ_{12} is the (variable) average value of the specific heat ratio appropriate for the compressor and depends on the relevant temperature range and pressure ratio. When this distinction is made between γ_{A} and γ_{12}, equation (11) should be modified to

$$\eta_{\mathrm{n,\,analyt}} = \frac{(\alpha/n)(1 - 1/x'^n) - (x' - 1)}{(\beta - 1)/n' - (x' - 1)} \tag{13}$$

by noting that x in Horlock and Woods' equation should be replaced by x', where

$$x' = x^{(\gamma_{\mathrm{A}}/\gamma_{12})[(\gamma_{12}-1)/(\gamma_{\mathrm{A}}-1)]} \tag{14}$$

Numerical calculations show that for the particular example being considered $\gamma_{12} = 1.388$ (in the neighbourhood of the optimum pressure ratio). This gives $x = x'^{1.0221}$. All values of ϕ_{n} calculated earlier by linearized analysis including those quoted in reference [**1**] do not employ this correction. Again, an apparently innocuous change in the indices has a substantial effect on the calculated value of ϕ_{n}.

In order to find the optimum condition, set $\partial\eta_{\mathrm{n,\,analyt}}/\partial x' = 0$. This gives

$$\left(\frac{\alpha}{n} - \frac{\beta - 1}{n'}\right)x'^{(n+1)} - \frac{\alpha(1+n)}{n}x' + \alpha\left(1 + \frac{\beta - 1}{n'}\right) = 0 \tag{15}$$

Equation (15) is solved by an iterative scheme. This gives, for $n = 0.9$ and $n' = 0.935$, $x'_{\mathrm{en,\,analyt}} = 2.12608$, $x_{\mathrm{en,\,analyt}} = 2.1618$ and $\phi_{\mathrm{n,\,analyt}} = 0.054$. $\varphi_{\mathrm{n,\,analyt}}$ is calculated from $(\eta_{\mathrm{n,\,analyt}} - \eta_{\mathrm{A}})/\eta_{\mathrm{A}}$. Table 4 shows the comparison of various models discussed in section 3.1.

3.2 Accuracy of a standard, approximate method for predicting the performance of gas turbines

Although it is desirable to use exact thermodynamic properties of air and combustion products, such as equations (7) to (10) used in the present computer program, often an approximate method of performance calculation is undertaken, particularly for hand calculations. The approximate method [**1, 7, 8**] is derived from the air-standard analysis by employing various average values of specific heat over appropriate range of temperatures.

Whittle makes the following interesting comment in his book (p. xi) [**8**]:

> When in jet engine design, greater accuracy was necessary for detail design, I worked in pressure ratios, used $\gamma = 1.4$ for compression and $\gamma = 1.33$ for expansion and assumed specific heats for combustion and expansion corresponding to the temperature range concerned. I also allowed for the increase of mass flow in expansion due to fuel addition (in the range 1.5–2%). The results, despite guesswork involved in many of the assumptions, amply justified these methods to

Table 4 Comparison of numerical and various analytical determinations of the 'n effect' (with $n = 0.9$ and $n' = 0.935$)

	ϕ_n	φ_n
Present numerical	0.048	− 0.0092
Reference [1]	0.04 [equation (11a)]	+ 0.019 [equation (11b)]
Present linear	0.0377 [equation (12a)]	− 0.0138 [equation (12b)]
Present non-linear analytical	0.054 [equation (15)] (with $x \rightarrow x'$ correction)	− 0.006 [equation (13)]

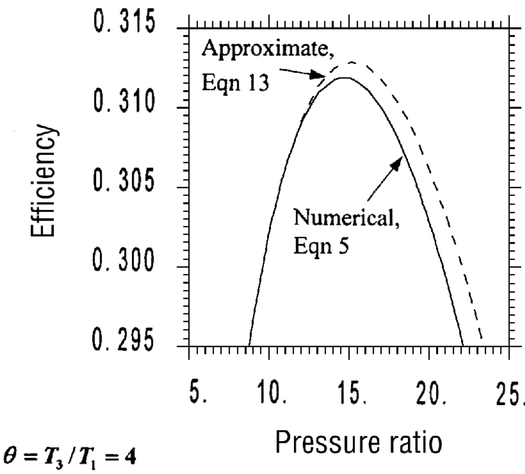
the point where I was once rash enough to declare that 'jet engine design has become an exact science'. (This statement was inspired by the fact that on the first test of the W2/500 engine every experimental point fell almost exactly on the predicted curves of performance.)

It is recognized that equation (13) is a good example of the approximate methods and its prediction, with exact values of n and n', would be the best that approximate methods can reach. The present context therefore offers an opportunity to assess the quantitative measure of accuracy of the standard approximate method, which is not available in the literature.

