Technical Note

Performance and optimization of gas turbines with real gas effects

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Abstract: The influence of various levels of mathematical modelling on gas turbine performance is systematically analysed. It is shown that internal combustion with real gas effects gives rise to an optimum turbine entry temperature which does not arise in a perfect gas analysis and has not been described previously in the literature. At any pressure ratio, the maximum possible efficiency with real gas effects is significantly lower (15–20 per cent) than the maximum possible value predicted by a perfect gas analysis. An explicit equation has been derived for determining the optimum pressure ratio as a function of turbine entry temperature and component efficiencies. It is shown that the optimum design depends very strongly on turbine and compressor efficiencies. It is demonstrated that the optimum relation between pressure ratio and turbine entry temperature depends strongly on whether the optimization is carried out at fixed pressure ratios or at fixed temperatures. All previous references seem to have considered only the latter method.

Keywords: gas turbine, internal combustion engine, aero engine, shaft power, optimization, optimum pressure ratio, optimum temperature, thermal efficiency

NOTATION

p_1	absolute pressure at inlet
Q_{cv}	lower calorific value of fuel
r_p	pressure ratio
R	$= r_p^{(\gamma-1)/\gamma}$
T_1	temperature at inlet
T_3	turbine entry temperature (TET)
γ	isentropic index of working fluid
η_c	isentropic efficiency of compressor
η_{Joule}	efficiency of the Joule cycle
η_t	isentropic efficiency of turbine
η_{th}	thermal efficiency

1 INTRODUCTION

An analysis for the performance of gas turbines is presented. The description begins with a very short introduction to the ideal Joule cycle. Several assumptions of the Joule cycle are then systematically removed leading to the final numerical solution for real engines with real components having losses with internal combustion of fuel and real gas properties including variable specific heat and dissociation. This route is taken to identify clearly the origin of various effects.

The MS was received on 12 October 2000 and was accepted for publication on 3 April 2001.

2 IDEAL JOULE CYCLE (IDEAL COMPONENTS, EXTERNAL HEAT TRANSFER, PERFECT GAS)

The analysis of the ideal Joule cycle is standard and can be found in many textbooks [**1**–**6**]. The cycle consists of two isentropic and two constant-pressure processes. The working fluid is a perfect gas. The compressor and the turbine are to be assumed ideal, i.e. reversible. If the pressure ratio is denoted by r_p, then the thermal efficiency of the cycle is given by

$$\eta_{Joule} = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}} \qquad (1)$$

Equation (1) shows that the thermal efficiency monotonically increases with pressure ratio and depends only on the pressure ratio. The maximum temperature in the cycle has no effect on the thermal efficiency. A higher turbine entry temperature, however, increases the specific power output and thus a smaller machine can deliver the required power.

3 TURBINES AND COMPRESSORS WITH LOSSES (EXTERNAL HEAT TRANSFER, PERFECT GAS)

The mathematical model can be made more realistic by introducing the isentropic efficiencies of the compressor and the turbine which are less than unity as dictated by the second law of thermodynamics ($\eta_c < 1$, $\eta_t < 1$). The working fluid is still assumed to be a perfect gas with constant specific heat capacities. The heat addition and rejection are still external. This analysis is also standard, and the thermal efficiency (η_{th}) of this cycle can be shown to be equal to

$$\eta_{th} = \frac{\dfrac{T_3}{T_1}\left(1 - \dfrac{1}{R}\right)\eta_t - \dfrac{R-1}{\eta_c}}{\dfrac{T_3}{T_1} - \dfrac{R-1}{\eta_c} - 1} \tag{2}$$

where

$$R = r_p^{(\gamma-1)/\gamma} \tag{3}$$

and T_1 is the temperature at the inlet and T_3 is the turbine entry temperature (TET).

3.1 Variation of η_{th} with r_p at fixed T_3/T_1

Equation (2) has been plotted in Fig. 1 in which η_{th} is shown as a function of r_p with the temperature ratio, T_3/T_1, as a parameter.