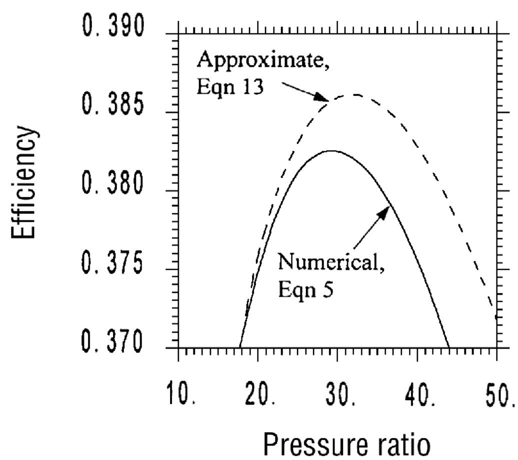
Figure 2 can be studied for the evaluation of accuracy of equation (13). The solid curves in Fig. 2 are calculated by equation (5) in conjunction with equations (7) to (10). Figure 2 shows that equation (13) predicts a higher maximum efficiency and higher optimum pressure ratio than the accurate numerical calculations (this explains the numbers found in Table 4). The error in the prediction of equation (13) grows as the turbine entry temperature increases. It should be noted that, in Fig. 2, exact values of n and n' are used at *each* point of calculation, determined by the present computer program. Obviously equations like (11) or (13) are of maximum worth as a predictive tool only if they work with reasonable accuracy with approximate (fixed) values of n and n'. Figure 2 shows that even when exact values of n and n' are used at each point of calculation, the prediction of equation (13) is not accurate, the error increasing with increasing pressure ratio and turbine entry temperature. Exact values of n and n' ensure that enthalpy differences are determined accurately for a given temperature difference in the approximate model; however, it is not possible then to simultaneously satisfy the entropy equation exactly, particularly when large pressure ratios and temperature differences are involved. [The average ratio of specific heats $\gamma_{av} = c_{p,av}/(c_{p,av} - R)$. The approximate theory assumes that the equation, $p/\rho^{\gamma_{av}} = \text{constant}$, would specify the isentropic condition. This would not give the same results as setting $s_{final} - s_{initial} = 0$ in equations (9) and (10).]

The above discussion shows that, although an average specific heat and specific heat ratio may be used for calculating turbine work, heat input and efficiency approximately, there is difficulty in using these equations [e.g. equation (11) or equation (13)] for calculating *small changes* in these quantities, for the very reason that the equations are approximate.

Other than the above difficulty regarding specific heat and the specific heat ratio, the approximate theories also



(a) $\theta = T_3/T_1 = 4$



(b) $\theta = T_3/T_1 = 5$

Fig. 2 The assessment of the accuracy of a standard, approximate method for predicting gas turbine performance. [For all calculations, $\eta_c = 0.8$, $\eta_t = 0.9$, $T_1 = 288$ K, no pressure loss. Exact values of n and n', determined from numerical simulation, are used at *each* operating point while evaluating equation (13), which is a standard, approximate method [1, 7, 8] for predicting gas turbine performance]

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have the difficulty of obtaining a good estimate of the fuel–air ratio. Following Whittle's suggestion, equation (13) could be enhanced by considering the added fuel mass that goes through the turbine. This gives

$$\eta_{\mathrm{n+f,\,analyt}} = \frac{[\alpha(1+f)/n](1-1/x'^{\mathrm{n}}) - (x'-1)}{(1+f)(\beta-1)/n' - (x'-1)}$$

Again, an approximate theory is of maximum worth as a predictive tool if it works with reasonable accuracy with approximate (fixed) value of f (reference [**1**] uses $f = 0.014$). The next section discusses the effect of assuming a fixed fuel–air ratio.

3.3 Comments on ϕ_{f}

Horlock and Woods [**1**] have assumed a constant value of the fuel–air ratio ($f = 0.014$) for the calculation of ϕ_{f}. Instead of determining x_{ef} from the linear perturbation analysis, it can also be determined directly from equation (6) by setting $(\partial\eta_{\mathrm{f}}/\partial x)_{f=\mathrm{constant}} = 0$. The result is

$$x_{\mathrm{ef,\,analyt}} = \frac{-B_{\mathrm{f}} - \sqrt{B_{\mathrm{f}}^2 - 4A_{\mathrm{f}}C_{\mathrm{f}}}}{2A_{\mathrm{f}}} \qquad (16)$$

where

$$A_{\mathrm{f}} = \alpha - \beta + 1$$
$$B_{\mathrm{f}} = -2\alpha$$
$$C_{\mathrm{f}} = \alpha[1 + (1+f)(\beta - 1)]$$

Equation (16) can be compared with equation (2). With the values of various parameters chosen, this gives $x_{\mathrm{ef,\,analyt}} = 2.0759$, $r_{\mathrm{ef,\,analyt}} = 12.89$ and $\phi_{\mathrm{f,\,analyt}} = 0.012$, thus agreeing with the results of reference [**1**]. However, numerical simulation shows that this value of ϕ_{f} is inaccurate by about 100 per cent (Table 3).

In order to understand the reason for the discrepancy, η_{f} is calculated from equation (6) in the computer program in two different ways: firstly, using accurate values of the fuel–air ratio at each pressure ratio (as done in Fig. 1) and, secondly, using a constant value $f = 0.014$ at all pressure ratios (as was done in the linear perturbation analysis). The result is shown in Fig. 3, which indicates that the peak of efficiency in the second case indeed occurs at a pressure ratio of 12.89, giving $\phi_{\mathrm{f},f=\mathrm{constant}} = 0.012$.

The moral is that although the actual change in the fuel–air ratio with the pressure ratio near the optimum point is small (being $f = 0.0142$ at $r = 10.93$ and $f = 0.0134$ at $r = 12.88$), this variation cannot be neglected while determining the optimum value of the pressure ratio. It is true that this inaccuracy would somewhat be masked by other dominant effects (such as the 'n effect') in the overall calculation of the optimum pressure ratio, but nevertheless the analytical theories are not too successful in predicting ϕ_{f} itself. It is expected that the error would increase with higher values of turbine entry temperature than was used for these example calculations.

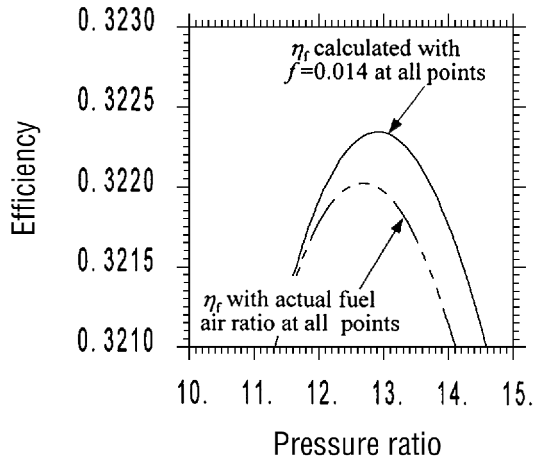

Fig. 3 Variation of gas turbine thermal efficiency as a function of pressure ratio; calculation of the 'f effect' in two ways. (For all calculations, $\theta = T_3/T_1 = 4$, $\eta_{\mathrm{c}} = 0.8$, $\eta_{\mathrm{t}} = 0.9$, no pressure loss)

3.4 Comments on φ_{f}

Equation (16) gives $x_{\mathrm{ef,\,analyt}} = 2.0759$. Using this value in equation (6) gives $\varphi_{\mathrm{f,\,analyt}} = 0.0235$, which is close to the numerically calculated value 0.0225. (The difference arises from the use of an approximate value for f in the analysis and the inaccuracy in x_{ef} as explained in section 3.3. Even though, for the chosen values of various parameters, φ_{f} does not show much sensitivity to the error in x_{ef}, this may not be a universal fact.) This shows that there is no physical explanation for why φ_{f} is calculated to be a negative quantity in reference [**1**] (see Table 3).

It has recently been traced [**6**] that there has been an algebraic error in the determination of φ_{f} from the general equation (66) of reference [**1**]. The reworking of the linear analysis gives $\varphi_{\mathrm{f,\,linear}} = +1.69f$ instead of $\varphi_{\mathrm{f,\,ref1}} = -1.25f$ (which is equation (51) of reference [**1**]). With this correction, the linear analysis gives $\varphi_{\mathrm{f,\,linear}} = 0.0237$, bringing this in line with present numerical calculations.

3.5 Validity of linear superposition

Horlock and Woods postulated (see reference [**1**], p. 251) that, since the perturbation analyses are linear, the overall values of ϕ and φ can be calculated as a simple sum of individual effects. Table 2 shows that, provided equations (5), (6), (1) and (3) are used for various efficiencies (thus ensuring that individual effects are determined accurately), this assumption is approximately true. However, if the expression $[(1+f)h_{A3} - h_{A2}]$ is used as the heat input in equation (6) [see the discussion after equation (6)], then Table 2 shows that $\varphi_{n+f} \neq \varphi_n + \varphi_f$. Thus, although following Horlock and Woods, the changes in the value of efficiency from the closed, air-standard cycle have been ascribed to the 'n effect', the 'f effect', etc.; in reality, the particular value of heat input used in the definition of rational overall efficiency of an open-circuit plant [equation (3)] is also responsible, to some extent, for the changes.

3.6 Effects of pressure loss and its combination with the 'n effect' and the 'f effect'

It is a straightforward matter to calculate the effects of pressure losses both in isolation as well as in conjunction with all other effects. The turbine pressure ratio is lower than the compressor pressure ratio by a factor $(1 - \sum \Delta p/p)$, where $\sum \Delta p/p$ is the total fractional pressure loss in the combustion chamber, turbine exhaust and other ducts. Assuming that $\sum \Delta p/p = 0.1$, as per reference [**1**], the present computer program predicts that the effect of pressure loss alone is given by $\eta_p = 0.28315$, $r_{ep} = 12$, $\phi_p = -0.008$ and $\varphi_p = -0.1008$. The pressure loss, with the assumed 10 per cent loss in the turbine pressure ratio, decreases the maximum efficiency significantly and decreases the optimum pressure ratio slightly. The combined effect $(n + f + p)$ is determined by the computer program as $\eta_o = 0.2889$, $r_e = 14.5$, $\phi = 0.047$ and $\varphi = -0.0826$. Since $\phi_n + \phi_f + \phi_p = 0.048 + 0.0071 - 0.008 = 0.0471$ and $\varphi_n + \varphi_f + \varphi_p = -0.0092 + 0.0225 - 0.1008 = -0.0875$, the principle of superposition is again found to be a reasonable assumption, provided that individual effects are determined accurately.