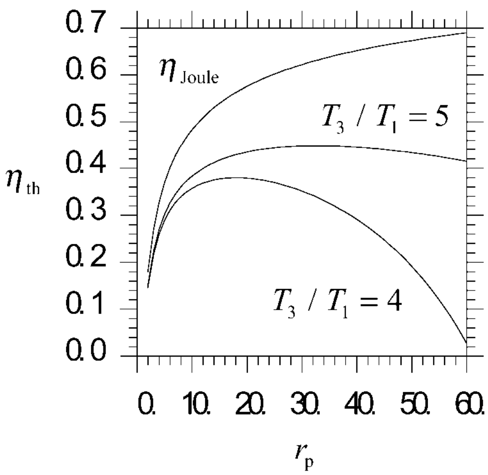

Fig. 1 Variation of thermal efficiency with pressure ratio at two temperature ratios ($\eta_c = \eta_t = 0.9$; perfect gas, $\gamma = 1.4$)

From Fig. 1 and similar calculations, the following conclusions can be drawn:

1. Component inefficiencies have a disproportionate effect on the thermal efficiency of the gas turbine cycle. A small amount of loss in the compressor and the turbine can reduce the engine efficiency substantially. This is because the gas turbine cycle has a poor work ratio, i.e. a large fraction of the turbine power is expended to drive the compressor. Thus, because of irreversibilities, as the compressor input increases and the turbine output decreases, the thermal efficiency of the engine is affected seriously.
2. In contrast to the Joule cycle, the thermal efficiency no longer increases monotonically with pressure ratio. Corresponding to a particular value of temperature ratio T_3/T_1, there is an optimum pressure ratio beyond which any further increase in pressure ratio decreases the thermal efficiency. The magnitude of the optimum pressure ratio depends also on the isentropic efficiencies of the compressor and the turbine.
3. The higher the temperature ratio T_3/T_1, the higher is the optimum pressure ratio.
4. Similarly to the efficiency, if the specific power output was plotted as a function of pressure ratio then also another optimum pressure ratio would result. The optimum pressure ratio for maximum specific power is lower than the optimum pressure ratio for maximum efficiency.

The magnitude of the optimum pressure ratio for maximum efficiency can be obtained by differentiating equation (2) with respect to R and setting $\partial\eta_{th}/\partial R = 0$. The principle is simple and has been suggested by Haywood (reference [**2**], p. 40), but it appears that the actual mathematical operation has not been carried out previously. The result is

$$r_{p,\text{optimum}}^{(\gamma-1)/\gamma} = \frac{-B + \sqrt{B^2 + 4AC}}{2A} \tag{4}$$

where

$$A = \frac{1}{\eta_c} - \frac{T_1}{T_3}\frac{1}{\eta_c} - \frac{\eta_t}{\eta_c}$$

$$B = 2\frac{\eta_t}{\eta_c}$$

$$C = \frac{T_3}{T_1}\eta_t - \eta_t + \frac{\eta_t}{\eta_c}$$

Equation (4) has been plotted in Fig. 2. It is found that the value of optimum pressure ratio depends strongly on the efficiencies of the components. Equation (4) has no

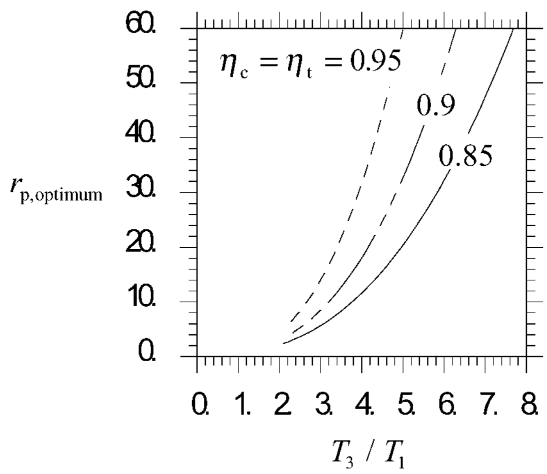

Fig. 2 Optimum pressure ratio as a function of temperature ratio: prediction of equation (4) (perfect gas, $\gamma = 1.4$)

physical solution when the turbine and compressor are fully reversible ($\eta_c = \eta_t = 1$); this is consistent with the conclusions made previously for the Joule cycle.

3.2 Variation of η_{th} with T_3/T_1 at fixed r_p

In an analogous manner to the derivation of equation (4), if equation (2) is differentiated with respect to T_3/T_1 and the result is set to zero [i.e. $\partial\eta_{th}/\partial(T_3/T_1) = 0$], no physically valid solution results. This shows that, at any fixed r_p, the thermal efficiency monotonically increases with increasing T_3/T_1 and does not show any maxima. It is, however, interesting to examine the behaviour of equation (2) as the temperature ratio becomes extremely large:

$$\lim_{T_3/T_1 \to \infty} (\eta_{th}) = \left(1 - \frac{1}{R}\right)\eta_t = \eta_t \eta_{Joule} \tag{5}$$

Equation (5) is plotted in Fig. 3 and shows that, in the limit $T_3/T_1 \to \infty$, the efficiency of the cycle with component losses is still less than the Joule cycle by a factor η_t—the turbine isentropic efficiency; the compressor isentropic efficiency η_c has, however, no effect on this limiting value.

The above discussion shows that, at a fixed temperature ratio, there is an optimum pressure ratio, but at a fixed pressure ratio there is no optimum temperature ratio as long as the working medium is a perfect gas. The thermal efficiency continuously improves with increasing temperature ratio (although the rate of improvement would diminish at higher temperatures). This would seem to suggest that as high a TET as possible should be used.

Obviously the temperature resistance of the material would set a practical limit to the TET. Use of cooling air allows the gas temperature to be higher than the melting point of the turbine blade material, but the use of cooling air also has a detrimental effect on the work output. At a fixed cooling technology, there is therefore a limit to what percentage of flow can be used for this purpose. Nevertheless, if the perfect material could be found, there is no limit to the TET set by thermodynamics.

4 REAL ENGINES (COMPONENTS WITH LOSSES, INTERNAL COMBUSTION WITH REAL GAS EFFECTS)

Now two more assumptions of the Joule cycle are relaxed: the external heat addition is replaced with internal combustion and the gases are not perfect. The engine ceases to work in a closed cycle. The fuel addition alters the mass flow slightly and the composition of combustion products is different from that of pure air. Moreover, the specific heat capacity of air itself changes significantly as there is a very large variation in temperature within a gas turbine power plant. (The specific heat at constant pressure of air increases by 25 per cent as the temperature changes from 300 to 2100 K. In addition to the temperature, the specific heat of combustion products depends on the fuel and fuel–air ratio [**7**].) Chemical dissociation might also come into play. An analytical formulation of the gas turbine cycle incorporating real gas effects might be possible by using the simple equations for the calculation of thermodynamic properties given by Guha [**7**]. However, a commercial computer program, GasTurb [**8**], has been used here for the prediction of performance of a real gas turbine.

It is found (Fig. 4) that no new physical principle is involved in the study of a real engine with varying pressure ratio as compared with what has been described

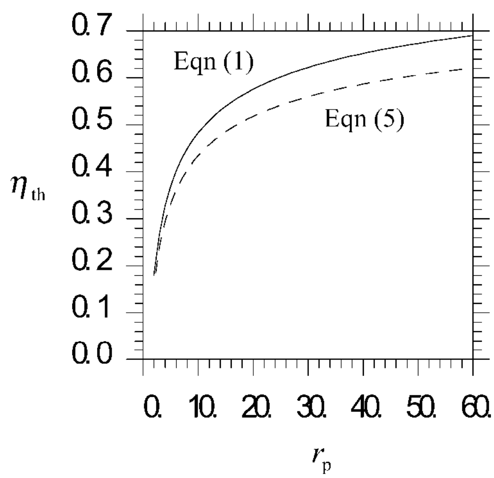

Fig. 3 Variation of thermal efficiency of a gas turbine with component losses as $T_3/T_1 \to \infty$ and its comparison with ideal Joule cycle efficiency (perfect gas, $\gamma = 1.4$, $\eta_t = 0.9$)