Similar to the derivation of equations (15) and (16), it is easy to formulate a more direct analytical procedure instead of the linear perturbation analysis. The thermal efficiency considering the pressure loss effect alone is given by

$$\eta_{p,\,analyt} = \frac{\dfrac{T_3}{T_1}\left(1 - \dfrac{1}{xk_p}\right)\eta_t - \dfrac{x-1}{\eta_c}}{\dfrac{T_3}{T_1} - \dfrac{x-1}{\eta_c} - 1} \tag{17}$$

where $k_p = (1 - \sum \Delta p/p)^{(\gamma_a - 1)/\gamma_a}$. It is assumed while deriving equation (17) that if the pressure ratio across the compressor is r then the pressure ratio across the turbine is $(1 - \sum \Delta p/p)r$. The maximum efficiency is determined by the condition $\partial \eta_{p,\,analyt}/\partial x = 0$. This gives

$$x_{ep,\,analyt} = \frac{-B_p - \sqrt{B_p^2 - 4A_pC_p}}{2A_p} \tag{18}$$

where

$$A_p = \frac{T_1}{T_3}\frac{1}{\eta_c} + \frac{\eta_t}{\eta_c} - \frac{1}{\eta_c}$$

$$B_p = -2\frac{\eta_t}{\eta_c k_p}$$

$$C_p = \frac{1}{k_p}\left(\frac{T_3}{T_1}\eta_t - \eta_t + \frac{\eta_t}{\eta_c}\right)$$

Equation (18) can be compared with equation (2). Using the assumed values for various parameters, equation (18) gives $x_{ep,\,analyt} = 2.0348$, $\phi_{p,\,analyt} = -0.0076$ (compared to the numerical solution -0.008 and $\phi_{p,\,ref1} = -0.007$). Equation (17) then gives $\varphi_{p,\,analyt} = -0.1007$ (compared to the numerical solution -0.1008 and $\varphi_{p,\,ref1} = -0.094$). An analytical theory, linear or non-linear, is thus quite successful in predicting the pressure loss effect $(\phi_p,\ \varphi_p)$. The present direct method, i.e. use of equations (17) and (18), is slightly more accurate than the linear perturbation analysis.

3.7 Optimum turbine entry temperature

In the air-standard analysis, if the turbine entry temperature is increased, keeping the pressure ratio fixed, then the thermal efficiency rises continuously and asymptotically to the product of Joule cycle efficiency and the turbine isentropic efficiency [**2**]:

$$\lim_{T_3/T_1 \to \infty} \eta_A = \eta_t \eta_{Joule} \tag{19}$$

In a real, open-circuit gas turbine, increasing turbine entry temperature shows new behaviour. At each pressure ratio, there exists an optimum turbine entry temperature above which any further increase in T_3 reduces the thermal efficiency. The concept of optimum turbine temperature is enunciated for the first time in reference [**2**], which systematically explains the reasons for its existence. This has important implications also for the optimization of aircraft engines [**9**]. In aircraft turbofan engines, the maximum thermal efficiency of the whole engine occurs at a lower temperature than that at which the maximum efficiency of the core engine occurs, and the existence of the optimum turbine entry temperature also gives rise to the existence of an optimum bypass ratio. The magnitudes of the optimum turbine entry temperature and optimum bypass ratio depend on the isentropic efficiencies of the compressors and turbines. At a fixed specific thrust and overall pressure ratio, the optimum bypass ratio decreases progressively as the component efficiencies are increased [**9**]. Reference [**9**] gives a new, comprehensive optimization method for aero gas turbine engines.

Figures 4a and b show the effect of increasing the turbine entry temperature at fixed pressure ratios. Each figure contains three curves. One of them is calculated

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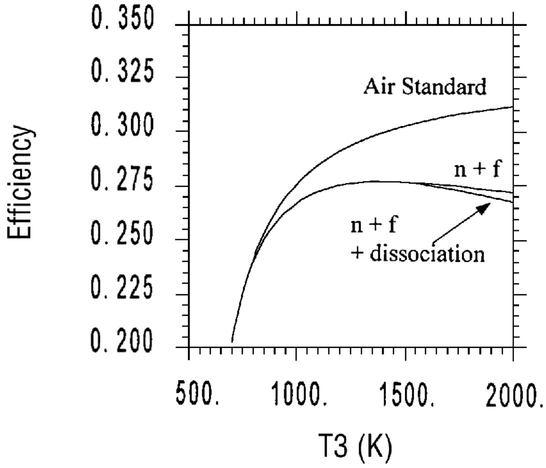
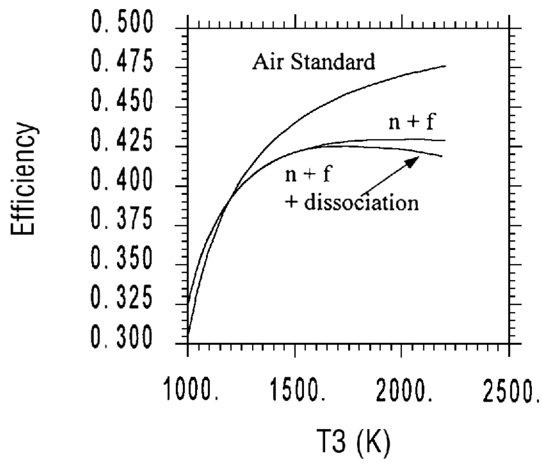


(a) Pressure ratio = 5



(b) Pressure ratio = 20

Fig. 4 Variation of gas turbine thermal efficiency as a function of turbine entry temperature. (For all calculations, $\eta_c = 0.8$, $\eta_t = 0.9, T_1 = 288\,\text{K}$, $p_1 = 1$ bar, no pressure loss)

from equation (1) giving the air-standard efficiency. The second is calculated from equations (3), (4), (7), (8), (9) and (10), as described in section 2.2.2. This curve thus shows the behaviour of a real, open-circuit gas turbine plant when all 'real' effects except dissociation are included. The third curve is calculated by GasTurb [**4**], which calculates the specific heat of air and combustion products as polynomials in temperatures. In addition, GasTurb considers the effects of equilibrium dissociation while calculating the temperature rise due to combustion. The temperature rise due to combustion is tabulated for a reference pressure as a function of the burner inlet temperature and fuel–air ratio. For pressures other than the reference value a correction factor is applied which again depends on the burner inlet temperature and fuel–air ratio (p. 149 of reference [**4**]).

The accuracy of the predictions of the present computer program can be judged from their general agreement with GasTurb predictions in Fig. 4. Figure 4 thus provides a validation of the present numerical code. (However, the 'n', 'f' and 'Δp' effects could not be studied separately by GasTurb, which the present purpose-built computer program makes possible.) Figures 4a and b also offer the opportunity of studying the separate effects of 'n', 'f' and dissociation in determining the optimum turbine temperature. At a low pressure ratio (Fig. 4a), the n + f effects themselves can cause the turning over of the efficiency curve defining the optimum turbine entry temperature. At a higher pressure ratio (Fig. 4b), the 'n + f effects' significantly slow down the change of thermal efficiency with temperature, but the effects of dissociation play an important role in defining the actual magnitude of the optimum temperature.

In most references on gas turbine performance, various efficiencies are plotted as a function of the pressure ratio, keeping the turbine entry temperature as a parameter (as done here in Figs 1 to 3). Figure 4 shows a new way of analysis (as done in reference [**2**]) in which efficiencies are plotted as a function of the turbine entry temperature, keeping the pressure ratio as a parameter. This representation, together with the modelling of all 'real' effects, was important in establishing the existence of an optimum turbine entry temperature (see Appendix 3).

4 CONCLUSIONS

Present numerical calculations, such as given in Fig. 1, show that both the 'n effect' and the 'f effect' increase the optimum pressure ratio at which maximum efficiency occurs, the influence of the 'n effect' in this regard being the dominant one. The 'n effect' decreases the maximum efficiency while the 'f effect' increases it, the combined effect being a slight increase in maximum efficiency. Pressure losses reduce the maximum efficiency significantly and reduce the optimum pressure ratio only slightly. The 'n effect' has the greatest influence in changing the optimum pressure ratio, and (depending on the loss coefficient) pressure losses have the greatest potential in changing the maximum efficiency. Numerical calculations at other values of T_3/T_1, η_c and η_t also showed these qualitative behaviours. The higher the ratio T_3/T_1, the greater is the difference between the efficiency predicted by the air-standard analysis and that predicted by the real 'cycle' analysis. The numerical calculations can be used as the benchmark solution to assess various analytical theories.

Inclusion of pressure losses in an analytical theory is the most straightforward part, and as expected the linear perturbation analysis gives reasonable approximate answers for this effect. Section 3.1 and the Appendices, however, explain that there are fundamental difficulties with the linear perturbation theory to predict the 'n effect'. The error in the linear analysis would grow as the turbine entry temperature increases over the value used in the example calculations. Instead of the linear perturbation analysis, the direct, more accurate, analytical relations derived here may also be used: equations (6) and (16) for the 'f effect' alone, equations (17) and (18) for pressure loss effects alone and equations (13) and (15) for the 'n effect' alone.

Developing an analytical theory for the 'n effect' is the most difficult part. Even though equations like (11) or (13) involving various average specific heats are frequently used to calculate approximate values of efficiency, work done or heat input, predictions of *small changes* based on them must be treated with caution. Section 3.2 and Fig. 2 enumerate the errors involved in this standard, approximate method [**1**, **7**, **8**] for predicting gas turbine performance, even when exact values of n and n' are used at each operating point. Obviously the approximate theories would be useful as a predictive tool only if they work with reasonable accuracy with approximate (fixed) values of n and n'; ϕ_n and φ_n are very sensitive to parameters like n and n', for whose accurate determination a computational program such as the present one is needed. It should be noted that the present numerical calculations are valid at all operating points (shown, for example, as complete curves in Fig. 1), whereas a perturbation analysis determines only the mathematical optimum point and does not reveal, for example, how flat or sharp the curves are near the optimum point. Actual design decisions depend on such knowledge.