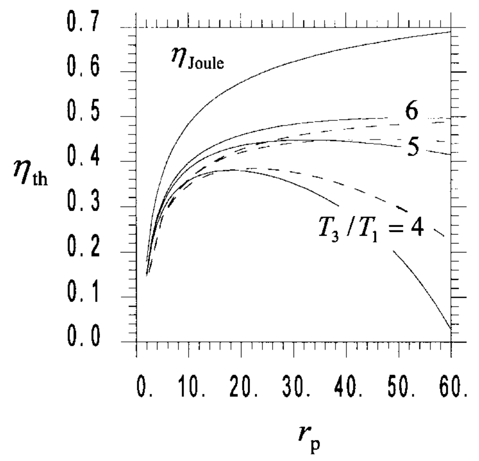

Fig. 4 Thermal efficiency versus pressure ratio: comparison of the perfect gas model, equation (2), and numerical calculations with real gas and internal combustion at $T_3/T_1 = 4, 5, 6$ (—, perfect gas with $\gamma = 1.4$; – – –, real gas with internal combustion, i.e. standard fuel in air, $T_1 = 288.15$ K, $p_1 = 1.013$ bar, $Q_{cv} = 43.12$ MJ/kg; both models use the same compressor and turbine efficiency, $\eta_c = \eta_t = 0.9$)

in Section 3. The optimal conditions occur at higher pressure ratios. The thermal efficiency is slightly different from what is predicted by perfect gas analysis, the difference increasing at higher temperature ratios. The broad physical picture conveyed by Figs 1 and 2, however, still remains valid. The effects of varying temperature ratio, at fixed pressure ratios, on the other hand, produce new phenomena.

The solid lines in Fig. 5 show the performance taking internal combustion and real gas effects, including dissociation, into account, while the dashed lines give the perfect gas limits at $T_3/T_1 \rightarrow \infty$. The following conclusions can be drawn:

1. Corresponding to each pressure ratio, there is an optimum temperature ratio. Any further increase in TET reduces the thermal efficiency. This limit is set by the thermodynamics of real gases; this has nothing to do with material limitations. It is observed in Section 3 that thermodynamics posed no such limit for perfect gases for which the thermal efficiency continues to rise with increasing temperature ratio. The occurrence of an optimum temperature ratio due to real gas thermodynamics has not previously been reported in the literature. (In order that the effects of real gases can be isolated, no additional losses have been assumed in the numerical calculations other than those in the compressor and turbine.)
2. The limiting thermal efficiency given by equation (5) is never reached. At any pressure ratio, the maximum possible efficiency with real gas is 15–20 per cent lower than the maximum possible value in a perfect gas as predicted by equation (5). The maximum possible efficiency increases with increasing pressure ratio.
3. The optimum temperature ratio (for maximum efficiency) increases with increasing pressure ratio. However, at higher pressure ratios, the curves of η_{th} versus T_3 become flatter. Thus choosing a value of T_3 slightly lower than the optimum might not affect the thermal efficiency significantly.
4. The performance with perfect gases, as discussed in Section 3, could be described completely in terms of the non-dimensional parameter—temperature ratio T_3/T_1. For real gases, the absolute value of T_3 is also important. The absolute level of pressure also becomes relevant for the calculation of dissociation.

It is to be noted that, although the thermal efficiency would decrease if T_3 is any higher than its optimum value, the specific power output continues to increase as T_3 is increased at a fixed pressure ratio. The value of optimum temperature would change slightly if the effects of blade cooling, pressure losses in the ducts and combustion chamber, etc. are taken into account.