Table 2 shows that the combined influence of the 'n effect' and the 'f effect' on the optimum pressure ratio is significant, but that on the maximum cycle efficiency is modest. It would be improper to conclude from the small values of ϕ and φ that the inclusion of 'real' effects is unimportant. The present numerical calculations shown in Figs 1 and 4 reveal that there are considerable differences in actual performance curves (efficiency versus pressure ratio and efficiency versus temperature) between the air-standard analysis and the open-circuit analysis with all 'real' effects. The numerical analysis of open-circuit gas turbine plants with non-perfect gases shows the existence of an optimum turbine entry temperature. Figure 4 shows the individual and combined contribution of 'n + f effects' and dissociation towards the establishment of the optimum temperature. The optimum temperature increases with an increasing pressure ratio. There is no counterpart of the optimum turbine entry temperature in the air-standard analysis with constant specific heat [**2**].

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APPENDIX 1

Typical variations of n and n' determined by numerical simulation of gas turbine performance

Figure 5 shows typical variations in n and n' with pressure ratio, at two different temperature ratios. These are calculated by the present computer program employing equations (3) and (4) and (7) to (10) and the method of section 2. Numerical calculations, not shown here, reveal that n and n' also depend on $\eta_c, \eta_t, \sum \Delta p/p, T_1, p_1$ and the fuel.

It is of historical interest to refer to Whittle's comment mentioned in section 3.2 of using $\gamma = 1.4$ for air and $\gamma = 1.33$ for combustion gases. The gas tables (e.g. by Jamison and Mordell) were based on these assumptions and widely used by the industry in the 1950s and 1960s. Since $c_p = \gamma R/(\gamma - 1)$ and since $R_g \approx R_a$ with kerosene as a fuel [**3**], these assumptions directly lead to the value $n = 3.5/4 = 0.875$, which forms the basis of the choice in reference [**1**].

It can be seen from Fig. 5 that at $\theta = 4$ and low pressure ratios (~ 7), $n = 0.875$, the value used in reference [**1**], is a good choice. Similarly, the ratio $2n'/(1 + n)$ is not very far from unity, a condition on which the linear relations of reference [**1**] are based. For

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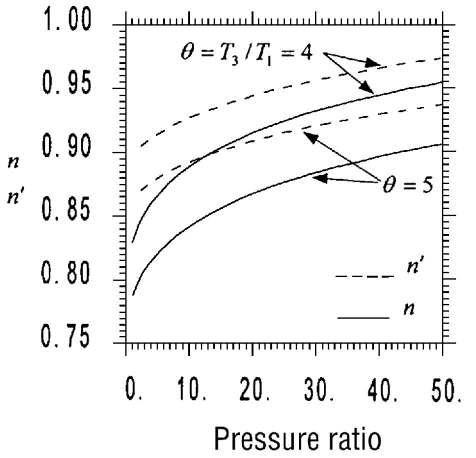

Fig. 5 Typical variations in n and n'. (For all calculations, $\eta_c = 0.8, \eta_t = 0.9, T_1 = 288\,\text{K}$, no pressure loss)

example, in the range of pressure ratio 12–15, Fig. 5 shows that representative values are $n = 0.9$ and $n' = 0.935$, for which $2n'/(1+n) = 0.9842$. However, Appendix 2 shows that this small departure from unity of the ratio has drastic consequences on φ_n (calculated by the linear perturbation analysis) and to a smaller extent on ϕ_n.

APPENDIX 2

Parametric analysis of the sensitivity of $\boldsymbol{\phi}_n$ and φ_n to parameters *n* and *n'*

The present computer program was used to determine the sensitivity of ϕ_n and φ_n to the two parameters n and n'. The analytical procedure would be of maximum use if the dependence were found to be weak. Table 5 gives a representative set of results. The following conclusions can be made:

1. Both ϕ_n and φ_n depend strongly on n and n'. Particularly, the dependence of φ_n is dramatic.
2. With approximate choices of n and n', there may be as high an error as 100 per cent in ϕ_n, but ϕ_n is always positive (for reasonable choices of n and n'); φ_n, on the other hand, changes sign.
3. If n is increased, keeping n' fixed, both ϕ_n and φ_n decrease.

Table 5 Sensitivity of ϕ_n and φ_n to parameters n and n'

n	$2n'/(1+n)$	n'	n'/n	ϕ_n [equation (12a)]	φ_n [equation (12b)]
0.87	0.97	0.9069	1.0425	0.0459	– 0.0397
0.87	0.98	0.9163	1.0532	0.0508	– 0.0197
0.87	0.99	0.9256	1.064	0.0557	– 0.0001
0.87	1	0.935	1.0747	0.0607	0.0192
0.87	1.01	0.9443	1.0855	0.0656	0.038
0.88	0.97	0.9118	1.0361	0.0408	– 0.0402
0.88	0.98	0.9212	1.0468	0.0456	– 0.0203
0.88	0.99	0.9306	1.0575	0.0505	– 0.0008
0.88	1	0.94	1.0682	0.0554	0.0183
0.88	1.01	0.9494	1.0789	0.0602	0.037
0.89	0.97	0.9166	1.0299	0.0358	– 0.0409
0.89	0.98	0.9261	1.0406	0.0406	– 0.0211
0.89	0.99	0.9355	1.0512	0.0454	– 0.0017
0.89	1	0.945	1.0618	0.0502	0.0173
0.89	1.01	0.9544	1.0724	0.055	0.0359
0.90	0.97	0.9215	1.0239	0.031	– 0.0417
0.90	0.98	0.931	1.0344	0.0357	– 0.022
0.90	0.99	0.9405	1.045	0.0404	– 0.0027
0.90	1	0.95	1.0556	0.0451	0.0162
0.90	1.01	0.9595	1.0661	0.0499	0.0348
0.91	0.97	0.9263	1.018	0.0262	– 0.0425
0.91	0.98	0.9359	1.0285	0.0309	– 0.0229
0.91	0.99	0.9454	1.039	0.0355	– 0.0038
0.91	1	0.955	1.0495	0.0402	0.0151
0.91	1.01	0.9645	1.0599	0.0448	0.0335
0.92	0.97	0.9312	1.0122	0.0216	– 0.0435
0.92	0.98	0.9408	1.0226	0.0262	– 0.024
0.92	0.99	0.9504	1.033	0.0308	– 0.0049
0.92	1	0.96	1.0435	0.0353	0.0138
0.92	1.01	0.9696	1.0539	0.0399	0.0321

4. If n' is increased, keeping n fixed, both ϕ_n and φ_n increase.
5. φ_n predominantly scales with $2n'/(1+n)$: $\varphi_n \approx f_1[2n'/(1+n)]$.
6. ϕ_n predominantly depends on n'/n: $\phi_n \approx f_2(n'/n)$.

APPENDIX 3

Optimization at a fixed pressure ratio versus optimization at a fixed turbine entry temperature

It is recalled that if the working medium is a perfect gas then, at fixed pressure ratios, there is no optimum turbine entry temperature (TET) and with an increasing TET the thermal efficiency asymptotically approaches a limiting value [**2**] $\lim_{T_3/T_1 \to \infty} \eta_A = \eta_t \eta_{Joule}$ [equation (19)]. The thermal efficiency continuously improves with an increasing temperature ratio (though the rate of improvement would diminish at higher temperatures). This would seem to suggest that as high a TET as possible should be used.

Obviously the temperature resistance of the material would set a practical limit to the TET. Use of cooling air allows the gas temperature to be higher than the melting point of the turbine blade material, but the use of cooling air has also a detrimental effect on the work output. At a fixed cooling technology, there is therefore a limit to what percentage of flow can be used for this purpose. Nevertheless, if the perfect blade material could be found, there is no limit to the TET set by thermodynamics if the working medium were a perfect gas. Thus the optimum TET discussed in this paper is set by the thermodynamics of *non-perfect* gases, and is separate from the well-known material restriction on the usable maximum temperature.

The solid lines in Fig. 6 show the performance taking into account the internal combustion and real gas effects, including dissociation. The dashed lines give the perfect gas limits given by equation (19). The following conclusions can be drawn:

1. Corresponding to each pressure ratio, there is an *optimum temperature* ratio. Any further increase in the turbine entry temperature reduces the thermal efficiency.
2. The limiting thermal efficiency given by equation (19) is never reached. At any pressure ratio, the maximum possible efficiency with real gas is 15–20 per cent lower than the maximum possible value in a perfect gas, as predicted by equation (19). The maximum possible efficiency increases with the increasing pressure ratio.
3. The optimum temperature ratio (for maximum efficiency) increases with the increasing pressure ratio. However, at higher pressure ratios, the curves of η_o versus T_3 become flatter. Thus, choosing a value of T_3 slightly lower than the optimum might not affect the thermal efficiency significantly.
4. The performance with the perfect gases can be described completely in terms of the non-dimensional parameter–temperature ratio (T_3/T_1). For real gases, the absolute value of T_3 is also important. The absolute level of pressure also becomes relevant for calculation of dissociation.

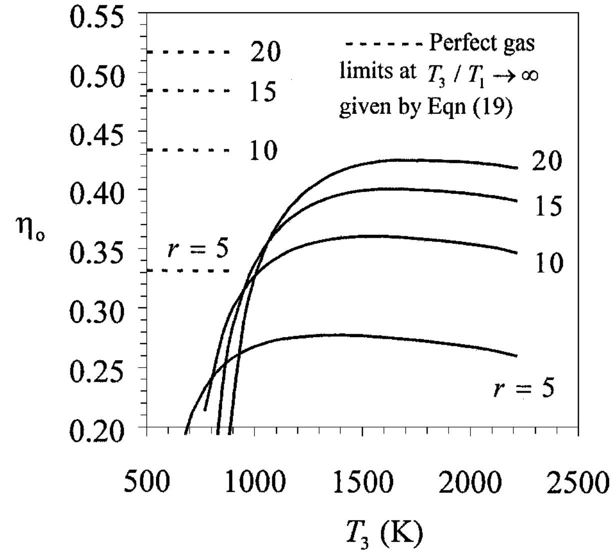

Fig. 6 Variation of thermal efficiency with the turbine entry temperature in a real gas turbine power plant including component losses, internal combustion and real gas properties. (Solid lines are calculated by GasTurb. $\eta_c = \eta_t = 0.9$, $T_1 = 288.15$ K, $p_1 = 1.013$ bar, $Q_{cv} = 43.12$ MJ/kg, $C_{14.3}H_{27.8}$ in air)

It is to be noted that, although the thermal efficiency would decrease if T_3 is any higher than its optimum value, the specific power output continues to increase as T_3 is increased at a fixed pressure ratio. The value of the optimum temperature would change slightly if the effects of blade cooling, pressure losses in the ducts and combustion chamber, etc., are taken into account.