5 CONCLUSION

The paper systematically analyses the effects of various levels of modelling on the predicted performance of a gas turbine power plant producing shaft power. An explicit relation, equation (4), has been derived that specifies the

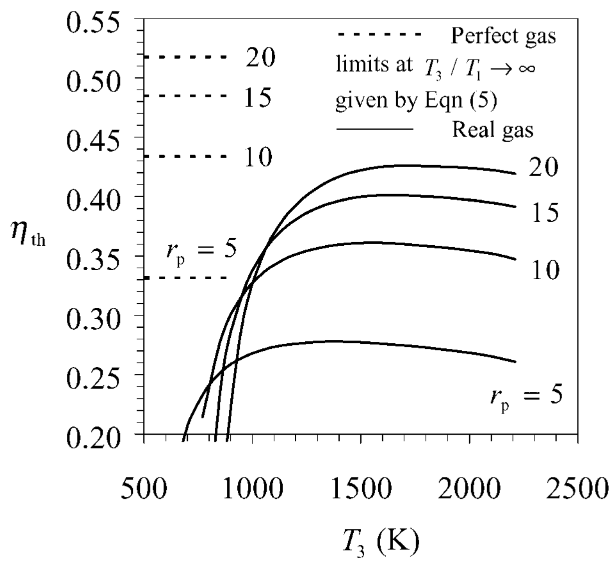

Fig. 5 Variation of thermal efficiency with TET in a real gas turbine power plant including component losses, internal combustion and real gas properties ($\eta_c = \eta_t = 0.9$; $T_1 = 288.15$ K; $p_1 = 1.013$ bar; $Q_{cv} = 43.12$ MJ/kg, standard fuel in air)

optimum pressure ratio as a function of temperature ratio. It is shown in Fig. 2 that the optimum pressure ratio depends very strongly on turbine and compressor efficiencies. For example, at $T_3/T_1 = 5$, improving the isentropic efficiencies from 0.85 to 0.95 increases the optimum pressure ratio by a factor of 3. Therefore, as the efficiencies improve, the optimum design of an engine would change considerably.

Numerical calculations have been presented for the performance of a real gas turbine with internal combustion of fuel and with real gas properties including dissociation. An optimum TET is discovered from these calculations (Fig. 5). [It is recalled from Section 3.2 that if the working medium is a perfect gas then, at fixed pressure ratios, there is no optimum TET and with increasing TET the thermal efficiency asymptotically approaches the limiting value given by equation (5).] This optimum TET, established for the first time in the present paper, is set by the thermodynamics of real gases and is separate from the well-known material restriction on the useable maximum temperature.

As a result, the usual maxim 'higher the TET, better is the performance' is found to be not necessarily true. This has important implications for the optimization of jet engines [**9**, **10**]. (Reference [**9**] describes a new methodology for the optimization of bypass engines in which the design specific thrust is determined from a direct operating cost analysis and the optimum values of overall pressure ratio, turbine entry temperature, bypass ratio and fan pressure ratio are determined concurrently that minimize specific fuel consumption at the fixed design specific thrust.)

The optimum TET increases with increasing pressure ratio. Figure 5 shows that, at any fixed pressure ratio, the