An interesting situation arises from the existence of the optimum turbine entry temperature. The optimization can be performed in two different ways: at each value of the TET, the pressure ratio is chosen such that the thermal efficiency is maximized (method I) and at each value of the pressure ratio, the TET is chosen such that the thermal efficiency is maximized (method II). Figure 7a shows the optimum relations between the TET and the pressure ratio obtained by these two methods and Fig. 7b shows the corresponding thermal efficiencies. (The calculations shown in Fig. 7 are performed assuming no blade cooling, but similar qualitative variations would be obtained with other assumptions about the level of cooling technology and amount of coolant.) Almost all previous references (except references [**2**] and [**9**]) seem to have considered

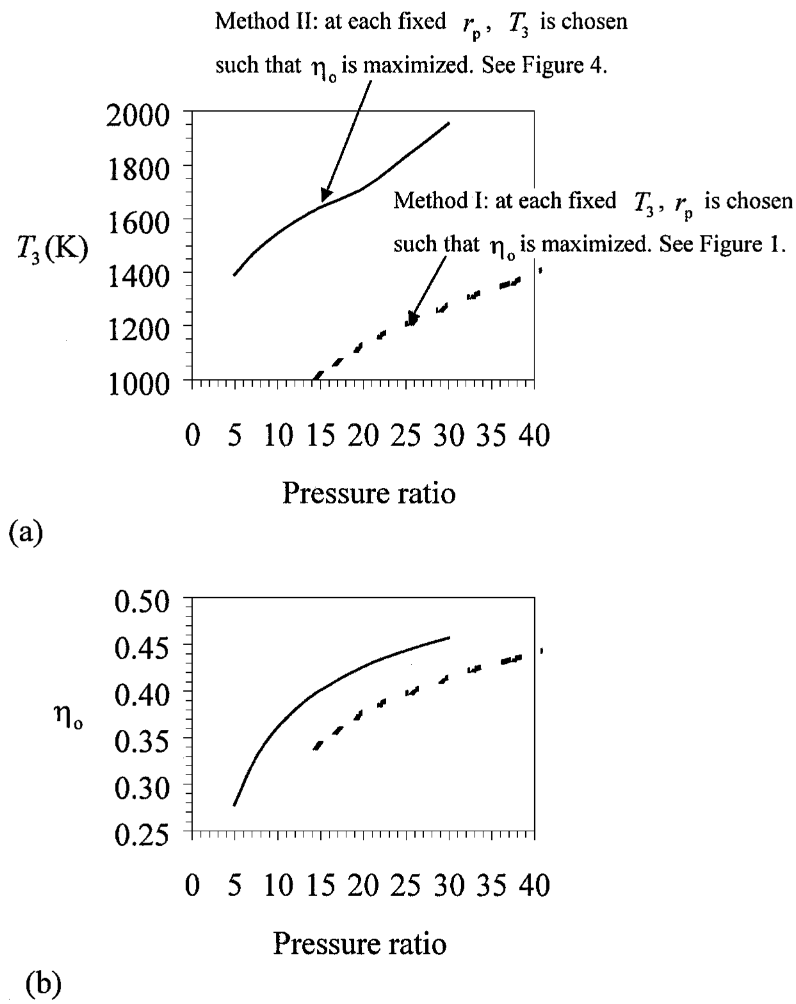

Fig. 7 Optimum relations between the pressure ratio and the turbine entry temperature, and corresponding thermal efficiencies obtained by two different numerical optimization methods, both performed by GasTurb ($\eta_c = \eta_t = 0.9$, $T_1 = 288.15\,K$, $p_1 = 1.013$ bar, $Q_{cv} = 43.12$ MJ/kg, $C_{14.3}H_{27.8}$ in air)

only method I. This approach only provides a partial picture. Figure 7a shows that the lower line obtained by method I does not have to be the preferred optimal relation between the pressure ratio and the TET that previous works seem to suggest. The only prohibition that can be specified with certainty is that, from considerations of thermal efficiency, the operating point should not lie below the lower line or above the upper line in Fig. 7a.

It is easy to see that if a value of pressure ratio r is optimum at a given temperature ratio r_T, then the same value of r_T is not the optimum temperature ratio if the pressure ratio was fixed at the same r. For example, with the assumed values of parameters as shown in Fig. 7, the optimum pressure ratio at $T_3 = 1150\,K$ is about 20 but the optimum T_3 at a pressure ratio of 20 is about 1700 K (the thermal efficiency is higher for the latter combination). The actual values adopted in a design would depend on whether the design is more limited by practically attainable values of the pressure ratio or that of the turbine entry temperature.