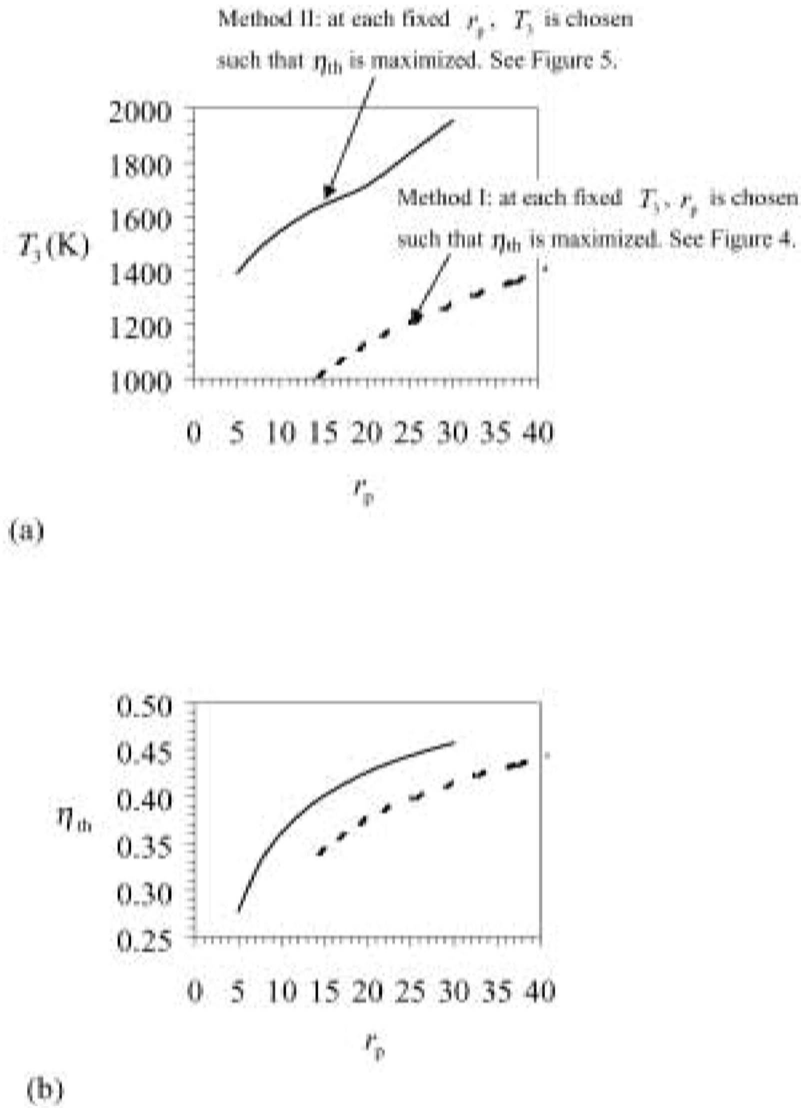

Fig. 6 Optimum relations between pressure ratio and TET, and corresponding thermal efficiencies obtained by two different numerical optimization methods both performed by GasTurb (data correspond to Figs 4 and 5) ($\eta_c = \eta_t = 0.9$; $T_1 = 288.15$ K; $p_1 = 1.013$ bar; $Q_{cv} = 43.12$ MJ/kg, standard fuel in air)

maximum possible efficiency with real gas is 15–20 per cent lower than the maximum possible value in a perfect gas as predicted by equation (5). The maximum possible efficiency increases with increasing pressure ratio.

An interesting situation arises from the existence of optimum temperature ratio, which may be understood by studying Figs 4 and 5 together. The optimization can be performed in two different ways: at each value of TET, the pressure ratio is chosen such that the thermal efficiency is maximized (method I), and at each value of pressure ratio, the TET is chosen such that the thermal efficiency is maximized (method II). Figure 6a shows the optimum relations between TET and pressure ratio obtained by these two methods and Fig. 6b shows the corresponding thermal efficiencies. (The calculations shown in Fig. 6 are performed assuming no blade cooling, but similar qualitative variations would be obtained with other assumptions about the level of cooling technology and amount of coolant.) All previous references seem to have considered only method I. Even the suggested design combinations of pressure ratio and TET for the core part of turbofan engines for aircraft propulsion follow method I (e.g. Fig. 21 of reference [**11**] and Fig. 4 of reference [**12**]). This approach only provides a partial picture. Figure 6a shows that the lower line obtained by method I need not be the preferred optimal relation between pressure ratio and TET which previous works seem to suggest. The only prohibition that can be specified with certainty is that, from considerations of thermal efficiency, the operating point should not lie below the lower line or above the upper line in Fig. 6a.

It is easily seen that if a value of r_p is optimum at a given temperature ratio r_T, then the same value of r_T is not the optimum temperature ratio if the pressure ratio was fixed at the same r_p. For example, with the assumed values of parameters as shown in Fig. 6, the optimum pressure ratio at $T_3 = 1150$ K is about 20 but the optimum T_3 at a pressure ratio of 20 is about 1700 K (the thermal efficiency is higher for the latter combination). The actual values adopted in a design would depend on whether the design is more limited by practically attainable values of pressure ratio or that of the TET.

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